### Robust Control of Reheat-Fan Engine

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#### Abstract

In this paper, reheat-fan engine is described as class of models constructed from nominal and uncertainty model for robust control. In this class of models, uncertainty model consists of structured and unstructured uncertainty, and each model is identified from nonlinear simulation using FFT and ML technique. Then, control requirements and augmented plant are specified.  $H_{\infty}$  controller satisfiying the control requirements is designed by using constant scaling matrix. Finally, efficacy of the  $H_{\infty}$  controller is showed by computer simulation.

#### 1 Introduction

One of the most important functions of jet engine controls is to obtain fast and accurate thrust modulation in response to pilot power-lever angle manipulation. To achieve this objective, engine controls control fuel flow to combustors and various variable geometries such as fun-inlet-guide-vane angle, compressor-stator-vane angle, and exhaust nozzle area. Since it is very difficult to measure thrust directly, fan speed, compressor speed, and turbine discharge temperature are used as engine thrust parameters. Among them, product of engine pressure ratio and engine air flow rate is most reliable thrust parameter. The engine pressure ratio is defined as the ratio of outlet pressure to inlet pressure, while the air flow rate is a function of corrective fun speed.

Large moment of inatia of the rotor assembly prevent from fast engine response, usually requiring several seconds for acceleration from idle speed to max thrust. For Engines which have variable exhaust nozzle area, it is possible to modulate the thrust by manipulating the engine pressure ratio, while keeping the fan speed constant. This allows very fast thrust modulation as far as fan stall margine is appropriate.

Since engine dynamics vary very much with thrust level, robust stability is required for closed loop system.

In this paper, we construct a control system shown in fig.1, and design a controller based on  $H_{\infty}$  control theory which modulate thrust by manipulating engine pressure ratio  $E_{PR}$  while keeping fan speed  $N_L$  constant and has robust stability for perturbation to engine dynamics. In this figure,  $W_F$  and  $A_7$  denote fuel flow and nozzle area, respectively. Then we evaluate control performance by computer simulation.

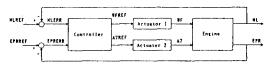


Figure 1: Control System

# 2 Modeling of Reheat-Fan Engine for $H_{\infty}$ Control

When we design a robust controller for an actual plant, it is usually necessary to obtain a set of plant models which cover the actual plant with actual uncertainty. This set for the rehear-fan engine is constructed from linear time-invariant nominal model and linear time-invariant/varying uncertainty model. In obtaining the set of plant models, evaluation of uncertainty is one of the most important step. In general, the plant uncertainties consist of structured uncertainties (uncertainties of plant parameters) and unstructured uncertainties (additive uncertainties and/or multiplicative uncertainties). In this paper, we evaluate the uncertainties of the reheat-fan engine using a mixed parametric and non-parametric system identification technique.

## 2.1 Modeling of the Reheat-Fan Engine around Equilibrium Points

### 2.1.1 State Space Structure of Reheat-Fan Engine Model

It is indispensable for parametric system identification to define previously state space structure of the plant, so we introduce the following structure for the reheat-fan engine.

In this state space model,  $[W_F, A_7]^T$ ,  $[N_L, E_{PR}]^T$ , and  $[x_1, x_2, x_3]^T = [N_H, N_L, 810.64 \times E_{PR}]^T$  denote control input, observed output, and state, respectively.

#### 2.1.2 Equilibrium Points for System Identification

Equilibrium points for system identification are listed below.

Parameter	Point 1	Point 2	Point 3
$W_F(\%)$	111.28	100.00	89.788
$A_{7}(\%)$	91.667	100.00	108.33
$N_H(\%)$	101.38	100.00	98.674
$N_L(\%)$	100.00	100.00	100.00
EPR(%)	111.19	100.00	90.114

Table 1: Equilibrium Points for System Identification

#### 2.1.3 Results of Identification

M-sequences of fuel flow  $W_F$  and nozzle area  $A_7$  are added to simulation model of the reheat-fan engine at three equilibrium points to record input/output data for both parametric system identification(based on maximum likelyhood method) and non-parametric system identification(based on FFT analysis). Resulting system matrices and input matrices by parametric identification at each equilibrium point are as follows...

