Adaptive Control of Gas Metal Arc Welding Process

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Abstract

Since the welding process is complex and highly nonlinear, it is very difficult to accurately model the process for real-time control. In this paper, a discrete-time transfer function matrix model for gas metal arc welding process is proposed. Although this linearized model is valid only around the operating point of interest, the adaptation mechanism employed in the control system render this model useful over a wide operating range. A multivariable one-step-ahead adaptive control strategy combined with a recursive least-squares method for on-line parameter estimation is implemented in order to achieve the desired weld bead geometries. Command following and disturbance rejection properties of the adaptive control system for both SISO and MIMO cases are investigated by simulation and experiment.

1. Introduction

Welding processes have been developed into an automated operation over the past decades. Although control systems for the welding torch motion (e.g., joint tracking and robotic welding) are now commercially available, process control systems have not been fully developed for many reasons such as complexity of the welding process and lack of reliable sensors. However, control of the welding process itself is also very important so that full automation of the process and, possibly, unmanned operation may be achieved.

Gas Metal Arc Welding (GMAW) is a complex, multi-energy domain process that is essential to many types of manufacturing. Weld quality features such as final metallurgy and joint mechanics are typically not measurable on-line for control; thus, some indirect way of controlling the weld quality is necessary. A comprehensive approach to in-process control of welding includes both geometric features of the bead (such as the cross section features width, depth and height) and thermal characteristics (such as the heat-affected zone width and cooling rate). The definitions of these features are illustrated in Fig. 1.

The nonlinearity of welding has been confirmed by several process modeling investigations. For Gas Tungsten Arc Welding (GTAW) process, Hardt et al. [1] developed a simple non-stationary first-order model, which showed both analytically and experimentally that the process parameters

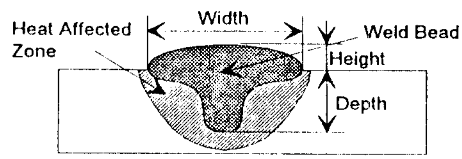

Fig. 1 Cross-section of weldment and definitions of weld bead geometries

were dependent both on the operating point of the process and on the boundary conditions. A transfer function matrix model for GMAW relating wire feedrate and travel speed to bead geometries, bead width and height was developed in [2]. In this case the nonlinearity was explicitly related to the inputs.

Recently, some researchers [3] modeled the welding process based on the artificial neural network (ANN). If enough training data are given, the ANN model can capture the process characteristics including nonlinearities and parameter couplings over large operating ranges. However, the ANN model requires a large amount of I/O data to accurately train the network and cannot describe the physics of the process.

Even though much research has been done for the modeling and analysis of welding processes, research on real-time welding control is a relatively recent development, mainly because of lack of reliable sensors for feedback control. Most of the early research was based on single-input, single-output nonadaptive control schemes. Recently, with the development of robust control algorithm, computers and sensors, more sophisticated control schemes (e.g., adaptive control, artificial neural network [3, 4], fuzzy control [5]) have been employed to improve performance of the welding control system.

One of the first adaptive control applications to welding was demonstrated in [6]. A Model Reference Adaptive Control (MRAC) scheme was used to regulate the back bead temperature of GMAW using travel speed as an input. Recently, an adaptive control systems for the full penetration GTAW problem was developed in [7]. In this case, both MRAC and Self Tuning Control (STC) were shown to be very effective in responding to both uncertain nominal parameters and severe parameter disturbances. The MIMO case of thermal control by using a deadbeat adaptive algorithm with parameter adjustment based on a projection algorithm for parameter identification was addressed in [8]. Hale and Hardt

[9] dealt with the nonlinearities of GMAW geometry control problem using a scheduled gain approach.

In this research, the transfer function matrix model for GMAW process addressed in [2] is further extended to include more accurate dynamic characteristics. In order to consider nonlinear and multivariable nature of the process, an adaptive control system with on-line parameter estimation is developed and tested for both SISO and MIMO cases. Some guidelines for designing this adaptive control system are also suggested.

