'93 KACC (1993.10.20~22)

Robust Model Matching Design using Normalized Left Coprime Factorization approach

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Abstract

In this paper, we propose a new design procedure of the Robust Model Matching (RMM) using the Normalized Left Coprime Factorization (NLCF) approach. The RMM aims at reducing the sensitivity of a given control system, but standard design procedures are not for robust stability. Therefore we try applying the robust stability condition based on NLCF to RMM procedure. We first formulate the RMM using the robust stability condition of NLCF approach, then we propose the new procedure of the RMM. The point is that the condition includes the measure of sensitivity of the RMM. In the proposed procedure, a cost function is determined through the condition and solved by H_∞control technique. Finally we show a design example and check the performance.

1. Introduction

The importance of the robust control is well-known. That is why we can not describe a real plant by formulas completely, so it is difficult for the controller to have the same performances for the real plant as it does for the formulas plant. The H_∞control, which evaluates performances of control systems by H_∞norm, has been applied to robust control, one of the reasons is that robust stability conditions are derived by using H_∞norm. Nowadays several solutions of the H_∞control have been developed and solving H_∞control problems has been getting easier [1] [2] [3].

On the other hand, the Robust Model Matching (RMM) has been proposed [4]. The RMM aims at reducing the sensitivity of a given control system by appending a compensator, so-called robust compensator, without changing reference responses. The effect of the RMM has been confirmed by laboratory experiments and industrial applications [5] [6] [7]. As for the robust stability of the RMM, Zhong *et al.* treated the problem in the case of minimum phase SISO plants and the authors showed heuristic design procedures of the robust compensator for robust stability [8] [9]. However, the robust stabilization of the RMM for unstructured uncertainties has not been considered.

In this paper, we propose a new design procedure of the robust compensator using the robust stability condition of the NLCF approach. We first formulate the RMM method using NLCF approach and propose the new procedure of the RMM. The point is that the robust stability condition includes the measure of the sensitivity of the RMM as well. Therefore in designing a robust compensator it is possible to evaluate both the robust stability and the sensitivity simultaneously. To do that, we determine a cost function through the condition and solve it via H_∞control techniques [1]. The proposed procedure is a kind of H_∞control from the point of view of the RMM framework. In the last part of this paper, we show a design example and check the performance of the control system.

2. Preliminaries

2.1 RMM

First of all, we review the RMM.

The RMM attaches an additional compensator to an existing control system, which is called the robust compensator. One of the characteristics of the robust compensator is that modes of the robust compensator have no connection with the reference responses if the plant is nominal. This mean that the robust compensator does not change feedforward property, but changes feedback property. Therefore we can design the robust compensator independently from the reference responses. Note that, for convenience, we do not describe the reference inputs in the following explanation, even if controllers have reference inputs.

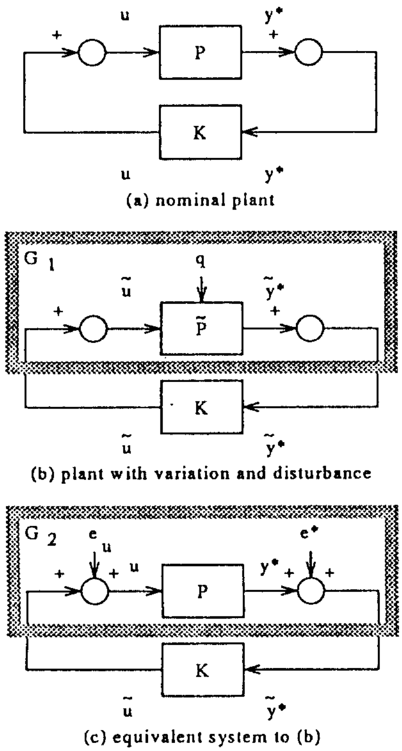

Fig. 1: The equivalent disturbances

The mechanism which the RMM reduces the sensitivities with is concerned with equivalent disturbances. Fig.1-(a) represents a control system, where P denotes a nominal plant, K denotes a controller, the vector $u \in R^m$ denotes the controller output and the vector $y^* \in R^l$ denotes the plant output. In Fig.1-(b), the plant has changed from P to $\tilde{P}$ and has put a disturbance q into $\tilde{P}$, and also the vector u and y^* has changed to $\tilde{u}$ and $\tilde{y}^*$, respectively. In Fig.1-(c), we introduce external inputs e_u and e^*. Note that we use P instead of $\tilde{P}$ and that the box G_2 in Fig.1-(c) is equal to the box G_1 in Fig.1-(b), when we determine e_u and e^* as follows,

$$e_u = u - \tilde{u} \tag{1}$$

$$e^* = \tilde{y}^* - y^*. \tag{2}$$

We call them the general equivalent disturbance. If we can reject them from the system in Fig.1-(c), it must have enough low sensitivity. That is the concept of the RMM.

