

## A Study on Optimal Trajectory Planning for a Dual Arm Robot

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### Abstract

In this paper, we design a time optimal controller of a dual arm robot to handle the object. Differently from the master-slave type robot, same priority is imposed on the both of the arms for effective handling the specified object. For finding a time optimal collision-free trajectory, a graphical method is applied for the robot with two degree of freedom. Some simulation results show the effectiveness of the proposed method.

### 1. Introduction

This paper proposes an algorithm on the collision free time optimal trajectory generation under some constraints imposed on the robot.

Recently, coordinated motion control of multiple arms of a robot has received increasing attention in recent years because of its potential application in assembly which requires complex motions or dexterity as well as handling large and heavy object beyond the capacity of a single manipulator. The major motivations for the use of the dual arm robot include its effectiveness to perform a specified job and its mechanical compactness that inherently results from its mechanical structure such as single controller and energy source. In the sense that multiple robots are appropriate for many tasks especially requiring complex motions or dexterity, control strategy for them have similar basis with that for multiple arm robot. Therefore, it is worth while to look back on the researches on both of those fields. Wang(1) proposed a simple strategy for the attitude control and arm coordination of a maneuverable space robot with dual arms. Shahinpoor(2) suggested a

near-optimal control algorithm for a highly nonlinear robot manipulator mode via a parameter sensitivity method and an optimal PD controller. Gilbert and Johnson(3) approached the collision-free optimal path planning problem by first expressing the collision avoidance. Shih(4) used an artificial potential field for path planning for two SCARA robot in the parameter space. Lee and Lee(5) dealt with collision free motion planning of two robots by classifying path requirement situations.

Considerable research has been done to find time-optimal trajectory for prespecified paths of each robot. Nevertheless, less effort has been devoted to find them for unspecified paths. Generally explicit solution for nonlinear system is unattainable. One way to make the problem tractable is to linearize the system. However, this method can't avoid the inherent error caused by the drastic approximation in its linearization.

This research focuses on finding time optimal trajectory for unspecified path of the tips of the both arms of the robot with graphical method. In spite of its exhaustive search for the optimal trajectories, this method provides the exact solution for the given system.

### 2. Mathematical Model

The dynamics equation of the robot under consideration can be derived using Lagrange's equation which takes the form of

$$(J_i + m_i l_i^2) \ddot{\theta}_i + \frac{1}{2} m_i g l_i \sin \theta_i = T_i \quad \text{for } i = 1, 2 \quad (1)$$

where  $J_i$ ,  $m_i$  and  $l_i$  represent the moment of inertia, mass and length of the  $i$ -th link of the robot.  $n$  is the number of degree of freedom of the system.

The vectors  $\theta = (\theta_1, \theta_2)^T$  and  $T = (\tau_1, \tau_2)^T$  are the joint angles and the applied actuator torques. The actuator's torque is constrained by the function of joint positions and velocities.

$$\tau_{i_{\min}}(\theta, \dot{\theta}) \leq \tau_i \leq \tau_{i_{\max}}(\theta, \dot{\theta}) \text{ for } i = 1, 2 \quad (2)$$

The feasible velocities and positions are also bounded by

$$\dot{\theta}_{i_{\min}} \leq \dot{\theta}_i \leq \dot{\theta}_{i_{\max}} \quad (3)$$

$$0 \leq \theta_i \leq \pi \quad (4)$$

### 3. Time Optimal Trajectory Planning

In current practice, manual trajectory planning fails to select the best paths or possible speeds along the paths. This results in lower productivity and reduces effectiveness in industrial application. Considerable research has been done to find an analytic solution to this problem for linearized model or nonlinear system.

#### 3-1 Collision region between two arms

The proposed method finds the collision region between two arms of the robot.