2. Process Modeling for Control

Among many possible outputs shown in Fig. 1, bead geometries, width and depth, are chosen as process outputs in this research. Because the process inputs must be able to regulate the chosen outputs, wire feedrate (f) and travel speed of the torch (v) (i.e., torch velocity) are selected as inputs. Even though these two inputs have similar effects on the outputs and cause strong coupling of I/O pairs, they are considered the best choices under the current configuration of GMAW setup. Note that the wire feedrate regulates the welding current, and thus the heat input into the weld.

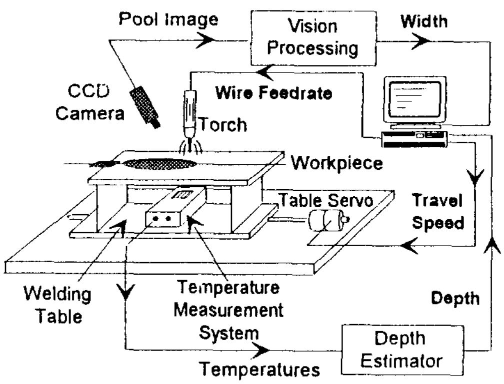

Fig. 2 Schematic for welding setup and measurement devices

In order to investigate the dynamics of the process, a series of open-loop welding experiments were performed using the welding setup in Fig. 2. The welding table with the workpiece clamped to it was set up to move in the welding direction, while the torch and measurement devices remain stationary. The experiments were conducted based on a bead-on-plate butt welding. The primary metals were low carbon steels which are 6.35 mm (1/4 in) thick. An Ar + 2% oxygen gas mixture was used as a shielding gas for this experiment.

The weld bead depth, which is the key geometric attribute of a major class of welds, is very difficult to directly measure, but a robust method to estimate the depth using temperature measurement was developed in [10, 11]. The bead width is measured on-line by real-time video image analysis [11].

Extensive open-loop step tests have shown that the GMAW process can be adequately modeled by the second-order dynamics [2]. Since the process is nonlinear, a locally linearized process model around the operating point is obtained. Based on these observations, the following discrete-time second-order transfer function matrix (TFM) model has been developed:

$$\begin{Bmatrix} y_1(z) \\ y_2(z) \end{Bmatrix} = \begin{bmatrix} \dfrac{\left(b_{11}+b_{12}\,z^{-1}\right) z^{-d_{11}}}{1+a_{11}\,z^{-1}+a_{12}\,z^{-2}} & \dfrac{\left(b_{13}+b_{14}\,z^{-1}\right) z^{-d_{12}}}{1+a_{11}\,z^{-1}+a_{12}\,z^{-2}} \\ \dfrac{\left(b_{11}+b_{12}z^{-1}\right) z^{-d_{21}}}{1+a_{21}\,z^{-1}+a_{22}\,z^{-2}} & \dfrac{\left(b_{23}+b_{24}\,z^{-1}\right) z^{-d_{22}}}{1+a_{21}\,z^{-1}+a_{22}\,z^{-2}} \end{bmatrix} \begin{Bmatrix} u_1(z) \\ u_2(z) \end{Bmatrix} \tag{1}$$

where $y_1(z)$ and $y_2(z)$ are the width and depth, $u_1(z)$ and $u_2(z)$ are the wire feedrate (f) and inverse travel speed or inverse velocity (v^{-1}), and d_{ij} is the time delay between the ith output and jth input. Note that the inverse travel speed is used instead of the travel speed in Eqn (1). This is because the leading coefficients b_{13} and b_{23} corresponding to $u_2(z)$ will have positive sign with the inverse travel speed as input; both width and depth increase as the inverse travel speed increases (i.e., the travel speed decreases).

It is noted that each output has been modeled to have common denominator dynamics; two process inputs have similar effects on each output from a physical point of view because both inputs are regulating the outputs through the amount of the heat input to the weldment. This common dynamics also simplifies the subsequent analysis and design; if each I/O pair had different second-order dynamics, the resulting transfer function matrix would include a fourth-order polynomial.