In the standard procedure of the RMM, the controller K is constructed as Fig.2. Here C denotes an existing controller and *Rcomp* denotes a robust compensator designed in the RMM procedure.

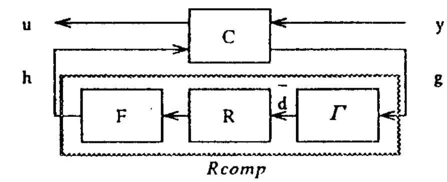

Fig. 2: Controller K via the standard construction of the RMM

The following list shows the standard design procedure of the robust compensator.

1. Calculate $P_{\bar{d}y^*}$, which is a transfer function matrix of P from $\bar{d}$ to y^*, as follows (in this paper we write a transfer matrix like this).

   Assume $\bar{M}$ and $\bar{N}$ are coprime polynomials such that

   $$P_{uy^*} = \bar{M}^{-1}\bar{N}. \tag{3}$$

   Then the $[P_{e_u y^*}, P_{e^* y^*}]$ has the factorization

   $$[P_{e_u y^*}, P_{e^* y^*}] = \bar{M}^{-1}[\bar{N}, \bar{M}]. \tag{4}$$

   Here $[\bar{N}, \bar{M}]$ is supposed to be row proper. The vector $\bar{d} \in R^l$ is defined as the external input of $\bar{M}^{-1}$ and called fundamental equivalent disturbance, which is illustrated in Fig.3.

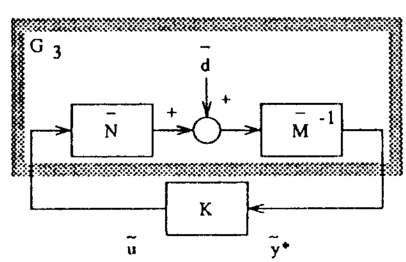

Fig. 3: The fundamental equivalent disturbance

The box G_3 in Fig.3 is equal to the box G_2 in Fig.1-(c), if the following equation is satisfied:

$$\bar{d} = [\bar{N}, \bar{M}] \begin{bmatrix} e \\ e^* \end{bmatrix} \quad (5)$$

Needless to say, $P_{\bar{d}y^*} = \bar{M}^{-1}$.

2. Calculate Γ which estimates the $\bar{d}$ from $g \equiv [\tilde{u}^T, \tilde{y}^{*T}]^T$, as follows.

   By Fig.1-(c), (4), (5) we have

$$\begin{aligned} \tilde{y}^* &= P_{uy^*}\tilde{u} + P_{uy^*}c_u + e^* \quad (6) \\ &= P_{uy^*}\tilde{u} + \bar{M}^{-1}\bar{d}. \quad (7) \end{aligned}$$

   Hence $\bar{d}$ is estimated by

$$\bar{d} = [-\bar{N}, \bar{M}]g. \quad (8)$$

   Then Γ is represented as

$$\Gamma = [-\bar{N}, \bar{M}]. \quad (9)$$

3. Minimize $\|T_{\bar{d}y} + T_{hy}R\|$ for R.

   Here, $\|\cdot\|$ means appropriate norm; T denotes the control system consisting P and C; h of C is selected by similar procedure to $\bar{d}$ of P; y is the controlled values which consists of the first mth elements of y^*.

4. Construct the robust compensator *Rcomp* from F, R, Γ.

   The transfer function matrix F has low pass filter property and is determined such that $C_{hu} \cdot F \cdot R \cdot \Gamma$ is proper.

2.2 Robust stability on the NLCF approach

The normalized left coprime factorization of P_{uy^*} is

$$P_{uy^*} = M^{-1}N \quad (10)$$

where N and M are coprime and stable proper transfer function matrices, and satisfy

$$N N^* + M M^* = I \quad {}^\forall s \in jR. \quad (11)$$

The factorization $[N, M]$ is unique to within left multiplication by a unitary matrix.

