'93 KACC (1993. 10. 20~22)

# A New Unbalance Compensation Methoa for Magnetically Supported Rotor

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**Absrtract**

In this paper we propose two feedfroward unbalance compensation algorithms, they accommodate changes of rotor dynamics including rotating speed. The first one determine the compensating signals by identifying system dynamics successively. Whereas, the second one is more primitive like PID algorithm without identifying system dynamics.

## 1 Introduction

It is a well known fact that magnetic bearings provide some unique features enabling them to realize some desired rotor system characteristics [1]. These are absence of mechanical wear and lublicants and adjustability of rotor dynamics. The magnetic bearing system as a dynamic system is subjected to disturbances such as system noise, aerodynamic or hydrodynamic forces and unbalance force. The unbalance force originates from the rotor's residual mass unbalance, which in practice can never be avoided. An important feature of unbalance is the fact that they manifest themselves as synchronous rotating force vectors. Control of vibration caused by unbalance mass is one of important topics related with magnetic bearings. Already many papers about unbalance compensation were presented, some uses model based-observer or notch filter. Interesting studies are those which use feedforward unbalance compensation [2]-[8]. The basic characteristic of the feedforward compensation is embodied in the fact that the feedforward controller is not taken up in the loop of the system, hence the rotor dynamic is not affected by the feedforward compensation. For feedforward compensation, signals are harmonic and synchronous to the rotational frequency of the rotor and the gain and phase of the harmonic signals are adjusted to suppress the vibration.

While these compensation methods proposed by the studies above show good control characteristic, the operating condition which these compensation method is applicable is limited to the case of steadily rotating at some specified speed. However, there exist some cases where the rotor rotates at variable speeds and also the case where unbalance mass changes its position and quantity suddenly. Therefore, compensation methods which are robust to changes of the rotor dynamics are desired.

In this paper we propose two feedfroward unbalance compensation algorithms, they accommodate changes of rotor dynamics including rotating speed. The first one determine the compensating signals by identifying system dynamics successively. Whereas, the second one is more primitive like PID algorithm without identifying system dynamics.

We implemented these two unbalance compensation algorithm on a experimental setup to evaluate its performance of suppressing rotor vibration induced by synchronous disturbances.

There are some criteria adopted for unbalance compensation, like minimization of the rotor displacement, minimization of the magnetic force. In this paper the criteria to pursue minimization of the rotor displacement is considered.

## 2 Experimental Setup

For the experiments, a setup, schematically shown in Fig.1,was available, consisting of rigid rotor suspended in two radial magnetic bearings, a direct drive rotor, eight-channel switched amplifiers, four eddy current displacement sensors, eight current sensors and a digital signal processor (TMS320C30 ).

The 9.6 kg weighing rotor, made of aluminum, is 78.6 mm in diameter and has length of 955 mm. Furthermore the rotor is laminated with soft-magnetic sheets and two sensor rings. A power of 0.75kW can be delivered by the direct-drive motor which is located symmetrical with respect to both radial magnetic bearings. The radial magnetic bearings have the following measure: outer and inner diameters are 164 mm and 80 mm, the width is 42 mm and the air gap is 0.7 mm. Eddy current sensors serve as displacement sensors and are no-collocated with the magnetic bearings. In order to detect the rotating angle of the rotor, an optical sensor is employed. A personal computer ( IBM-PC/AT ) features as the host computer.

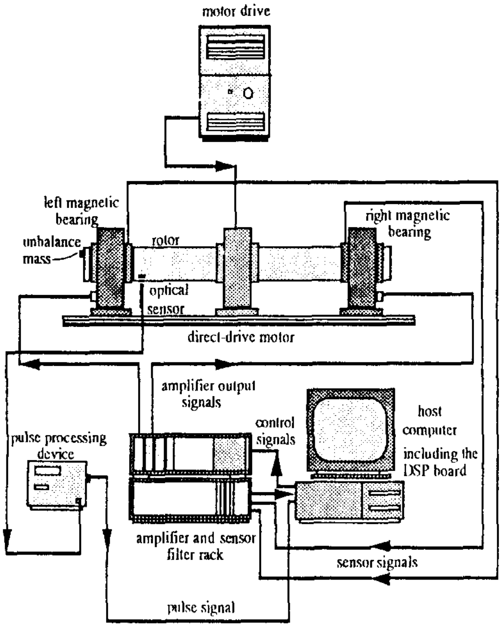


Fig. 1 Experimental Setup

## 3 . Unbalance Compensation Algorithm with Successive Identification

### 3.1 Derivation of Iterative Compensating Algorithm

Fig.2 illustrates on the basis of a block diagram the control underlying the applied feedforward compensation for our experiments. If the synchronous disturbance caused by the rotor unbalance doesn't exist, PD controller $G_c(j\omega)$ is enough to stabilize the rotor. In order to compensate the effects of the synchronous disturbance, sinusoidal compensation signal is imposed here.