Equilibrium Point-1

$$[A_1,B_1] = \left[ \begin{array}{ccc|c} -4.256 & -1.192 & -0.7380 & 2.370 & 0.52704 \\ 6.725 & -9.126 & -1.822 & 0.9864 & 4.998 \\ -2.796 & 12.22 & -14.99 & 0.8049 & -13.33 \end{array} \right]$$

Equilibrium Point-2

$$[A_2, B_2] = \begin{bmatrix} -4.479 & -0.5832 & -0.2854 & 2.333 & 0.6074 \\ 7.634 & -7.433 & -5.868 & 1.304 & 3.196 \\ -1.705 & 9.145 & -15.12 & 0.8846 & -11.66 \end{bmatrix}$$

Equilibrium Point-3

$$[A_3,B_3] = \begin{bmatrix} -4.365 & -0.6723 & -0.3363 \\ 7.088 & -6.557 & -4.601 \\ -2.410 & 7.584 & -14.31 \\ \end{bmatrix} \begin{bmatrix} 2.374 & 0.7485 \\ 1.366 & 3.444 \\ 0.9461 & -9.619 \\ \end{bmatrix}$$

Gain plots of the resulting transfer functions from  $A_7$  to  $N_L$  by parametric and non-parametric identification at equilibrium point-2 are shown in fig.2.

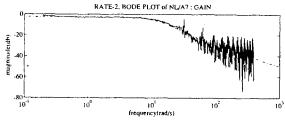


Figure 2: Results of Identification

#### 2.2 Evaluation of Uncertainties

We evaluate structured and unstructured uncertainties from preceding results of system identification at three equilibirium points. We regard a parameter perturbation of system matrix and input matrix as structured uncertainty and an additive perturbation to the plant as unstructured uncertainty. As a result, the following set of plants is considered.

$$\mathcal{P} = \{ P_{\Delta_1} + \Delta_2 W_2 : \Delta_1 \in \mathcal{B}_{\Delta_1}, \|\Delta_2\|_{\infty} \le 1 \}$$

$$\mathcal{B}_{\Delta_1} = \{ \Delta_1(t) = [\delta_1(t), \delta_2(t), \dots, \delta_{15}(t)] : |\delta_t(t)| \le 1 \}$$

In the above definition of  $\mathcal{P}$ ,  $P_{\Delta_1} = [A_{\Delta_1}, B_{\Delta_1}, C_{\Delta_1}, D_{\Delta_1}]$ , which corresponds to the plant with structured uncertainty, is given as follows.

$$A_{\Delta_1} = \begin{bmatrix} A11 + \delta_1(t)\Delta A11 & A12 + \delta_2(t)\Delta A12 \\ A21 + \delta_4(t)\Delta A21 & A22 + \delta_5(t)\Delta A22 \\ A31 + \delta_7(t)\Delta A31 & A32 + \delta_8(t)\Delta A32 \end{bmatrix}$$

$$A13 + \delta_5(t)\Delta A13 \\ A23 + \delta_6(t)\Delta A23 \\ A33 + \delta_9(t)\Delta A33 \end{bmatrix}$$

$$B_{\Delta_1} = \begin{bmatrix} B11 + \delta_{10}(t)\Delta B11 & B12 + \delta_{11}(t)\Delta B12 \\ B21 + \delta_{12}(t)\Delta B21 & B22 + \delta_{13}(t)\Delta B22 \\ B31 + \delta_{14}(t)\Delta B31 & B32 + \delta_{15}(t)\Delta B32 \end{bmatrix}$$

$$C_{\Delta_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1.2336 \times 10^{-3} \end{bmatrix}$$

The nominal system matrix  $A_{\Delta_1=0}=A=[Aij]$  and the nominal input matrix  $B_{\Delta_1=0}=B=[Bij]$  are given below. The uncertainty  $\Delta A=[\Delta Aij]$  of the system matrix and the uncertainty  $\Delta B=[\Delta Bij]$  of the input matrix are also given below.