The perturbation class associated with NLCF uncertainty is defined as

$$D_{C_\epsilon} \equiv \{\Delta = [\Delta_N, \Delta_M]; \Delta \in RH_\infty^{p\times(m+p)}; \|\Delta\|_\infty < \epsilon\}. (12)$$

The perturbed plant is denoted by

$$\tilde{P}_{uy^*} = (M + \Delta_M)^{-1}(N + \Delta_N) \quad (13)$$

and illustlated in Fig.4.

The controller K stabilizes $\tilde{P}_{uy^*}$ for all $\Delta \in D_{C_\epsilon}$, if and only if

(a) K stabilize P_{uy^*}

(b) $$\left\| \begin{bmatrix} K(I - P_{uy^*}K)^{-1}M^{-1} \\ (I - P_{uy^*}K)^{-1}M^{-1} \end{bmatrix} \right\|_\infty \leq \epsilon^{-1}. \quad (14)$$

The largest number of ϵ $(= \epsilon_{max})$ is given by

$$\epsilon_{max} = (1 - \|[N, M]\|_H)^{1/2} \quad (15)$$

which is called the maximum stability margin. Here $\|\cdot\|_H$ denotes the Hankel norm.

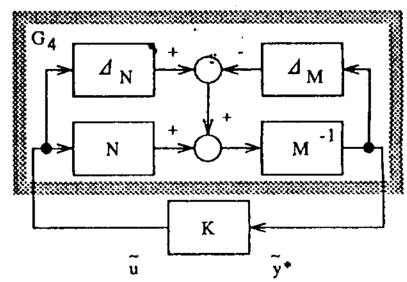

Fig. 4: The NLCF perturbation

3. RMM Using NLCF Approach

The block diagram in the box G_4 in Fig.4 is interpreted such that the signal associated with the NLCF uncertainty is put into the nominal plant. We notice that the interpretation is analogous to the idea of the equivalent disturbance $\bar{d}$ in the box G_3 in Fig.3. From the point of view of that notion, we propose another equivalent disturbance and reformulate the RMM to apply the NLCF approach. Then we propose a new procedure of the RMM and clarify the class of the controller via this procedure.

3.1 Re-formulation about the RMM

We commence to define another equivalent disturbance d from Fig.4 as

$$d \equiv [\Delta_N, -\Delta_M] \begin{bmatrix} \tilde{u} \\ \tilde{y}^* \end{bmatrix}. \quad (16)$$

The relation between d and $\bar{d}$ is concerned with the relation between $[N, M]$ and $[\bar{N}, \bar{M}]$, because

$$\tilde{y}^* = P_{uy^*}\tilde{u} + \bar{M}^{-1}\bar{d} \quad (17)$$
$$= P_{uy^*}\tilde{u} + M^{-1}d \quad (18)$$

hence

$$\bar{M}^{-1}\,\bar{d} = M^{-1}\,d. \quad (19)$$

The left coprime factors of a polynomial matrix can be transformed to the left coprime factors of a stable proper rational matrix by left multiplication of a stable proper rational diagonal matrix. The left coprime factors of the stable proper rational matrix can be normalized by left multiplication of a unimodular matrix. Therefore there exists such a matrix H of the left multiplication as

$$[N, M] = H\,[\bar{N}, \bar{M}]. \quad (20)$$

By (19),(20),

$$d = H\bar{d}. \quad (21)$$

Similarly to (8), the following equation is derived:

$$d = [-N,\, M]g \qquad \text{by (18)} \quad (22)$$
$$= H[-\bar{N},\, \bar{M}]g \qquad \text{by (20)} \quad (23)$$
$$= H\Gamma g \qquad \text{by (9)}\,. \quad (24)$$

Therefore d is estimated by putting H in the standard *Rcomp*.

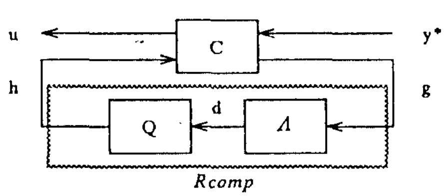

Fig. 5: Controller K via new construction of the RMM

Now, we re-construct the *Rcomp* by using d in Fig.5. Here, Λ is equal to $H \cdot \Gamma$ and Q is a stable proper rational function matrix. We don't need F in this construction because Λ and Q are proper. We choose Q instead of R to achieve the RMM design.

3.2 Cost function

The objective of this paper is to apply the robust stability condition (14) to the RMM. Note that since the signal d is input of M^{-1}, the transfer function matrix in (14) is of the whole control system from d to g. Though the transfer function matrix of a whole control system from a equivalent disturbance vector to a controlled value vector is the measure of the sensitivity on the RMM, it is included by the matrix in (14). We consider that by taking the matrix as the cost function of the RMM, it is possible to evaluate both the robust stability and low sensitivity simultaneously.