Although the dynamics are assumed to be common to each output, the gains (b's) are modeled to be different for each I/O pair in Eqn (1). . Another important point is that each numerator polynomial should have at least two coefficients in order to compensate for a fractional time delay which cannot be represented by $z^{-d_{ij}}$, where d_{ij} is an integer.

3. Adaptive Control Algorithm

Since welding is inherently a nonlinear, simple nonadaptive feedback control schemes cannot be successfully applied. Nonlinearities observed in the process significantly change the process parameters (a's and b's) depending on the operating points. These factors necessitate the use of adaptation (e.g., on-line parameter estimation) in the controller design.

One of multivariable adaptive control algorithms developed for discrete-time system [12, 13] is a multivariable one-step-ahead adaptive control algorithm (also referred to deadbeat adaptive algorithm in some literature). This algorithm is applicable to discrete-time multi-input, multi-output (MIMO) deterministic linear systems. It has a simple structure but guarantees global and asymptotical stability of an output tracking system with bounded input sequences. Since it is based on a transfer function model, it requires only information on process inputs and outputs for feedback without an explicit state space description. This feature is suitable for control of the welding process because some of the state variables in the state-space representation are not physical quantities and, thus, can only be observed at best.

3.1 Generalized One-Step-Ahead Control

A general MIMO linear discrete-time system with m inputs and m outputs can be described by

$$\mathbf{A}(q^{-1})\,\mathbf{y}(k) = \mathbf{B}(q^{-1})\,\mathbf{u}(k) \tag{2}$$

where $\mathbf{y}(k)$ and $\mathbf{u}(k)$ are output and control vectors, respectively. In general, there exist time delays between I/O pairs (i.e., $z^{-d_{ij}}$ in Eqn (1)). Since each delay is generally different from each other, a new output-dependent delay is defined as follows:

$$d_i = \min_{1 \le j \le m} d_{ij} \qquad i = 1, \cdots, m \tag{3}$$

If we define $\mathbf{D}(q) = \text{diag}[q^{d_1}, ..., q^{d_m}]$ as a delay matrix, then $\mathbf{B}(q^{-1})$ of Eqn (2) can be represented as

$$\mathbf{B}(q^{-1}) = \mathbf{D}^{-1}(q)\left(\mathbf{B}_0 + \mathbf{B}_1 q^{-1} + \cdots\right) = \mathbf{D}^{-1}(q)\,\mathbf{B}'(q^{-1}) \tag{4}$$

Note that $\mathbf{B}_0$ is the matrix consisting of the leading coefficients. With this output-dependent delay, the *predictor form* can be given by

$$\bar{\mathbf{y}}(k) = \alpha(q^{-1})\,\mathbf{y}(k) + \beta(q^{-1})\,\mathbf{u}(k) \tag{5}$$

where $\bar{\mathbf{y}}(k)$ is the future output and

$$\alpha(q^{-1}) = \alpha_0 + \alpha_1 q^{-1} + \alpha_2 q^{-2} + \cdots$$
$$\beta(q^{-1}) = \beta_0 + \beta_1 q^{-1} + \cdots = \beta_0 + q^{-1}\beta'(q^{-1}) \tag{6}$$

This predictor form enables the future outputs to be expressed in terms of the outputs and inputs up to time k (i.e., current time).

Consider the following cost function

$$\begin{aligned} J(k) = &\{\bar{\mathbf{y}}(k) - \bar{\mathbf{y}}^*(k)\}^T \{\bar{\mathbf{y}}(k) - \bar{\mathbf{y}}^*(k)\} + \\ &\{\mathbf{u}(k) - \mathbf{u}(k-1)\}^T \mathbf{R}\, \{\mathbf{u}(k) - \mathbf{u}(k-1)\} \end{aligned} \tag{7}$$

where $\mathbf{R}$ is a control weighting matrix which is positive definite. The first term in Eqn (7) represents the cost due to tracking errors, while the second term represents the cost due to large control signals. With the first term alone, the control system can achieve the desired output in just one step, but it may produce an excessively large control signal and sometimes cause instability. In addition, in the subsequent closed-loop control system with the first term alone, the closed-loop poles tend to cancel process zeros and thus it cannot be used for nonminimum-phase process. The matrix $\mathbf{R}$ in the second term has been introduced to overcome these problems [13]. Minimizing the cost function with respect to the control $\mathbf{u}(k)$ yields the following *generalized one-step-ahead control* law:

$$\begin{aligned} \mathbf{u}(k) = &\left[\beta_0{}^2 + \mathbf{R}\right]^{-1} \cdot \left\{\beta_0 \left\{\bar{\mathbf{y}}^*(k) - \alpha(q^{-1})\mathbf{y}(k) \right.\right. \\ &\left.\left. -\beta'(q^{-1})\,\mathbf{u}(k-1)\right\} + \mathbf{R}\;\mathbf{u}(k-1)\right\} \end{aligned} \tag{8}$$

where $\bar{\mathbf{y}}^*(k)$ is the desired future output vector, which is assumed to be known in advance and bounded for all time. On the other hand, the closed-loop control system can be derived from the cost function as follows:

$$\begin{aligned} &\left[\mathbf{B}'(q^{-1}) + (1 - q^{-1})\mathbf{B}'(q^{-1})\beta_0{}^{-1}\mathbf{R} \cdot \right. \\ &\qquad \left. \mathbf{B}'(q^{-1})^{-1}\mathbf{A}(q^{-1})\right] \bar{\mathbf{y}}(k) = \mathbf{B}'(q^{-1})\,\bar{\mathbf{y}}^*(k) \end{aligned} \tag{9}$$

where $\mathbf{B}'(q^{-1})$ is defined in Eqn (4).

3.2 Parameter Estimation

As mentioned earlier, the parameters of the welding process are changing depending on both the operating conditions and the past thermal history. Thus, the parameters cannot be determined accurately before the welding process and on-line parameter estimation is required to achieve consistent control. Among numerous on-line parameter estimation methods, a recursive least-squares (RLS) algorithm was selected. The forgetting factor was used in the RLS method. With this forgetting factor, the RLS method works more effectively for the system with time-varying parameters by discarding the old data. On the other hand, the data collected from the experiments are usually contaminated by high frequency noise. Low pass filtering of the data prior to parameter estimation improves performance since the welding process is inherently slow and thus the low frequency portion is more important. The same low pass filters must be used for both input and output signals.

3.3 Multivariable One-step-Ahead Adaptive Control

The multivariable generalized one-step-ahead control law combined with the recursive least-squares parameter estimation scheme constitutes the multivariable generalized one-step-ahead adaptive control algorithm. The convergence and stability properties were proved in [12].

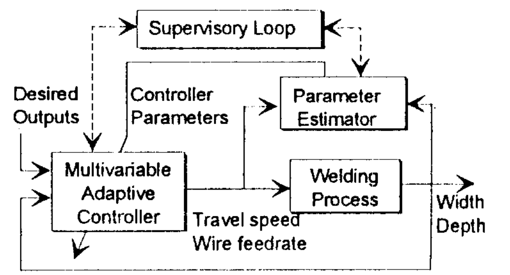

Fig. 3 Multivariable adaptive control (Direct approach)

In this paper, only direct approach where the controller parameters are estimated directly by reparameterizing the process model in advance is considered. Figure 3 shows the block diagram for the direct approach, where the parameter estimator identifies the controller parameters from the I/O data every sampling period. A supervisory loop primarily monitors the parameter estimator, improves its performance and prevents its malfunction.

3.4 Guidelines for Control Design

Since the closed-loop poles tend to cancel the process zeros in the one-step-ahead (OSA) control scheme without control weights, the locations of the transmission zeros are very important to the stability of the control system in that case. This is no longer true in the generalized OSA control law with control weights where the closed-loop poles can be chosen arbitrarily to some extent, but *a priori* information on the transmission zeros is still important to the controller design because the values of the control weighting factors may be determined by the transmission zeros. In general, the following guidelines can be used in determining control weighting factors.

a) no or small control weights for well-damped zeros (i.e., zeros well inside the unit circle),
b) relatively large control weights for poorly-damped zeros (i.e., zeros inside but near the unit circle),

Another important point in determining control weights is their effect on the closed-loop poles. As shown in the left hand side of Eqn (9), the closed-loop poles depend on the choice of the control weighting matrix **R**; therefore, the closed-loop stability should be the first consideration when selecting **R**.