#### 2.2.1 Evaluation of Structured Uncertainty

The nominal system matrix  $\Lambda$  and the nominal input matrix B are computed as

$$Aij = \frac{1}{2} [\max_{k} A_k ij + \min_{k} A_k ij],$$
  

$$Bij = \frac{1}{2} [\max_{k} B_k ij + \min_{k} B_k ij],$$

and the uncertainty  $\Delta A$  of the system matrix and the uncertainty  $\Delta B$  of the input matrix are computed as

$$\begin{split} \Delta Aij &= \frac{1}{2}[\max_{k} A_{k}ij - \min_{k} A_{k}ij], \\ \Delta Bij &= \frac{1}{2}[\max_{k} B_{k}ij - \min_{k} B_{k}ij], \end{split}$$

respectively.

#### 2.2.2 Evaluation of Unstructured Uncertainty

The additive uncertainty is computed as differences of frequency responses between results of parametric identification  $P_p(j\omega)$  and those of non-parametric identification  $P_{np}(j\omega)$ ,

$$|P_{np}(j\omega) - P_p(j\omega)|,$$

thereby an weighting for the additive uncertainty,  $W_2^{\bullet}$ , is defined as follows.

$$W_2 = \begin{bmatrix} 0.04 & 0.2 \\ 0.065 & 0.2 \end{bmatrix}$$

The additive uncertainties to the transfer functions from  $W_F$  to  $N_F$  and (1,1) element of  $W_2$ ,  $W_2$ 11, are shown in fig.3.

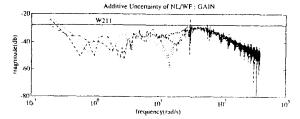


Figure 3: Additive Uncertainties

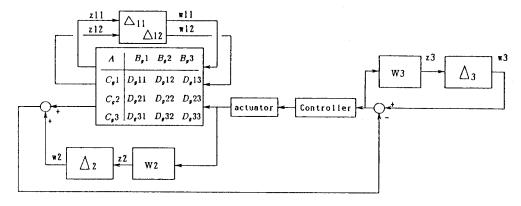


Figure 4: Augmented Plant

#### 2.3 Actuator Model

Fuel metering valve( $act_1$ ) and nozzle actuator( $act_2$ ) are integrated in the simulation program. Engine fuel flow is governed by the fuel metering valve and nozzle area is adjusted by the nozzle actuator. Models of these actuators used in this simulation are the following.

$$act_1 = \frac{1}{0.002s^2 + 0.12s + 1},$$

$$act_2 = \frac{1}{0.2778s^2 + 0.1867s + 1}$$

#### 3 Control Requirements

In this section, we state control requirements for the reheatfan engine and present an augmented plant for design of  $H_{\infty}$ controller satisfying such requirements.

#### 3.1 Requirement for Robust Stability

Requirement for robust stability is as follows: closed loop system shall be internally stable for every plant in  $\mathcal{P}$ .

#### 3.2 Requirements for Nominal Performance

Requirements for nominal performance are as follows: 1) fan speed,  $N_F$ , shall be kept constant, 2) engine pressure ratio,  $E_{PR}$ , shall track reference value. Using frequency weigting  $W_3$  and sensitivity function S of the closed loop, requirement 1) and 2) can be described as follows.

$$||W_3S||_{\infty} \leq 1$$

#### 3.3 Augmented Plant

The augmented plant, G, for design of  $H_{\infty}$  controller is shown in fig.4. In fig.4,  $P_g = [A_g, B_g, C_g, D_g]$  are defined by the following equation.

$$F_{\mathbf{u}}(P_{\mathbf{g}}, \Delta_1) = P_{\Delta_1},$$

In the augmented plant, transfer function from  $w_{11}$  to  $z_{11}$  and from  $w_{12}$  to  $z_{12}$  represent robust stability requirements for parameter perturbation to system matrix and input matrix, respectively. And transfer function from  $w_2$  to  $z_2$  represents robust stability requirement for additive uncertainty to the plant. On the other hand, transfer function from  $w_3$  to  $z_3$  represents nominal performance requirement. Here af-

ter,  $F_l(G, K)$  means closed loop system constructed from augmented plant G, and controller K.