Suppose that T denotes the control system consisting of P and C, that is to say, in the case K consists of C. The cost function for the RMM is denoted as

$$G = \| W\,[T_{dg} + T_{hg}Q] \|_\infty \quad (25)$$

where

$$W = \begin{pmatrix} I_m & 0 & 0 \\ 0 & w & 0 \\ 0 & 0 & I_{l-m} \end{pmatrix} \quad (26)$$

w : stable proper-rational matrix

Now, $T_{dg} + T_{hg}Q$ denotes the transfer function matrix from d to g of the whole control system consisting of P, C and *Rcomp*, that is to say, in the case K consists of C, Λ and Q. The matrix W weights y in g.

Becausse both T_{dg} and T_{hg} are stable, (25) has the same formula as the model matching problem in [1]. Suppose α and γ is as follows:

$$\alpha = \inf G \quad (27)$$
$$\gamma < \alpha. \quad (28)$$

From the solutions of the model matching problem, we can obtain Q satisfying

$$\| W\,[T_{dg} + T_{hg}Q] \|_\infty \le \gamma. \quad (29)$$

3.3 The class of controllers

The Plant P_{uy^*} has a right coprime factorization

$$P_{uy^*} = \hat{N}\hat{M}^{-1} \quad (30)$$

where $\hat{N} \in R^{l\times m}$ and $\hat{M} \in R^{m\times m}$ are proper and stable. Then there exist X and Y satisfying

$$X\hat{N} + Y\hat{M} = I \quad (31)$$

where $X \in R^{m\times l}$ and $Y \in R^{m\times m}$ are proper and stable.

An existing controller C in Fig.5 is parametrized by the formula

$$C_{y^*u} = -(Y - \hat{Q}N)^{-1}(X + \hat{Q}M). \tag{32}$$

Here the parameter $\hat{Q}$ is proper and stable, and (32) is a left coprime factorization as well. Suppose h is the input vector of $(Y - \hat{Q}N)^{-1}$, the whole controller K consisting C, Λ and Q is represented by the same formula as (32):

$$K_{y^*u} = -(Y-(\hat{Q}-Q)N)^{-1}(X+(\hat{Q}-Q)M). \tag{33}$$

Therefore we conclude that the class of all the controllers K via the RMM in Fig.5 is equal to the set of *all* proper real-rational's stabilizing P_{uy^*}.

Note that the formula (33) implies that the order of K in the case of realizing C and *Rcomp* separately is higher than in the case of realizing them simultaneously by the order of Λ.

3.4 Summary of the procedure

We summarize the proposed procedure of the RMM.

Step 1 Calculate NLCF $[N, M]$ of P_{uy^*}.

Step 2 Set Λ as $[-N, M]$.

Step 3 Obtain Q satisfying (29).

Step 4 Construct *Rcomp* separately from C, or construct *Rcomp* and C simultaneously like (33).

4. Example

We show a design example using the proposed procedure.

We use the plant in [10]:

$$P_{uy^*} = \frac{\begin{pmatrix} s(s+1)^2 & s(s-1)^2 \\ (s+1) & (s+1)(s-1) \end{pmatrix}}{s(s+1)(s-1)} \tag{34}$$

where $y = y^*$, that is, we select the controlled value as all the measured values.

Suppose r represents a reference input vector. The existing controller is as follows:

$$C_{ru} = \frac{\begin{pmatrix} 2(s-13) & 2(-s^2+2s+15) \\ -2(3s+7) & 2(s+3)^2 \end{pmatrix}}{(s^2+s+2)} \tag{35}$$

$$C_{y^*u} = \frac{\begin{pmatrix} -s^2+24s+25 & 2(s^2-10s-15) \\ 6s^2+19s+13 & -2(3s^2+10s+9) \end{pmatrix}}{s^2+s+2}. \tag{36}$$

Then we have the transfer function matrix T_{ry} consisting of P and C:

$$T_{ry} = \begin{pmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{2}{s+2} \end{pmatrix} \tag{37}$$

and the characteristic polynomial $D(s)$ of T:

$$D(s) = (s+1)(s+2)(s+3)(s^2+s+2). \tag{38}$$

In the following, we get the controller K in Fig.5 via the above RMM procedure.

