'93 KACC (1993. 10. 20~22)

Nonlinear Feedback Control of a Electromagnetic Suspension System using a Digital Signal Processor

°Sungjun Joo, Jijoon Byun, Hyungbo Shim, Jinheon Seo
Dept. Electrical Eng. Seoul National University

Abstract

A feedback linearization controller for EMS system is implemented using DSP. In this paper, we show that given EMS system is input-state linearizable and satisfies some robustness condition. Also we derive feedback linearization controller for given system. Finally, some experiments are performed to demonstrate the performance of the proposed controller - especially, comparing this with the classical state feedback controller using linear perturbation.

I. INTRODUCTION

Feedback Linearization is an approach to the design of nonlinear controllers, which has attracted a great of research interests in recent years. However, actual applications resulting in the implementation of such control algorithms have been few, due to the limitations of feedback linearization method and the computational requirements. The new algorithm has a number of important limitations - the full state has to be measured and no robustness is guaranteed in the presence of parameter uncertainty. Active research has been carried out to overcome the limitation of parameter uncertainty by means of outer loop design technique [12], and adaptive control schemes [5], [7].

In this paper, we consider the control of an attraction type ElectroMagnetic Suspension (EMS) system which consists of rail and single magnet. The basic model of the system and the state feedback control by linear perturbation of the operating point were investigated in [9]. Due to the nonlinearity and open-loop instability of the attraction type EMS system, it is difficult to derive a stabilizing controller under all operating conditions using the classical frequency-domain and state variable feedback methods. We propose a controller based on the nonlinear feedback linearization method. We show that an EMS system satisfies the condition for robustness and therefore the control algorithm based on feedback linearization is applicable to the industrial implementation.

In practice, digital control law is inevitable. However, discretization can destroy linearity [1] [3] [6] [11], since the control input should be constant between sampling times. The method which is usually employed is that of neglecting the error due to sampling by increasing the sampling frequency. In general, a feedback linearization controller requires more computing power to implement than the classical state feedback controller. To enable experimental evaluation of feedback linearization without the limitations imposed by current microprocessor-based controllers, we construct a digital control system based on a TMS320C31 DSP. Due to the computational power of DSP, the high speed sampling is possible and the sampling error can be neglected. •

The experimental setup consists of a single magnet, a rail, a PWM amplifier, sensors of gap, acceleration and current and a TMS320C31 -floating point digital signal processor. The algorithm of digital filter, integration of sensor value, and feedback linearization control are performed in 3[Khz] sampling time by DSP.

In the next section, we review mathematical preliminaries. We report on an implementation of a applicable feedback linearization controller in Section III, and compare the proposed controller with the classical state feedback controller in Section V. The description of the experimental setup is presented in Section IV, and conclusions are summarized in Section VI.

II. MATHEMATICAL PRELIMINARIES

Consider the nonlinear single-input-single-output system

$$\dot{x} = f(x) + g(x)u, \; x(0) = x_0, \qquad (2.1)$$

where $x \in R^n$, $f(x)$ and $g(x)$ are real smooth vector fields on R^n, and u is a control function.

If the system (2.1) is input state linearizable [10], the following coordinate transformation and input transformation can be obtained :

$$z = T(x) \qquad (2.2)$$

$$u = \left(\frac{1}{L_g L_f^{n-1} T}\right)\left(-L_f^n T + v\right) = \alpha(x) + \beta(x)v \qquad (2.3)$$

such that z, v satisfy

$$\dot{z} = Az + Bv$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & 1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ 1 \end{pmatrix} \qquad (2.4)$$

The objective is to find a control u which stabilizes the system (2.1) under parameter variations.

III. Mathematical Model of EMS System and Feedback Linearizing Controller

A) Mathematical Model Of EMS System

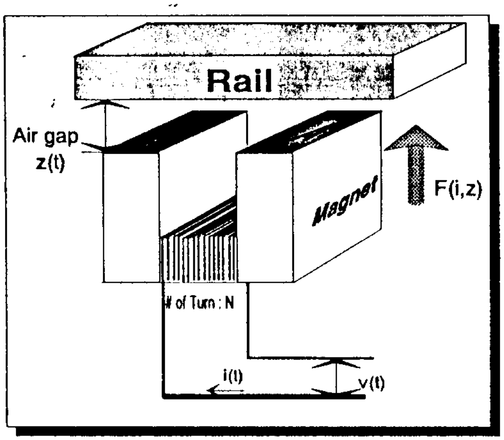

Fig.3.1 Electromagnet Configuration

From [9] ,the electromagnetic force of attraction in Fig.3.1 is given by

$$F(i,z) = \frac{B^2 A}{\mu_0} = \frac{\mu_0 N^2 A}{4}\left[\frac{i(t)}{z(t)}\right]^2 \qquad (3.1)$$