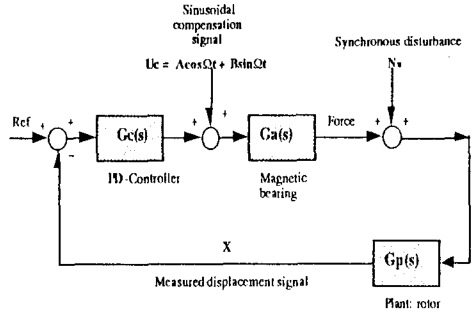


Fig. 2 Bolock Diagram of the feedforward commpensation scheme

The gain and phase of the sinusoidal compensation signal, which corresponds A and B parameters in Fig.2, are determined by identifying rotor dynamics and unbalance mass.

From Fig.2 a relation between the compensation signal U and the displacement signal X can be derived in the frequency domain by

$$X(j\omega) = G_1(j\omega)U(j\omega) + G_2(j\omega) \tag{1}$$

where $\omega$ is rotating frequency of the rotor, and $G_1(j\omega)$ and $G_2(j\omega)$ are frequency transfer function.

In the above equation we can define X and U as vectors, but in the followings we consider X and U as scalar complex variables in order to make the following explanation more understandable. Since signal $X(j\omega)$ can be measured and $U(j\omega)$ is known, $G_1(j\omega)$ and $G_2(j\omega)$ can be determined after imposing on the system two different compensation signals $U_1(j\omega)$ and $U_2(j\omega)$ as follows.

$$G_1(j\omega) = \frac{X_2(j\omega) - X_1(j\omega)}{U_2(j\omega) - U_1(j\omega)} \tag{2}$$

$$G_2(j\omega) = \frac{X_1U_2(j\omega) - U_1X_2(j\omega)}{U_2(j\omega) - U_1(j\omega)} \tag{3}$$

where $X_1$ and $X_2$ are displacement signals when $U_2(j\omega)$ and $U_1(j\omega)$ are imposed on the system respectively. Control signal $U_3$ that renders the displacement signal $X_3$ to zero is determined by

$$U_3 = U_2 - \frac{(U_2 - U_1)X_2}{X_2 - X_1} \tag{4}$$

This equation means that if previous two consecutive signals and the resultant displacement signals are known, control signal to suppress unbalance vibration can be suppressed. It is readily understandable that actual unbalance vibrations needs to iterate the above procedure considering the effects of disturbances and fluctuations of rotor dynamics. Therefore, control signals to suppress the unbalance vibration are determined by the following iterative formula.

$$U_{k+1} = U_K - \frac{X_k}{G_1} \tag{6}$$

$$G_1(j\omega) = \frac{X_2(j\omega) - X_1(j\omega)}{U_2(j\omega) - U_1(j\omega)} \tag{7}$$

It should be noticed that the above formula is obtained under the condition that Eq.(1 ) is satisfied. Therefore, once the compensating signal is imposed on the system, the resultant displacement signal should be measured after some while.

It is interesting to note that after some iteration of Eq.(6) , (7). The transfer function $G_1(j\omega)$ becomes almost constant value. Therefore, as one practical unbalance compensation algorithm we can employ iterative formula Eq.(6), (7) unless the following two conditions are satisfied.

(1) The frequency transfer function converged to some constant value.

(2) The displacement of the rotor is sufficiently suppressed.

And we employ iterative formula Eq.(6) if the both conditions are satisfied.

In the above the variation of the rotating frequency is not considered.

The above results are readily extended to the case with the variation of the rotating frequency. If the variation of the rotating frequency is prominent, Eq.(1 ) can be represented by

$$X(j\omega) = G_1(j\omega)U(j\omega) + \omega^2 G_2(j\omega) \tag{8}$$

where the second term on the right-hand side term is represented with $\omega^2$. We considering the external force caused by the unbalance mass.