In "Step 1", we get

$$N = \frac{\begin{pmatrix} 0.577(s+1.98)(s^2+1.44s+1.54) & 0.577(s-1.75)(s^2+1.17s+0.738) \\ -0.146(s+1.70)(s-1.35) & 0.854(s^2+1.51s+1.05) \end{pmatrix}}{(s+1.84)(s^2+1.34s+0.857)} \tag{39}$$

$$M = \frac{\begin{pmatrix} 0.577(s+1.17)(s+1)(s-0.747) & -0.0138s(s+90.5) \\ -0.146(s+4.21)(s+1) & s(s+1.58)(s+0.178) \end{pmatrix}}{(s+1.835)(s^2+1.340s+0.8570)}. \tag{40}$$

In "Step 2", we set Λ by (39), (40) immediately.

In "Step 3", we get the *sub-optimal* Qs. Here we set the weighting matrix W as

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & w & 0 \\ 0 & 0 & 0 & w \end{pmatrix}. \tag{41}$$

Here for convenient, suppose w is a positive number. We take several numbers as w and obtain Qs for each w and appropriate γ respectively.

In "Step 4", we construct K such that we realize *Rcomp*, add it to C, and reduce the order of K by the order of Λ.

We show some results in Table 1, where T denotes the control system consisting of P and C; ${}_RT$ denotes the control system consisting of P, C and *Rcomp*; $\|T_{dg}\|_\infty$ and $\|{}_RT_{dg}\|_\infty$ are the measures of the robust stability; $\|T_{dy}\|_\infty$ and $\|{}_RT_{dy}\|_\infty$ are the measures of the sensitivity. From Table 1, we recognize the values of $\|{}_RT_{dg}\|_\infty$ and $\|{}_RT_{dy}\|_\infty$ are smaller than $\|T_{dg}\|_\infty$ and $\|T_{dy}\|_\infty$ respectively. Table 1 shows that the larger the value of w is, the smaller the value of $\|{}_RT_{dy}\|_\infty$ is. But when the value of w is large enough, enlarging w does not change $\|{}_RT_{dg}\|_\infty$ and $\|{}_RT_{dy}\|_\infty$ so much.

We show the some step responses of y to r when the pole -1 of the plant becomes -2 in Fig.6,

			$\|T_{dg}\|_\infty$	$\|T_{dy}\|_\infty$
			10.61	2.252
w	α	γ	$\|{}_RT_{dg}\|_\infty$	$\|{}_RT_{dy}\|_\infty$
1	2.225	2.230	2.229	1.902
2	3.314	3.320	2.780	1.475
3	4.125	4.130	3.428	1.266
4	4.859	4.860	3.965	1.142
5	5.633	5.640	4.374	1.077
10	10.23	10.24	5.214	1.011
50	50.04	50.05	5.632	1.002
100	100.02	101.0	5.645	1.002

Table 1: Results of design

where "nominal" denotes the plant is nominal, "w=1" and "w=10" denote the controller is the K where $w = 1$ and $w = 10$ respectively and "only C" denotes the controller consists of only C. Except ${}_RT_{ry}(2,1)$, "w=1" is closer to "nominal" than "only C" and "w=10" is closer than "w=1". It is the same order as the values of $\|T_{dy}\|_\infty$ and $\|{}_RT_{dy}\|_\infty$s. It implys $\|{}_RT_{dy}\|_\infty$ has the property that it indicates the sensitivity of the control system.

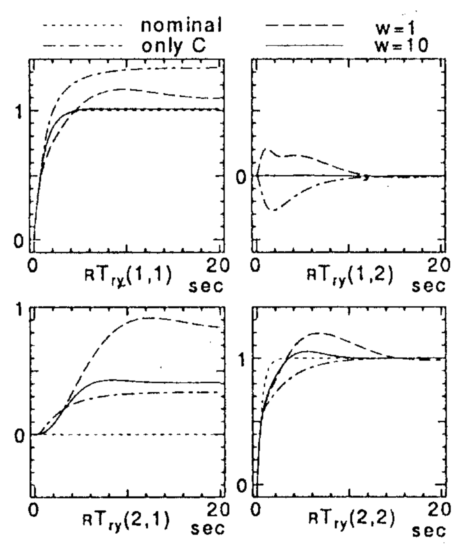

Fig. 6: Step respunses in the pole -1 of P becomes -2

5. Conclusion

We proposed the design procedure of the RMM which aims at achieving both the robust stability and the sensitivity. We made use of the robust stability of the NLCF approach to do that. And we showed the example and the effect of the proposed procedure.

Although we didn't mention about the steady-state property in this paper, we intend to achieve it in the near future.

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