Since the system under consideration is square (i.e., the same number of inputs and outputs), the transmission zeros and closed-loop poles can be computed by taking the determinant of each side of Eqn (9); the transmission zeros are found as the roots of $\det \mathbf{B}'(q^{-1}) = 0$ and the closed-loop poles are computed as the roots of $\det[\mathbf{B}'(q^{-1}) + \ldots \mathbf{A}(q^{-1})] = 0$.

However, for the complex system, selecting **R** analytically is often difficult because of computational complexity. Therefore, proper values of the control weighting factors are often found by simulation or experiment by trial and error.

4. Control Experiments

In order to evaluate the performance of the adaptive control system, a series of closed-loop control experiments were conducted. Both SISO and MIMO control systems are developed and implemented.

4.1 SISO Cases

There are four combinations of the input-output pairs: feedrate-width, feedrate-depth, travel speed-width, and travel speed-depth. Although all four combinations are equally important since each output must be controllable from each input in a multivariable control system, only a few results are shown here.

Choice of Control Weighting Factors: The importance of the control weighting factor to the generalized one-step-ahead (OSA) adaptive control was mentioned previously and is experimentally verified in Fig. 4. Figure 4(a) plots the depth

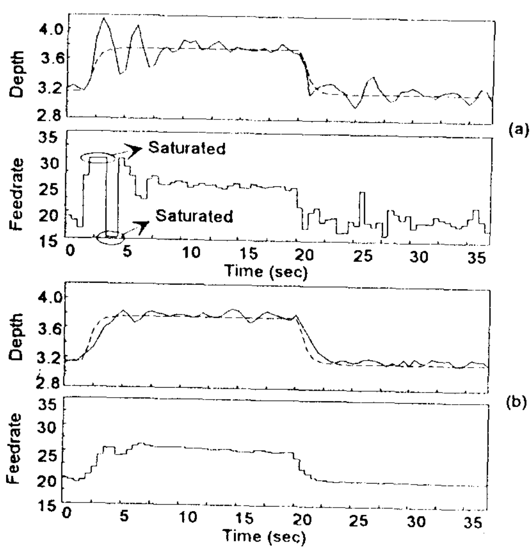

Fig. 4 Depth responses of OSA adaptive controller for (a) no control weight, and (b) control weight = 0.5 (Input; feedrate (cm/s), Output; depth (mm), $T_s = 0.5$ sec)

responses when the process is subjected to step changes in reference command without a control weight. The feedrate was used as the control input, while the travel speed was kept constant. When no control weight was given, the response shows a large overshoot and a undershoot immediately after step changes in a reference command. In the absence of control weight, the OSA control algorithm tries to achieve the desired output in a single step, which leads to a large variation in the control signal and an actuator saturation. In an attempt to reduce this large control effort, the control weighting factor of 0.5 was introduced as shown in Fig. 4(b); inputs and outputs are normalized so that they are of about the same magnitude (typically, ≈1). The overshoot observed in Fig. 4(a) is significantly reduced and the output maintains good command-following. Additionally, the control signal shows a reasonable variation.

Command Following: The command following performance by the adaptive controller is compared to two nonadaptive controllers in Fig. 5 for feedrate-width pair. Figure 5(b) shows the response of the digital PI controller in the following form:

$$u(z) = \left[K_P + \frac{K_I}{1 - z^{-1}}\right]e(z) \qquad (10)$$

where K_P and K_I are the proportional and integral gains, respectively, and $e(z)$ is the error signal defined as $y^*(z) - y(z)$. It is observed that the width response cannot follow the step change in reference command quickly enough, though the steady-state performance is relatively good. This slow response is because the PI controller is operated based on the error signal between the reference command and the measured output. Since there are time delays of two samples (delay of 1 sec) and a computational delay of one sample in the width control, the control action is always delayed by at least three samples. This delay can be slightly reduced by adjusting the gains (e.g., increasing the proportional gain K_P), but at the expense of a possible overshoot. When compared to the PI controller, the OSA adaptive controller shows much faster time response (Fig. 5(a)). This is because the OSA adaptive controller is based not on the error signal but on the predictor form which accounts for the time delays including a computational delay. In the width control case involving three samples of time delay, for example, the control signal at time