Two consecutive equations under different rotating frequency $\omega_1$, $\omega_2$ becomes

$$X_1(j\omega_1) = G_1(j\omega_1)U(j\omega_1) + \omega_1^2 G_2(j\omega_1) \tag{9}$$

$$X_2(j\omega_2) = G_1(j\omega_2)U(j\omega_1) + \omega_2^2 G_2(j\omega_2) \tag{10}$$

Here we assume that $G_1(j\omega)$ and $G_2(j\omega)$ are almost constant in the frequency domain between ($\omega_1$ $\omega_2$). Then the following iterative algorithm is obtained.

$$U_{k+1} = \left(U_K - \frac{X_k}{G_1}\right)\left(\frac{\omega_{k+1}}{\omega_k}\right)^2 \tag{11}$$

$$G_1 = \frac{X_2 - X_1\left(\frac{\omega_1}{\omega_2}\right)^2}{U_2 - U_1\left(\frac{\omega_1}{\omega_2}\right)^2} \tag{12}$$

### 3.2 Experimental results 1

We applied the proposed algorithm to our experimental setup.

Unbalance Compensation was executed on the left side magnetic bearing.

Renewals of control signals were executed at every two seconds. And the frequency response $X(j\omega)$ was obtained with DFT ( Digital Fourier Transform) algorithm. The unbalance mass was inherent one on the test rotor.

Fig. 3 shows that the unbalance vibration of rotor was suppressed sufficiently small , the displacement of the rotor was less that 20 micrometers.

Without the compensating signals, the amplitude of unbalance vibration was mm. Fig. shows that how well the unbalance vibration is suppressed at every rotating frequency under the variation of rotating frequency. The rotating frequency was increased from rpm until rpm in seconds. It can be recognized that the unbalance vibration is suppressed at every rotating frequency but the performance was not satisfactory.

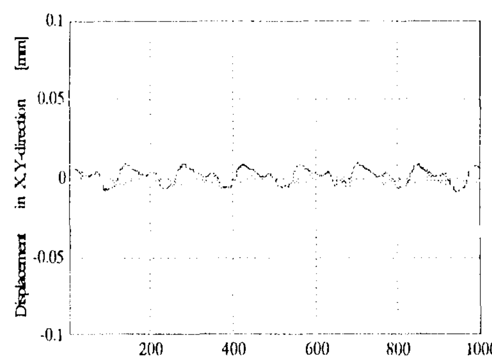


Fig. 3 Displacement of Rotor with Compensation at a Rotational speed of 89 rad / sec

We also checked if the iterative formula of Eq.(6 ) works at faster iterating speed. First we renewed the compensating signal at every two seconds. After the frequency transfer function was converged, we renewed the compensating signal at every rotation of the rotor without renewing the frequency transfer function.

Fig. 4 shows how fast the unbalance mass could be compensated by our algorithm. In this figure a virtual unbalance mass was imposed on the rotor by the magnetic bearings in order to simulate the turbine blade loss. A transient vibration occurred, but the vibration was suppressed after three rotation of the rotor.

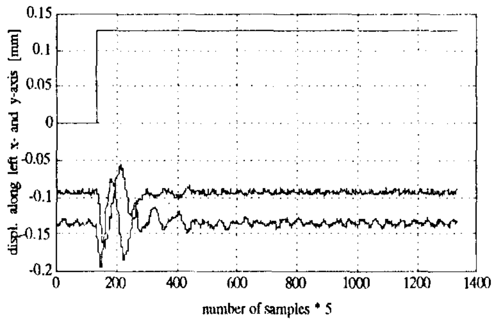


Fig.4 Rotor Response Directly after applying a sudden sinusoidal disturbance at a Rotational speed of 120 rad /sec

## 4 Unbalance compensation without Identification.

### 4.1 Concept of Unbalance Compensation without Identification.

The vibration caused by the unbalance on the rotor is sinusoidal. But if we measured unbalance vibrations of the rotor on the coordinates ( $x_r$, $y_r$ )which synchronously rotates with the rotor, the unbalance vibration is similar to the step response of the two dimensional system as shown in Fig. 5