#### 4 Design of $H_{\infty}$ Controller

In this section, we present design procedure for  $H_{\infty}$  controller with constant scaling matrix .

#### 4.1 Frequency Weighting for Sensitivity Function

Frequency Weigthing  $W_3$  for sensitivity function S is given as follows.

$$W_3 = \begin{bmatrix} \frac{500.0}{623.75s+1} & 0\\ 0 & \frac{500.0}{623.75s+1} \end{bmatrix}$$

## 4.2 Design of $H_{\infty}$ Controller Using Constant Scaling Matrix

#### 4.2.1 Procedure of Design

Scaling matrix is used for the design of  $H_\infty$  controller to reduce conservatism of robust stability. The design algorithm preceeds as follows.

Step1: Compute  $H_{\infty}$  controller  $K_j$  for the augmented plant  $G_j$ .

Step2: Compute scaling matrix  $S_i$  that minimize

$$||S_{i}^{-1}F_{l}(G_{j},K_{j})S_{j}||_{\infty}$$

using numerical optimization technique. Since structured uncertainties are time-varying, the following scaling matrix with constant elements is used.

$$diag[s_1, s_2, \ldots, s_{15}, s_{16} \cdot I_{2 \times 2}, I_{2 \times 2}], \quad s_j \in R$$

**Step3**: Calculate scaled augmented plant  $G_{j+1}$  by  $S_j$  as follows.

$$G_{j+1} = diag[S_j^{-1}, I_{2\times 2}] \cdot G_j \cdot diag[S_j, I_{2\times 2}]$$

Step4: If the following inequality does not hold, then continue Step1 to Step3, otherwise stop the algorithm.

$$||F_1(G_{j+1},K_j)||_\infty \leq 1$$

#### 4.2.2 $H_{\infty}$ Controller

Numerical computation of  $H_{\infty}$  controller is based on the Glover-Doyle 2-Riccati-Algorithm [2]. Also, final scaling marix is as follows.

 $[s_1, s_2, \dots, s_{16}] = [11.251, 10.482, 8.6090, 11.143, 7.1491, 3.7596, 8.2065, 2.4842, 8.8482, 25.821, 28.702, -37.764, 10.660, 24.741, 7.9199, 1.0342]$ 

Bode plot of the resulting  $H_{\infty}$  controller is shown in fig.5.

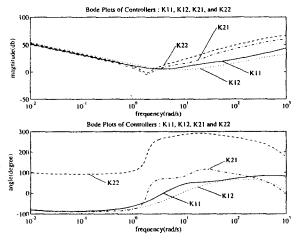


Figure 5: H<sub>∞</sub> Controller

#### 5 Simulation

 $E_{PR}$  and  $N_L$  responses to  $E_{PR}$  step reference at three equilibrium points are shown in fig.6 and fig.7, respectively.

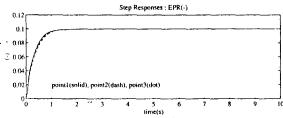


Figure 6: EPR Responses

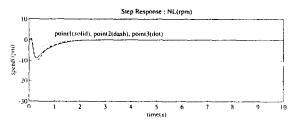


Figure 7: N<sub>L</sub> Responses

#### 6 Conclusion

In this paper, reheat-fan engine was described as class of models constructed from nominal model and uncertainty model for robust control. In this class of models, uncertainty model consisted of structured and unstructured uncertainty, and each model was identified from nonlinear engine simulation using FFT and ML technique. Control requirements and augmented plant were then specified.  $H_{\infty}$  controller satisfying the control requirements was designed by using constant scaling matrix. Control performance was evaluated by computer simulation and efficacy of the  $H_{\infty}$  controller was recognized.

#### References

- R.Watanabe, M.kurosaki, T. Kusakawa, K. Uchida, M. Fujita H<sub>∞</sub> Control of Gasturbine Engines for Helicopters, Proc. in American Control Conference, 1123/1127 (1993)
- [2] K.Glover, J.C.Doyle State-Space Formulae for Stabilizing Controllers that Satisfy a H<sup>∞</sup> Norm Bounds and Relations to Risk Sensitivity, Systems and Control Letters, 11-2, 162/172 (1988)