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# A Figure Categorization Structure for Imagery and Conceptualization

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## Abstract

In an intelligent man-machine interface, it is very effective to support human thinking and to be in communication in some intuitive fashion. For this, sharing experience between the party concerned, human operator(s) and the interface is essential. It is also necessary to keep mutual understanding in some conceptual levels. Here in the present paper, figures which are an aspect of concepts and form a basis of mental image are discussed.

## Introduction

A method for categorizing figures is proposed here and its mathematical aspect is described. Figures are categorized in some hierarchical manner, and the method of sectional linearization (called just "linearization" in the sequel) is employed which provides the one to one correspondence between linearized portions of the figure and its attributes. This means that every portion of a figure has its own name as an attribute in order to distinguish it from other portions in a linguistic manner. A concept is understood as its denotation (extension). In this sense, experienced real objects (as figures) are related to verbal representation of concepts. A concept is kept in a human's brain as its internal (mental) representation. From this point of view, a concept is described by both verbal representation and image representation. Recently this aspect of concept is a topic in the field of cognitive science. The ideas proposed here which is based on the fact that concepts and knowledge are formed basically by experiencing is in conformity with that aspect. Here,

[Definition0]a category is considered to be the intermediate grouping of objects (figures) which must be passed through before establishing conceptualization of those objects.

That is, a category is a tentative grouping using some similarity(-ies) between objects or figures for final conceptualization. And some objects are omitted from that specific category when they are found to be distinguished in some points through experience accumulation. And the rest form a concept. Otherwise, a category is a concept as it is. The mathematical definition of category is employed as a way for understanding the proposed categorization structure. The notions of filter and filter base also play a fundamental role in the present paper, which are for the notion of convergence in an abstract sense. A filter base is shown to form a topology which is used to evaluate the typicality of a specific object as a member of the denotation of a concept. It is also shown that a filter base is a category in the above sense.

## Categorization Process

For an object, the visual stimulus, to his eye, a human judges if it is a,
1)plane figure,
or
2)3-dimensional figure.
Then he processes the information in detail. There are illusory stimulus in the real world. So at this moment, sometimes he makes misunderstanding because of the mistake in the above stage. He may perceive 3-dimensional figure as plane figure, or vice versa. In that case, he comes to be aware of the contradiction in his cognition. For example, a cube and a hexagon may be the case. As this example implies, the contour plays an important role in recognizing a figure. So here, a hierarchical categorization method is described, for classifying and conceptualizing concepts in the level of contour. More complicated figures including 3-dimensional ones are in consideration as part of scene understanding where the meaning of a scene is represented from the system's own experience using bilateral translation between images and words.[6] And this method is used in the framework of experience sequence proposed by one of the authors.[1,3,4,5]

Any contour is regarded as an ordered n-tuple of line segments, and each segment represents an attribute of that contour; i.e., for instance, "top," "bottom," and "side" in the case of a box. Many of parts of something are not named. In the above example of box, the naming of parts is in some abstract level. So the naming can be applicable to any other box-like object when viewed as a contour of its side view. As the above example shows, a concept is possible to always be attached to any part of an object at least in the abstract conceptual level in understanding it. Thus the bilateral translation between images and words is available as is described in Reference[6].

## Mathematical Consideration for Building Figure Classification Hierarchy

Related things may be found in Reference[7]. Fig.1 shows the hierarchy of category by the present categorization method.
Convex-concave category: Convex-concave category is the feature extracted by the categorization of a linearized contour in the basis of convexity and concavity, viewing the contour's features locally. This is the first procedure in the course of hierarchical categorization of a figure for cognition. For a convex node, "+" is assigned and for a concave node, "-." The length of the string of "+" and "-" thus obtained is called the length of a category.

Simplified category: A simplified category is obtained when for any convex-concave category, a succesion of the same symbol is replaced by only one of them. Thus, a simplified category is a single "+" (or single "-" instead, as will be described), or "+-," or any finite repetition of "+-." Thus the length of a simplified category is 1, or 2k, k=1,2,3,.... A single "+" denotes a circle in an abstract fashion. Of course any convex figures like triangle, rectangle, pentagon, etc has the simplified category of "+." Consider a circle as a linearized contour. The length of each line