Also the inductance is

$$L(z) = \frac{\mu_0 N^2 A}{2Z(t)} \qquad (3.2)$$

If R is the total resistance of the electric circuit, then the equation of electric dynamics is

$$V(t) = Ri(t) + \frac{d}{dt}[L(z,i)i(t)]$$

$$= Ri(t) + \frac{\mu_0 N^2 A}{2Z(t)}\frac{di(t)}{dt} - \frac{\mu_0 N^2 A i(t)}{2[Z(t)]^2}\frac{dz(t)}{dt} \qquad (3.3)$$

Mechanical dynamics is

$$m\ddot{z} = -F(i,z) + mg. \qquad (3.4)$$

The mathematical model for the EMS system is given below

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{\mu_0 N^2 A}{4m}\left(\frac{x_3}{x_1}\right)^2 + G \\ \frac{x_2 x_3}{x_1} - \frac{2R}{\mu_0 N^2 A}x_3 x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{2x_1}{\mu_0 N^2 A} \end{bmatrix} V(t) \qquad (3.5)$$

where $x_1 = z$ (vertical airgap) $[m]$
$x_2 = \dot{z}$ (vertical velosity) $[m/\sec]$
$x_3 = i$ (magnet current) $[A]$
A : area of horizontal section of magnet $[m^2]$
N : number of turn
$\mu_0 = 4\pi \times 10^{-7} [H/m]$
$G = 9.8 [m/\sec^2]$

B) Feedback Linearization

To derive a feedback linearizing control law which can be used in practical application i.e. maglev transportation, it is necessary that the conditions for feedback linearization [10] hold for the EMS system not only at the nominal parameter values but also under parameter variations. Especially, a mass is the most changeable parameter in the EMS system for the maglev transportation

We first show that the EMS system is input state feedback linearizable.

i) Linear Independence of $\{g, ad_f(g), ad_f^2(g)\}$

$$g = \begin{pmatrix} 0 \\ 0 \\ \frac{2}{\mu_0 N^2 A}x_1 \end{pmatrix}$$

$$ad_f(g)=\begin{pmatrix}0\\ \frac{1}{m}\frac{x_3}{x_1}\\ \frac{4R}{(\mu_0N^2A)^2}x_1^2\end{pmatrix}$$

$$ad_f^2(g)=\begin{pmatrix}-\frac{1}{m}\frac{x_3}{x_1}\\ 0\\ \frac{4R}{(\mu_0N^2A)^2}x_1x_2-\frac{1}{m}\left(\frac{x_3}{x_1}\right)^2+\frac{8R^2}{(\mu_0N^2A)^3}x_1^3\end{pmatrix}$$

From the above, it can be seen that $\{g, ad_f(g), ad_f^2(g)\}$ is linearly independent in U_{x_0},
where $U_{x_0}=\{(x_1,x_2,x_3)|x_1>0, x_3\neq 0\}$

ii) Involutivity Condition

Lie bracket of $g, ad_f(g)$ is obtained as

$$[g, ad_f^1(g)]=\begin{pmatrix}0\\ \frac{2}{m\mu_0N^2A}\\ 0\end{pmatrix}$$

The EMS system is involutive if there exist scalar a, b such that

$$a\begin{pmatrix}0\\ 0\\ \frac{2}{\mu_0N^2A}x_1\end{pmatrix}+b\begin{pmatrix}0\\ \frac{1}{m}\frac{x_3}{x_1}\\ \frac{4R}{(\mu_0N^2A)^2}x_1^2\end{pmatrix}=\begin{pmatrix}0\\ \frac{2}{m\mu_0N^2A}\\ 0\end{pmatrix} \quad (3.6)$$

Suitable a, b is obtained as follows

$$a=-\frac{4R}{(\mu_0N^2A)^2}\frac{x_1^2}{x_3},\ b=\frac{2}{\mu_0N^2A}\frac{x_1}{x_3}$$

Next, we show the conditions for a feedback linearization still hold for that EMS system under the system parameter - mass - change. Choose the coordinate transformation $z=T(x)$ as follows

$$\begin{pmatrix}z_1\\ z_2\\ z_3\end{pmatrix}=\begin{pmatrix}x_1\\ x_2\\ -\frac{\mu_0N^2A}{4m}\left(\frac{x_3}{x_1}\right)^2+G\end{pmatrix} \quad (3.7)$$