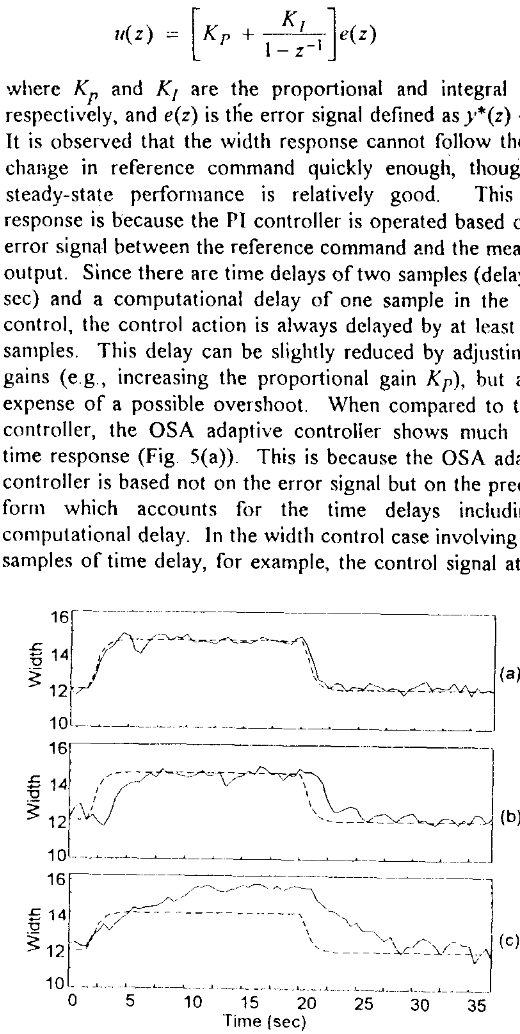

Fig. 5 Comparison of OSA adaptive controller to nonadaptive controllers (a) OSA adaptive controller, (b) PI controller, and (c) nonadaptive OSA controller (Input; feedrate (cm/sec), Output; width (mm), Travel speed = 7 mm/sec, $T_s = 0.5$ sec)

kT_s is commanded to track the reference input at time $(k+3)T_s$, thus leading to good tracking for the step change.

Figure 5(c) shows the performance of the *nonadaptive* generalized OSA controller. The parameters used for the controller were obtained from the operating condition corresponding to the portions before t = 2 sec and after t = 22 sec in Fig. 5(c). As a result, tracking performance for the portion between t = 2 and t = 22 sec was very poor. This indicates that the performance of the OSA controller depends strongly upon the accuracy of the parameters.

Disturbance Rejection: In arc welding, various forms of disturbances may exist (e.g., thickness change, change of material properties, parameter drifts of the welding machines, etc.). The disturbances need to be rejected in most cases since they adversely affect the performance of control systems. Figure 6 shows the disturbance rejection performance of the OSA adaptive controller for travel speed-width pair. Step changes in the feedrate were used as the disturbance. At the time t = 5 sec, the feedrate decreases from 22 to 18 cm/sec, thus resulting in a quick decrease in the width output. The control parameters then adapt to the new parameters; the controller decreases the travel speed to compensate for the decreased width. A similar trend is observed in the disturbance introduced at time t = 25 sec.