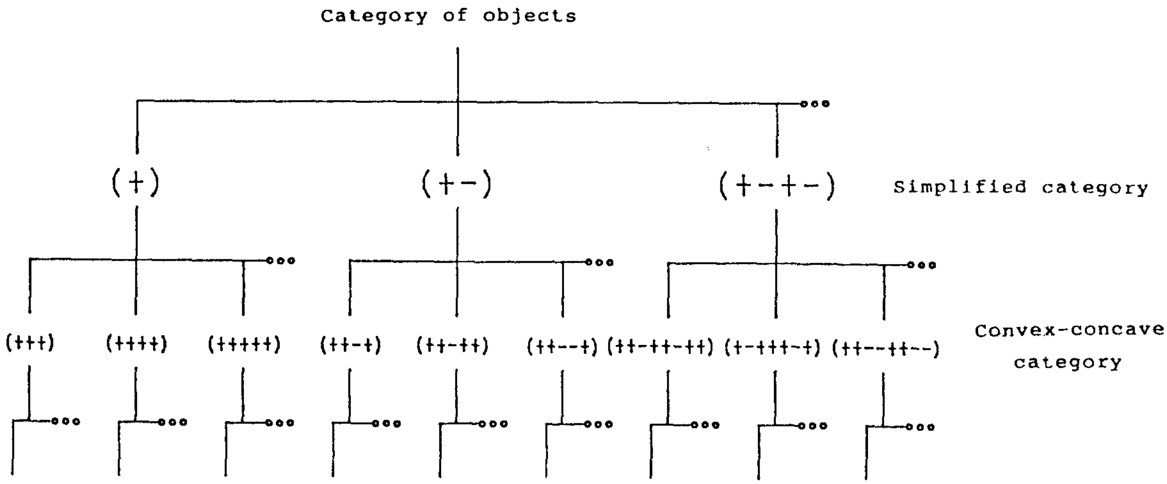


Fig.1 Schematic of the hierarchy of category.

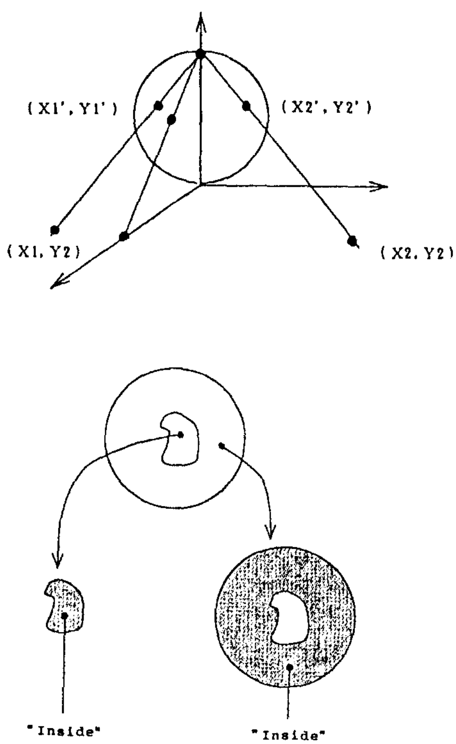


Fig.2 Schematic of one-point compactification. The (x,y) plane can be regarded as a sphere, adding a point to the sphere, which corresponds to the infinity. "Inside" and "outside" depends on the viewpoint.

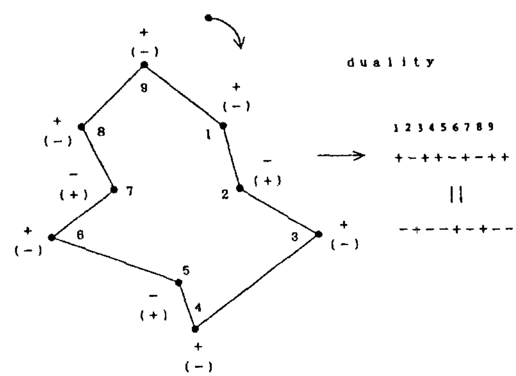


Fig.3 Duality in simplified ( or convex-concave) category.

Starting from node (1) +-++-+-++

Starting from node (9) ++-++-+-+

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Starting from node (2) -++-+-+++

Fig.4 Equivalent categories using the example in Fig.3.

segment depends on the accuracy in which the circle is approximated. Its length becomes smaller with the accuracy of approximation. Taking the limit yields a real circle. In this sense, a circle is the limit of polygons. Thus, it is said that any polygon belongs to the simplified category of "+." Using the morphisms described later, the simplified category of "+-+-," for example, is also obtained as a comprehensive categorization of a variety of figures which has two portions of succession of " -" in its contour.

Duality in category: "-" can be the simplified category for those convex figures. Which is used, "+" or "-," depends on from which direction the contour is viewed. That is, if a square is viewed from outside that square, then at every node (vertex), two edges are connected in a dull angle of 270degrees which shows concavity, though it is viewed from inside, then it has the angle of 90degrees which is judged to be convex.

As was seen in the discussion right above, "+" and "-" posses the same meaning as a conceptualized circle. The above discussion may be better understood in Fig.2. That is, Fig.2 shows the schematic of one-point compactification of a plane. In this sense, a (an unbounded) plane can be considered to be a sphere. So suppose that the circle (or any closed curve) is cut off from the surface of the sphere. Then the circle itself is a figure (contour), and also the rest of the sphere. (To be precise, the operation of closure must be taken to either figure.) This is the schematic description of the discussion on the viewpoint related to the problem of "+" or "-" above. And now it must be convinced that "+" and "-" plays the same role in considering categories of figures.

[Definition1] In the case of the conceptualized circle of "+," such viewpoint of categorization is called "+" basis, and for "-" circle, "-" basis.