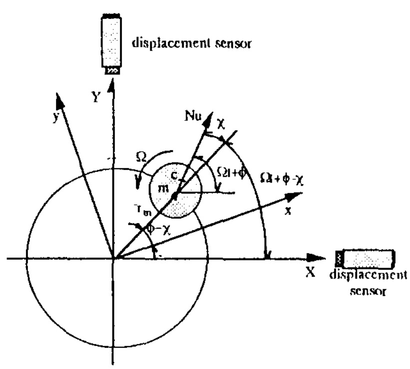


Fig. 5 Rotating Coordinate

It is well known that conventional PI algorithm ( Proportional Integral) and PID algorithm is powerful tool to control systems with only a few knowledge about controlled system. Furthermore, one important feature of PI and PID control is diminishing the offset error. Therefore, one algorithm to compensate the unbalance vibration can be

1) Measure the coordinates $x_r$, $y_r$ of the rotating rotor.
2) Determine the control signals $U_{xr}$ and $U_{yr}$ along $x_r$ axis and $y_r$ axis using a conventional PI algorithm. One example is

$$U_{xr} = -K_p x_r(k) - K_i \sum_{n=0}^{k} x_r(n) \tag{13}$$

$$U_{yr} = -K_p y_r(k) - K_i \sum_{n=0}^{k} y_r(n) \tag{14}$$

3) The control signals $U_{xr}$ and $U_{yr}$ should be rotated with the rotating rotor.

As a result, the magnetic bearing generates synchronously rotating force. The rotating force can be easily generated by imposing sinusoidal control signals on the magnet bearings.

While this algorithm is not difficult to implement on the magnet bearing system using DSP, some considerations are needed. One important thing to consider is how often to renew the above control signals. If the control signals are renewed fast, the rotor dynamic is effected. Therefore, one reasonable method is to renew the control signals relatively slow so that PI controller and unbalance compensator can be designed independently.

In the followings we propose one algorithm to renew the compensating signals $U_{xr}$ and $U_{yr}$ per several rotations.

It is interesting to note that average values of $x_r$ and $y_r$ during one rotation of the rotor can be calculated by the Fourier Transformation of the displacement data X and Y measured by the displacement sensor.

This fact is understandable by considering the following example where sensor signals are obtained by

$$X(t) = x_r\cos(\omega t) + y_r\sin(\omega t) \tag{15}$$

$$Y(t) = x_r\sin(\omega t) - y_r\cos(\omega t) \tag{16}$$

The above equation denotes a relation between the rotor displacements ($x_r$, $y_r$) on the rotating coordinates and the sensor signals (X(t),Y(t)). The rotor displacement are obtained by the following Discrete Fourier Transform algorithm

$$x_r = \frac{1}{2N} \sum X_k\cos(\omega t_k) \tag{17}$$

$$y_r = \frac{1}{2N} \sum Y_k\sin(\omega t_k) \tag{18}$$

where one period during one rotation is discretized into N discrete time.

The following is the proposed algorithm which is executed at every several rotations:

1) Measure displacement signals $X_k$ and $Y_k$ during several rotations.
2) Obtain the average displacement signals ($x_r$, $y_r$) in the rotating coordinates by Eq.( 13 ), (14).
3) Renew the control signals on the rotating axis by Eq.( 17 ), (18).
4) Impose the control signals on the magnetic bearings by the following equation

$$U_X = U_{xr}\cos(\omega t + \phi) + U_{yr}\sin(\omega t + \phi) \quad (19)$$
$$U_Y = U_{xr}\sin(\omega t + \phi) + U_{yr}\cos(\omega t + \phi) \quad (20)$$

where f is design parameter to improve stability of this algorithm.

This algorithm has three design parameters to effect the stability of this algorithm.

## 4.2 Convergence of the second algorithm

The convergence of the proposed algorithm should be considered.

Here we assume that renewal of control signal is executed slow enough that the steady vibration is obtained. Furthermore, only the control algorithm with integral term is considered in order to make the analysis easier.

A simplified model of magnetic bearings is .

$$\ddot{x} + c\dot{x} + kx = au + A\cos\omega t + B\sin\omega t \quad (21)$$
$$\ddot{y} + c\dot{y} + ky = av + A\sin\omega t - B\cos\omega t \quad (22)$$

where x and y are displacements in the x and y coordinates respectively, and u and v are control signals and terms with coefficients A or B are disturbances caused by unbalance mass.