Then, we obtain state equations

$$\begin{bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\end{bmatrix}=\begin{bmatrix}z_2\\ z_3\\ L_f^nT+L_gL_f^{n-1}Tu\end{bmatrix} \quad (3.8)$$

where $L_gL_f^{n-1}T=\frac{R}{m}\frac{2x_3^2}{x_1}$, $L_f^nT=\frac{1}{m}\frac{x_3}{x_1}$

If m change into λm, where $\lambda\in R^+$, we consider a new coordinate transformation $y=\hat{T}(x)$ as follows;

$$\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}=\begin{pmatrix}x_1\\ x_2\\ -\frac{\mu_0N^2A}{4\lambda m}\left(\frac{x_3}{x_1}\right)^2+G\end{pmatrix} \quad (3.9)$$

When we use the coordinate transformation (3.8) instead of (3.9), the Lie derivatives change into

$$L_gL_f^{n-1}\hat{T}=\frac{R}{\lambda m}\frac{2x_3^2}{x_1},\ L_f^n\hat{T}=\frac{1}{\lambda m}\frac{x_3}{x_1}$$

and we obtain the state eqautions

$$\begin{bmatrix}\dot y\\ \dot y_2\\ \dot y_3\end{bmatrix}=\begin{bmatrix}y_2\\ y_3\\ L_f^n\hat{T}+L_gL_f^{n-1}\hat{T}u\end{bmatrix}=\begin{bmatrix}x_2\\ x_3\\ \frac{1}{\lambda}L_f^nT+\frac{1}{\lambda}L_gL_f^{n-1}Tu\end{bmatrix} \quad (3.10)$$

From the Eq. 2.3 and Eq. 3.8, the following feedback linearization control law is obtained for the EMS system with the parameter unchanged

$$u=-\frac{mx_1}{x_3}\left(v-\frac{R}{m}\frac{2x_3^2}{x_1}\right) \quad (3.11)$$

From the Eq. 2.3 and Eq. 3.10, the control law for the system with the parameter changed is obtained as follows:

$$u=-\frac{\lambda mx_1}{x_3}\left(v-\frac{R}{\lambda m}\frac{2x_3^2}{x_1}\right) \quad (3.12)$$

Now, we substitute (3.11) instead of (3.12) for (3.10), and then following linear state equation can be obtained

$$\begin{bmatrix}\dot y_1\\ \dot y_1\\ \dot y_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{1}{\lambda}\end{bmatrix}v \quad (3.13)$$

So, the conditions for feedback linearization hold for the EMS system under mass- parameter variation.

C) Design Of Linear Control Law

Determine a new input v as follows;

$$v=k_1(z_1-r)+k_2z_2+k_3z_3 \quad (3.14)$$

$$v=k_1(x_1-r)+k_2x_2+k_3\left(-\frac{\mu_0N^2A}{4m}\left(\frac{x_3}{x_1}\right)^2+G\right)$$
$$=k_1(x_1-r)+k_2x_2+\lambda k_3\left(-\frac{\mu_0N^2A}{4\lambda m}\left(\frac{x_3}{x_1}\right)^2+G\right)+k_3(1-\lambda)G$$
$$=k_1(y_1-r)+k_2y_2+\lambda k_3y_3+k_3(1-\lambda)G \quad (3.15)$$

where r : operating gap

When we substitute (3.15) for v in (3.13), then the closed loop system can be written as follows;

$$\begin{bmatrix}\dot y_1\\ \dot y_2\\ \dot y_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\ \frac{1}{\lambda}k_1&\frac{1}{\lambda}k_2&k_3\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ -\frac{r}{\lambda}k_1+G(1-\lambda)k_3\end{bmatrix} \quad (3.16)$$

The characteristic equation of (3.16) can be obtain as

$$S^3-k_3S^2-\frac{1}{\lambda}k_2S^1-\frac{1}{\lambda}k_1=0 \quad (3.17)$$

If the bound of λ is given, an appropriate $[k_1, k_2, k_3]$ can be determined.

In Eq 3.16, the reference r changes into

$$\hat{r} = \frac{r}{\lambda} - G(1-\lambda)\frac{k_3}{k_1} \tag{3.18}$$

Therefore the steady state error of the system (3.16) is

$$\left| r - \hat{r} \right|$$

IV. EXPERIMENTAL SETUP

To provide flexibility and to simplify the hardware structure of the controller, all the blocks within the dashed line box in Fig.2 are implemented in software.