For a category of "+" basis, that category can be transformed into that of "-" basis without changing the category at all, when "+" is replaced by "-" and vice versa. Their difference is in just the interpretation of convexity. Thus the following definition is obtained:

[Definition2] The "+" basis and "-" basis are called dual to each other.

A schematic is shown in Fig.3. In Fig.3, the categorization is started at the node(1). There is no restriction in doing this. Thus, the node(9) can be the starting point,and the node(8) can be, and so on to the node(2). Hence the following characteristics holds:

[Definition3] Categories are equivalent to each other, if the category in the form of a string of "+" and "-" is rewritten by putting the last symbol at the top of the string and by shifting the rest to the right by one.

The schematic for this operation and the examples of equivalent categories are shown in Fig.4.

The procedure of categorization can be discussed within the framework of categorization in the mathematical sense[8], as will be described. An example is shown in Fig.5a. Suppose that there is at least one block of succession of the same symbols in a convex-concave category. And define a morphism to replace two successive such symbols ("++" or "--") into one (i.e., "++ → +" or "-- → -"). Continuing this operation one at a time within the same block or in one of other blocks obtains a simplified category. That is, here the morphism is to obtain a category of the length of k-1 from the one of the length of k.

[Theorem] In the course of obtaining the eventual simplified category by the above operation, a unique simplified category is obtained independent of the category is obtained independent of the order to apply the operation, starting from a specific convex-concave category.

An example of the equivalence of the order of applying the operation is shown in Fig.5a and 5b. Although the order of application of the operation is different from each other, the eventual simplified category is the same. As the example shows, it is readily seen that the Theorem holds. Since the application of the operation can be considered in every block of successive "+" or "-" independently, change in the order of application of the operation as the overall procedure does not make sense in building the simplified category.

The morphism here can be defined by using the following functions. For convenience, the maximum length of convex-concave category is 2N. Then for a convex-concave category of length k, $k \leq 2N$, assign cardinal numbers, 1 to k, to the symbols in the string of category from the leftmost one to the rightmost one. This procedure is defined by the function n:

$$n(\text{the i-th symbol from the left}) = i, \quad 1 \leq i \leq k. \quad \ldots (1)$$

Letting the i-th symbol be $s_i$, the gunction n can be written as:

$$n(s_i) = i. \quad \ldots (2)$$

As the function n is bijective, symbol $s_i$ can be regarded as the number i. Suppose that the symbol i and the next symbol, i+1, are the same. (If i=k, then i+1 is defined to be i+1=k+1=1). That is, the symbol k and the symbol 1 are considered to be neighboring to each other forming a loop. Let the function f be defined as:

$$f(j) = s_j, \quad \text{if } j \leq \min\{i, i+1\},$$
$$f(j) = sj+1, \quad \text{if } \min\{i, i+1\} < j \leq k-1, \quad \ldots (3)$$

where i+1=1 if i=k. Then the composite of n and f yields a category of k-1 from a category of length k.

For the filter base of an experience sequence[9] can be shown to be a category.

[Proposition] The filter base of an experience sequence is a category.

(Proof) The filter base of an experience sequence $(a_k)_n$ be denoted $\mathcal{B}$. Let $B_i$, $B_j \in \mathcal{B}$, $i \leq j \leq N$, and let the function r,

$$r: B_i \to B_j, \quad \ldots (4)$$

be defined as follows: For an element $a_k$ of $B_i$, if $B_j$ has the same element $a_k$, then r assigns $a_k$ to the same $a_k$. If not, r assigns $a_k$ to any one element of $B_j$. Now let the function $f_i$ be an r which is a function of $B_i$ to $B_j$:

$$f_i: B_i \to B_{i+1}. \quad \ldots (5)$$

Then the composable triples, the composition of $f_i$, $f_{i+1}$, and $f_{i+2}$, $i \leq 1$, is associative. A function $i_d$,

$$i_d: B_i \to B_i \quad \ldots (6)$$

can also be defined, and it is the identity. Thus defining as above, the filter base of an experience sequence is a category. Q.E.D.

As is described in Reference[5], a filter denotes the typicality or representativity as the extensive idea for typicality. Together with the above proposition, typicality denotes a category.

## Conclusion

Described here is the categorization methodology in a hierachical structure for figures. As is also described above, a concept has the aspects of both verbal and imagerial levels. Hence, the ideas here are not only for figures but also for words in an indirect sense. A filter base is used in evaluating objects using concepts experienced in the past. And the representativity or typicality is understood in the framework of category.

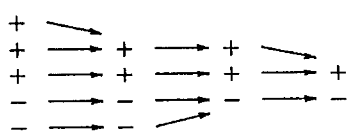

Fig.5a

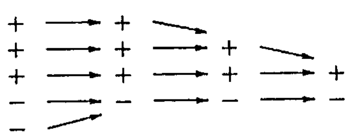

Fig.5b

Fig.5 Starting with the same category always gets the same simplified category independent of the course.

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