Introducing complex variables Z defined by x+yi, the above equation becomes

$$\ddot{z} + c\dot{z} + kz = aw + Dc^{-i\omega t} \quad (23)$$

After some algebraic manipulation we obtain

$$z_{k+1} = (\mathbf{I} - \mathbf{K}G(j\omega))z_k \quad (24)$$
$$\mathbf{K} = K_i c^{-i\phi} \quad (25)$$

Therefore, the convergency condition is that all eigenvalues of $\mathbf{I} - \mathbf{K}G(j\omega)$ are inside of the unit circle on the complex plane.

The above condition mentions that how to adjust control parameters $K_i$ and $\Phi$.

## 4.3 Experimental Results of the Second Algorithm

We applied our second algorithm on the magnetic bearings mentioned in Chap. 2.

In Fig. 6 displacement data in the x and y directons are shown where unbalance was compensated with the second algorithm. In these experiments, control signals are renewed every one rotation. In order to test the applicability under the speed variation, the rotating speed is increased as shown in Fig. 7 , where rotating speed is increased from 10 rot/sec until 90 rot/sec in 10 seconds. Vibrations cause by this speed increase are shown in Fig. 8.

From these figures, we can concluide that the second compensating algorithm yields a superior performance with respect robustness.

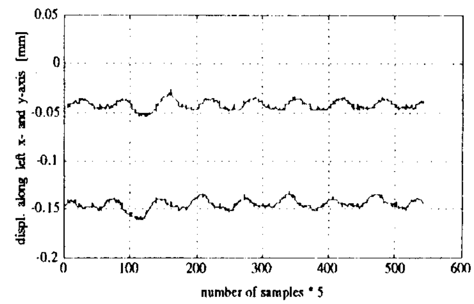


Fig. 6 Displasement of the Rotor at a Rotational speed of 847 rad / sec

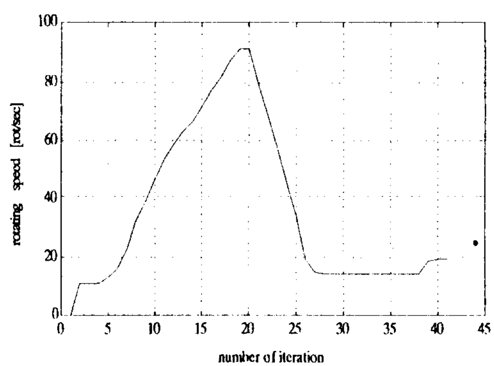


Fig. 7 Variation of the Rotating Speed

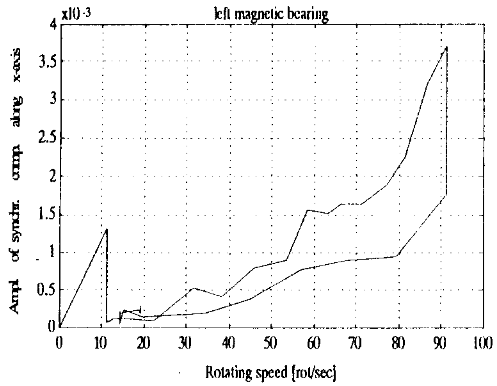


Fig. 8 Amplitude of the Displacement Signal under the Various Rotating Speed

## 5 Conclusion

In this paper two kinds of feedforward compensation methods, as an alternative control method to the feedback method, are proposed to attenuate the synchronous unbalance response of magnetically suspended rotors. The feedforward compensation signals are harmonic and synchronous to the rotational frequency of the rotor, and applied in an open-loop fashion to the feedback control currents.

The first algorithm proposed is a method with succesive identifications. Our experiments show that the first adaptive algorithm is capable to converge to the optimal compensation signal in one to two iterations.

The issue of blade losses is also examined as the resulting sudden change in the mass unbalance distribution may gives rise to intolerable rotor vibration and structural damage to the rotor system. It is found that an on-line identification of the transfer function between the compensation signal and the measuerd signal may remedy this problem.

In addition we propose the second promising novel feedback control based unbalance compensation method. The principle of this method is grounded on the consideration of two different coordinate systems: the coordinate system fixed to the sensors and a coordinate system rotating synchronously with the rotor. The feedback control consists of independent Proportional Integral controllers for all four axes. Our method shares some of the advantages of the feedforward concept of which the superfluity of any dynamic model of the AMB system is most attractive. In comparison with the feedforward approach the presented method is more robust against (very) fast speed changes. Rotational speeds up to 9000 rpm have been succesfully reached.

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