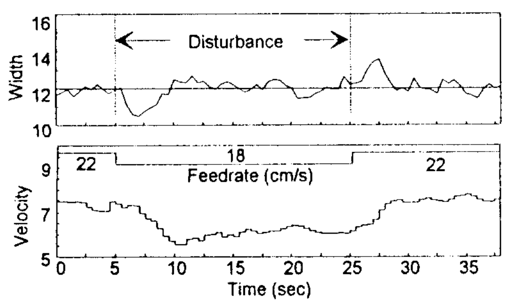

Fig. 6 Disturbance rejection of OSA adaptive controller (Input; travel speed (mm/s), Output; width (mm), Disturbance; feedrate)

4.2 MIMO Cases

The same control experiments as in the SISO case were extended to the two-input, two-output case. The control weighting factors are also important as in the SISO case. With no control weights, either unstable or highly oscillatory behavior was observed. In addition to the proper magnitudes of the control weights, the proper balance between two control weights are important as well in the MIMO case. Since the control weight tends to decrease the control signal, excessively large control weight reduces the corresponding control signal more than the other; as a result, the whole process is dominated by the other control signal, which is undesirable. In most cases, it is desirable to achieve equal participation from both control inputs, so that a good control performance may be accomplished. The proper level and balance can be found either by simulation or by experiment.

Command Following: Figure 7 shows the output responses and control inputs of direct adaptive controller. With proper control weights, both control signals show reasonable variations. Both output responses are shown to follow the reference commands reasonably well even with the step changes in the commands.

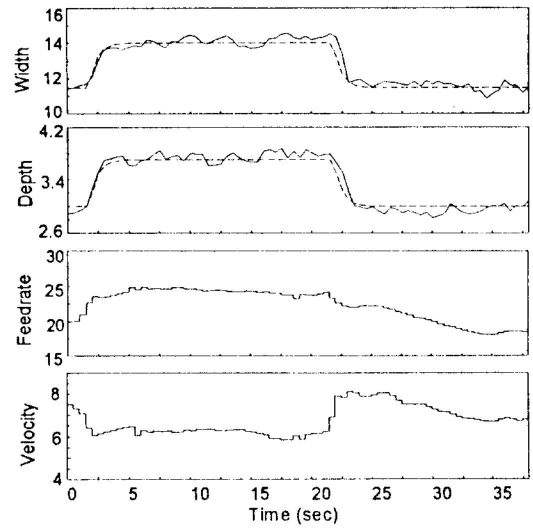

Fig. 7 Output responses and control inputs of multi-variable direct OSA adaptive controller

Control Range: In the MIMO case, the *control range* is also an important issue ([2, 9]). In gas metal arc welding process, two outputs (i.e., width and depth) are highly coupled partly because two inputs (i.e., feedrate and travel speed) have similar effects on each output. Therefore, there is a limit to the range which combinations of the two control inputs can achieve. For example, an increase in the width while decreasing or maintaining a constant depth cannot be attained with any combination of the control inputs. No control algorithm can overcome this inherent problem since the limitations are due to the welding process and machines used. Therefore, the desired bead geometries must be in the achievable control range. The adaptive controller cannot provide good command following performance when desired bead geometries deviate greatly from the achievable control range.

Disturbance Rejection: The output coupling may make the disturbance rejection difficult. If the disturbance affects one output much more than the other, then it is difficult to reject this disturbance. For example, a certain disturbance tends to increase the width with no change in the depth. Then, the control signals must decrease the width while maintaining the depth to achieve the desired bead geometries. As mentioned before, such a combination of the control inputs are hard to find because of the strong I/O coupling. However, if the disturbance affects both outputs and they are in the achievable control range, then the adaptive controller can reject it by adapting to new process parameters as in the SISO case.

5. Conclusions

It is, therefore, very difficult to obtain the process model accurately in advance and on-line parameter estimation and adaptive control algorithm are required. A one-step-ahead adaptive control strategy combined with a recursive least-squares method has been investigated. Command following and disturbance rejection properties of the adaptive control system for both SISO and MIMO cases are investigated by simulations and experiments. The adaptive control system shows reasonable performance on command following for both SISO and MIMO cases. This system also shows good disturbance rejection in the SISO case, but is shown to be of limited success in the MIMO case due to inherent lack of output decoupling in the GMAW process. Process modification to decouple the process outputs to a great extent is underway

Even though a truly independent control of the outputs is difficult to implement due to a strong output coupling inherently existing in the process, a control system for simultaneous control of bead width and depth was successfully implemented in this paper.

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