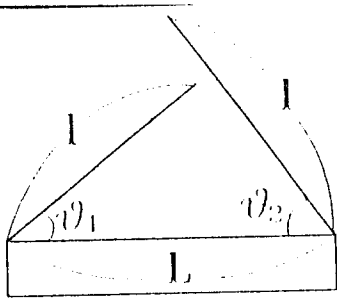


Fig. 1. The configuration of the robot under consideration

The collision region,  $R_c(\theta_1, \theta_2)$  can be described as

$$R_c(\theta_1, \theta_2) = \{(\theta_1, \theta_2) : l_1 \cos \theta_1 + l_2 \cos \theta_2 \geq L, \tan^{-1} \frac{l_2 \sin \theta_2}{L - l_1 \cos \theta_1} \geq \theta_2, 0 < \theta_i \leq \pi \text{ for } i = 1, 2\} \quad (5)$$

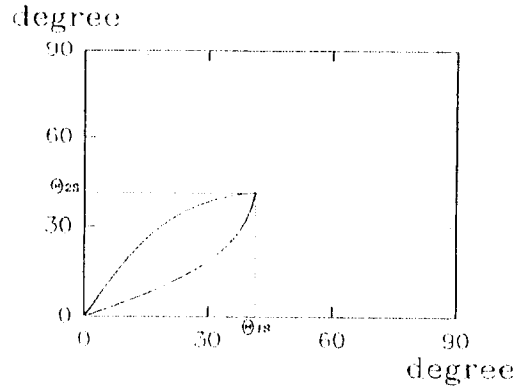


Fig.2. Collision region for  $l_1 = l_2 = 6$ , and  $L = 8$

#### 3-2 Time-optimal trajectory

For a manipulator with multiple degree of freedom with actuator constraints, either control theory and exhaustive search approaches can be applied to find minimum time trajectory. However, the solution can be obtained by neither of them satisfactorily because of the nonlinearity of the system. The time optimal trajectory of each tip for specified states can be obtained as shown in (6) by using maximum acceleration and deceleration.

Let the traveling time of each arms be  $T_1$  and  $T_2$ .

then total traveling time of the robot is defined as

$$T = \max(T_1, T_2) \quad (6)$$

The problem can be defined to find the optimal trajectory and the corresponding time  $T$  for which the initial states of the tips of the arms reach their final states.

##### 3-2-1 Simple case

For the robot under consideration, the positions of the tips of the robot are uniquely determined by the corresponding joint angles due to its inherent structure. Therefore, in some where the movement direction of each arm need not to be changed for its movement, the algorithm suggested by Bien and Lee(7) can be directly applied to find the optimal trajectory. Here, the switching states of its acceleration and deceleration can be found by constructing the switching curves in the  $\theta - \dot{\theta}$  phase plane.

### 3-2-2 General case

The existence of the collision free final configuration of the robot guarantees that of optimal trajectories. The procedure to find them can be flowcharted as in fig. 3.

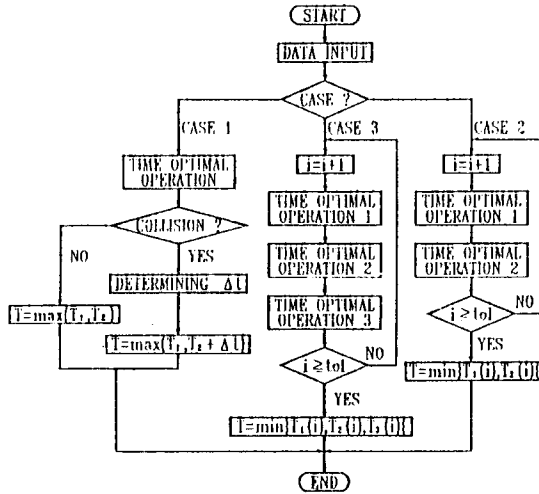


Fig. 3. Flowchart for finding time optimal trajectories

### 4. Simulation

For case study, we assume that  $\theta_1(t_0) \leq \theta_2(t_0)$ ,  $\theta_1(t_0) \leq \theta_1(t_f)$ ,  $\theta_2(t_0) \geq \theta_2(t_f)$  without loss of generality.

The situation is classified as shown belows.

Case ① :  $\theta_{1s} \leq \theta_{1f}$  and  $\theta_{2s} \geq \theta_{2f}$

Case ② :  $\theta_{1s} < \theta_{1f}$  and  $\theta_{2s} > \theta_{2f}$

Case ③ :  $\theta_{2s} > \theta_{2f}$  and  $\theta_{1s} > \theta_{1f}$

The result of case ① is shown in fig. 4 - fig. 7.