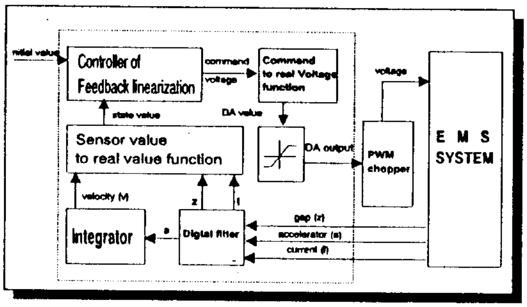

Fig.2. Block diagram of feedback linearizing controller

A) Implementation Of Measurement Function

The measured quantities (vertical gap (z), acceleration (a), current (i)) which have a voltage unit, are sampled by analog to digital converters. Since we can get only AD output values in the form of integer number, functions which transform AD output values into a physical values are needed. We can find these functions through the open loop test.

Digital Filter

The success of feedback linearization depends on acquirement of accurate physical values. Due to the environments with a high level of electromagnetic interference, signal conditioning filters are required. We found that the frequency band of measurement noises are 600[Hz] to 3K[Hz] by some practical experiments. Hence, we designed the 23-th order low pass FIR filter [8] which has 400[Hz] passband frequency and 550[Hz] stopband frequency.

Integrator of Acceleration

The velocity of magnet is obtained by integrating its acceleration value. The DC offset in the sensor must be minimized to get the accurate sensor values. In the implemented system, all sensors are calibrated every time the system is started. During the calibration, the AD converters are read hundreds of times to obtain the average offset while the system is not enabled. During normal operation, the offset is subtracted from the measured quantities. However, the DC offset in the sensor of acceleration are variable according to the voltage of PWM chopper. We use $\frac{1}{S+0.005}$ instead of $\frac{1}{S}$, to prevent saturation of the integrator. To obtain the coefficients of digital integrator, we used bilinear transformation of $\frac{1}{S+0.005}$. The equation of bilinear transformation is given as follows

$$S = \frac{2}{T_f}\frac{1-z^{-1}}{1+z^{-1}} \tag{4.1}$$

where T_f is sampling period. The algorithm of digital integrator is similar to that of IIR filter. The algorithm of digital integrator should be

$$H(z) = \frac{b[0]+b[1]z^{-1}}{a[0]+a[1]z^{-1}} \tag{4.2}$$

$$y[n] = \{b[0]x[n] + b[1]x[n-1] - a[1]y[n-1]\}/a[0] \tag{4.3}$$

B) Classical Control Law & Feedback Linearization

Classical state feedback using the Linear perturbation method at the operating point

From [9], the control law is obtained as follows

$$u = V_0 - [k_1(x_1 - r) + k_2x_2 + k_3(i_0 - x_3)] \tag{4.4}$$

where

$$V_0 = i_0 R$$

$$mG = \frac{\mu_0 N^2 A}{4}\left[\frac{i_0}{r}\right]^2 \tag{4.5}$$

Feedback linearization

We can obtain the control law by using Eq.3.11 and Eq 3.14

$$u = -\frac{mx_1}{x_3}\left[k_1(x_1 - r) + k_2x_2 + k_3\left(-\frac{\mu_0 N^2 A}{4m}\left(\frac{x_3}{x_1}\right)^2 + G\right) - \frac{R}{m}\frac{2x_3^2}{x_1}\right] \tag{4.6}$$

V. EXPERIMENTAL RESULTS

Some tests were conducted to characterize the performance of feedback linearization. First, to compare the performance of new algorithm with the classical state feedback method, the same operating reference gap (7.5[mm]) and initial gap (13[mm]) are applied to the system. Second, the system parameter (mass) changes from 9[kg] to 13[kg]. Third we change the reference gap (r) in Eq.4.4 and Eq.4.6 from 9 [mm] to 8[mm] and then to 7[mm] and 6[mm]. The control input voltage is limited within 0[v] to 80[v], and 3[mm] thick sheet of rubber which protects magnet from impingement are attached on the top of the magnet.