Fig.4 shows the movement of the two arms without considering the collision. The result in the above case is depicted on the configuration space in fig. 5. By applying time delay to the movement of the arm 1, the optimal trajectory is drawn as in fig. 6. Fig. 7 describes the time delay on the  $t - \theta$  plane. By comparatively simple procedure, the time optimal trajectories are obtained as shown in the simulation results.

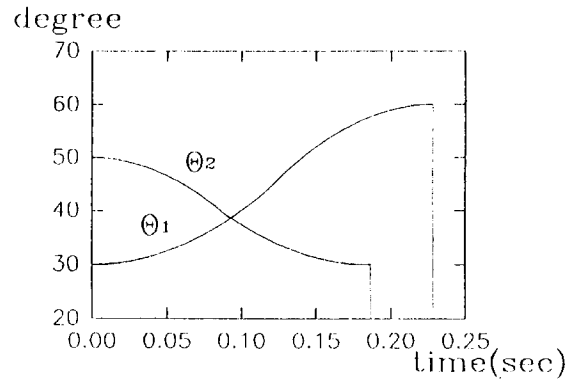


Fig. 4 Movement of two arms in case 1 without time delay

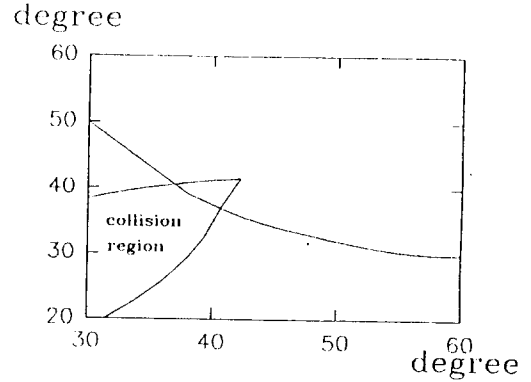


Fig. 5 Time optimal trajectory of case 1 with collision

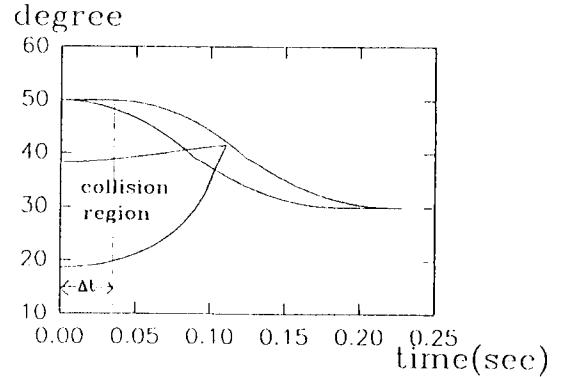


Fig. 6 Determination of time delay to avoid collision

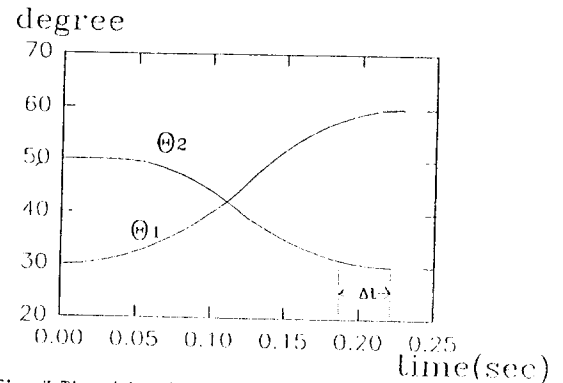


Fig. 7 Time delay for the movement of arm 1

## 5. Conclusion

One of the major advantages of the proposed algorithm is that the exact solution can be found for given nonlinear system under some complex constraints. As shown in simulation results, although the proposed graphical survey can't avoid the exhaustive search for the solution, it can be extended to the case where there exist some obstacles within the workspace of the robot.

Hopefully, the fast development of computer technology will make the proposed algorithm be more attractive in future. The proposed algorithm can be modified by considering the safety region to ensure collision avoidance practically. The research on dual arm with higher degree of freedom for each arm is on the way.

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