The control algorithm of the feedback linearization and classical state feedback structure is shown in Fig 5.1

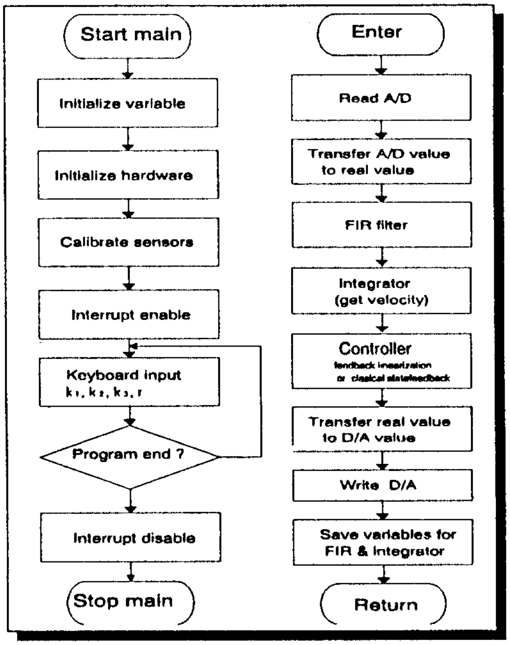

Fig 5.1 Flowchart of main and interrupt program

A) Classical State Feedback Control Using Linear Perturbation

i) Transient response

The feedback control gains k_1, k_2, k_3 were chosen using trial and error method. The chosen gains provide fairly good response. Fig.5.2 shows the transient responses of the gap sensor signal and control input signal when the classical control algorithm in Eq.4.4 and Eq.4.5 is used. The change of the gap sensor signal from 13[mm] to 7.5[mm] has been observed(the voltage unit does not have any meaning). The first overshoot of gap sensor signal is 3[mm] which is the upper limit of gap value. The second overshoot is 10[mm] and the system comes to steady state in about 2.0 [Sec].

ii) Mass change

The effect of mass parameter change in the EMS system was tested in Fig.5.3. While the EMS system is in steady state, 4[kg] ball is dropped on the magnet softly. The EMS system comes to steady state in about 1[Sec] .

iii) Reference gap change

A reference r like staircase input is applied to EMS system. As shown in Fig 5.4, it decreases from 9 [mm] to 8[mm], and then to 7[mm]. It further decreses to 6[mm], and then to 5[mm] . When the reference input is changed from 6[mm] to 5[mm], the classical controller does not keep the EMS system stable. Since linearization around operating point is used in the classical controller, the linear approximation fails in case where the operating point changes considerably.

B) Feedback Linearizing Controller

i) Transient response

We determined by simulation the values of k_1, k_2, k_3 in Eq.4.6 which provide good performance. With these values of k_1, k_2 and k_3 the results of simulation are very close to the actual system, We determined k_1= -216000, k_2= -3960, k_3 = -180 and the other conditions were the same as Fig 5.2.

Fig.5.5 shows that feedback linearizing controller has better dynamic performance than that of classical controller. The first overshoot is 4.5[mm], which is smaller than that of the classical controller, and the second overshoot is 9[mm]. The system comes to steady state in about 0.8 [Sec].

ii) Mass- parameter change

Fig 5.6 shows the gap sensor signal and control input signal with changing parameter when the feedback linearization is used. The mass of magnet changes from 9[Kg] to 13[Kg]. A steady state error is observed. Since in Eq. 3.18, the operating gap changes from 8[mm] to 9 [mm].

iii) Reference gap change

The same staircase reference input as Fig 5.4 is applied into EMS system. As shown in Fig 5.4, it decreases from 10[mm] to 9[mm], and then to 8[mm]. It further decreses to 6[mm], and then to 5[mm]. The feedback linearization controller keeps the EMS system stable for all reference inputs. Owing to exact linearizing technique, EMS system can keep the property of linearity in global operating region.

VI. CONCLUSION

The implementation of feedback linearization control on DSP based system is presented. A robustness condition is proposed for feedback linearizable system under parameter variation. We also showed that the EMS system is feedback linearizable and it satisfies the condition for robustness. A feedback linearization controller for EMS system was proposed and experiments were performed using a TMS320C31 DSP. Some tests were conducted for experimental comparison of the feedback linearization with the classical state feedback. The experimental results demonstrate that the feedback linearization controller shows better performance than that of the classical state feedback controller and it is robust with respect to disturbance and parameter variations, though some steady-state errors appear.

VII. REFERENCES

[1] J.W. Grizzle and P.V. Kokotovic, "Feedback Linearization of Sampled-Data Systems", IEEE Trans. Automatic Contr., vol. 33, no. 9, pp. 857-859, 1988

[2] A. Isidori, "Nonlinear Control Systems" , Springer - Verlag, 1989

[3] B. Jakubczyk and E.D. Sontag, "The Effect of Sampling on Feedback Linearization", Proc. 26th IEEE Conf. Decision Contr., pp.1374-1379, 1987

[4] J.H.Jin,"Controller Development for an EMS using Nonlinear Feedback Linearization", M.S.Thesis, Seoul National Univ.,1991

[5] I. Kanellakopous, P.V. Kokotovic, and A. Stephen Morse, "Systematic Design of Adaptive Controllers for Feedback Linearizable Systems", IEEE Trans. Automatic Contr., vol. 36, no. 11, pp. 1241 -1253, November, 1991

[6] H.G. Lee, A. Arapostathis and S.I. Marcus, "On the Digital Control of Nonlinear Systems", Proc. 27th IEEE Conf. Decision Contr., pp. 480-481, 1988

[7] R. Marino, I. Kanellakopoulos, and P. V. Kokotovic, "Adaptive trcking for feedback linearizable SISO systems",in Proc. 28th IEEE Conf. Decision Contr., Tampa, FL,Dec. , pp.1002-1007 , Dec., 1989

[8] Alan V. Oppenheim & Ronald W. Schafer,"Discrete-Time Signal Processing", Prentice-Hall, 1989

[9] P.K.Sinha, "Electromagnetic Suspension :Dynamics & Control", IEE control engineering series :v.30 1987

[10] J.J.E. Slotine and W. Li, "Applied Nonlinear Control", Prentice-Hall, 1991

[11] E.D. Sontag, "Orbit Theorems and Sampling" in Algebraic And Geometric Methods in Nonlinear Control Theory, M. Fliess and M. Hazewinkel (Eds.), Reidel, pp. 441-486, 1986

[12] M.W. Spong , "Modeling and Control of Elastic Joint Robotsl", Trans. ASME,J of Dyn.Sys.,Meas. ,and Cont., Vol. 109, PP310-319,April,1986

[13] M.W. Spong and M. Vidyasagar, "Robot Dynamics and Control", John Wiley & Sons, 1989

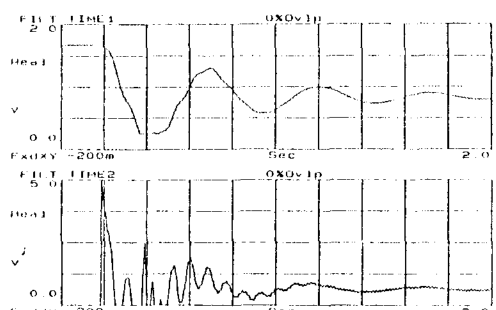

Fig 5.2 Transient response of classical state feedback controller

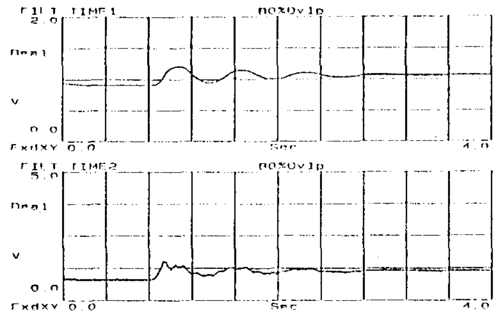

Fig 5.3 Response of classical controller with mass change

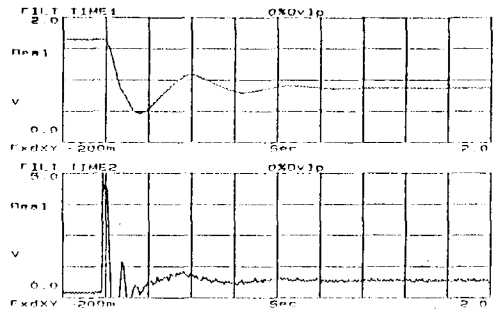

Fig 5.4 Transient response of Feedback Linearizing controller

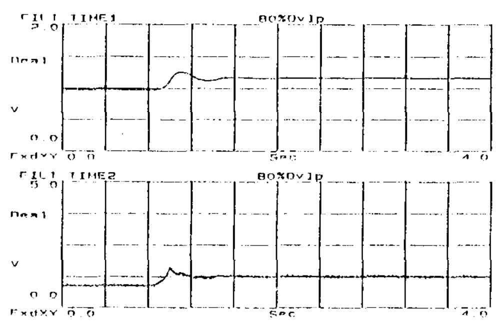

Fig 5.5 Response of feedback linearizing controller with parameter (variation : m: 9[kg] λm: 14[kg])