'93 KACC (1993.10.20~22)

Identification of Volterra Kernal of Nonlinear Systems by Use of M-sequence

Hiroshi Kashiwagi*, Sun Yeping* and Eiji Nishiyama**
* Faculty of Engineering, Kumamoto University, Kumamoto 860, JAPAN
** Kumamoto National College of Technology, Kumamoto 861-11, JAPAN

Abstract A new method is proposed for obtaining Volterra kernals of a nonlinear system by use of pseudorandom M-sequences and correlation technique. M-sequence is applied to a nonlinear system and the crosscorrelation function between the input and the output displays not only the linear impulse response of the linear part of the system, but also crosssections of the Volterra kernals of nonlinear system. Simulations are carried out for up to 3rd order Volterra kernal, and the results show a good agreement with the theoretical considerations.

1 Introduction

Identification of a nonlinear system is an important task for controlling dynamical behavior of a system, since the system to be identified includes nonlinear portions in general.

A nonlinear dynamical system is, in general, described by use of Volterra series expansion, each term containing so called Volterra kernals. So the measurement of Volterra kernals attracts attentions of many researchers [1)-4)] .

Barker et al[1)-4)] proposed the use of pseudorandom signals, especially antisymmetric M-sequence, for obtaining 2nd-order Volterra kernals with restricted conditions .

The authors propose here a new method for obtaining not only the linear impulse response, but also Volterra kernal of nonlinear system simultaneously. A pseudorandom M-sequence, specially chosen beforehand, is applied to the nonlinear system, and the crosscorrelation function between the input and the output is calculated. Then the linear impulse response together with several crosssections of the Volterra kernals are obtained. The computer simulations are carried out for nonlinear system having up to 3rd order Volterra kernal , and the results show a good agreement with the theoretical considerations.

2 Principle of the method

A nonlinear dynamical system is, in general, described as follows.

$$y(t) = \sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} g_i(\tau_1,\tau_2,\cdots\tau_i) \times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)d\tau_1 d\tau_2\cdots d\tau_i \qquad (1)$$

where $u(t)$ is the input, and $y(t)$ is the output of the nonlinear system, and $g_i(\tau_1,\tau_2,...)$ is called Volterra kernal of i-th order.

When we take the crosscorrelation function between the input $u(t)$ and the output $y(t)$, we have,

$$\phi_{uy}(\tau) = \overline{u(t-\tau)y(t)}$$
$$= \sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} g_i(\tau_1,\tau_2,\cdots\tau_i) \times \overline{u(t-\tau)u(t-\tau_1)\cdots u(t-\tau_i)}d\tau_1 d\tau_2\cdots d\tau_i \qquad (2)$$

where $\phi_{uy}(\tau)$ is the crosscorrelation function of $u(t)$ and $y(t)$ and ———— denotes time average.

The difficulty of obtaining $g_i(\tau_1,\tau_2,\cdots\tau_i)$ from $\phi_{uy}(\tau)$ is , in generqal, due to the difficulty of getting $(i+1)$th moment of the input $u(t)$, because the n-th moment of the signal is very difficult to obtain for actual signals.

Here we will show that when we use an M-sequence as an input to the system, the n-th moment of $u(t)$ can be easily obtained by use of so-called "shift and add property" of the M-sequence. So we can obtain the Volterra kernal $g_i(\tau_1, \tau_2, \cdots \tau_i)$ from simply measuring the cross-correlation function between the input and output of the nonlinear system.

The $(i+1)$th moment of the input M-sequence $u(t)$ can be written as

$$\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)} = \begin{cases} 1 & \text{(for certain } \tau) \\ -1/N & \text{(otherwise)} \end{cases} \quad (3)$$

where N is the period of the M-sequence. When we use the M-sequence with the degree greater than 10, $1/N$ is smaller than 10^{-3}. So Eq.(3) can be approximated as a set of impulses which appear at certain τ's.

Eq.(3) is due to the so-called shift and add property of the M-sequence; that is, for any integer $k_1, k_2, \cdots k_{i-1}$ (suppose $k_1 < k_2 < \cdots, k_i$), there exists a unique $k_i(modN)$ such that

$$u(t)u(t+k_1)u(t+k_2)\cdots u(t+k_{i-1}) = u(t+k_i) \quad (4)$$

Therefore Eq.(3) becomes unity when

$$\tau_1 = \tau - k_1, \tau_2 = \tau - k_2, \cdots \tau_i = \tau - k_i \quad (5)$$

Therefore Eq.(2) becomes

$$\phi_{uy}(\tau) = \sum_{i=1}^{\infty} g_i(\tau - k_1, \tau - k_2, \cdots \tau - k_i) \quad (6)$$

Since $g_i(\tau_1, \tau_2, \cdots \tau_i)$ is zero when any of τ_i is smaller than zero, each $g_i(\tau - k_1, \tau - k_2, \cdots \tau - k_i)$ in Eq.(6) appear in the crosscorrelation function $\phi_{uy}(\tau)$ when $\tau > k_i$. If we denote the k_i of i-th Volterra kernal g_i as k_i^*, and if each $k_i^*(i = 1, 2, \cdots)$ are sufficiently apart from each other(say, more than $50\Delta t$, where Δt is the time increment of the measurement time), we can obtain each Volterra kernal $g_i(\tau - k_1, \tau - k_2, \cdots, \tau - k_i)$ from Eq.(6). Volterra kernals $g_i(\tau_1, \tau_2, \cdots \tau_i)$ are obtained as a set of cross-sections along 45 lines in $(\tau_1, \tau_2, \cdots, \tau_i)$ plane as shown in Fig.1. In order for this to be realized, we have to select M-sequence as described later.

3 Measurement of 2nd Volterra Kernal

An example of obtaining second Volterra kernal by this method is shown here.

The system to be identified is assumed to have up to second Volterra kernal which is actually realized as shown in Fig.2, where $g(t)$ is the impulse response of the linear part of the system. Then the output $y(t)$ can be written as

$$\begin{aligned} y(t) &= z(t) + z^2(t) \\ &= \int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1 + \{\int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1\}^2 \\ &= \int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1 \\ &+ \int_0^\infty \int_0^\infty g(\tau_1)g(\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \end{aligned} \quad (7)$$

Therefore Volterra kernals are as follows in this case.

$$\begin{aligned} g_1(\tau_1) &= g(\tau_1) \\ g_2(\tau_1, \tau_2) &= g(\tau_1)g(\tau_2) \end{aligned} \quad (8)$$

When we use the M-sequence having the characteristic polynomial of $f(x) = 36073$(in octal notation, 13 degree), k_i's in Eq.(4) are

$$k_1 = 73, k_2 = 75$$

Therefore

$$\phi_{uy}(\tau) = g(\tau) + g_2(\tau - 73, \tau - 75) + g_2(\tau - 146, \tau - 150) + \cdots \quad (9)$$

Fig. 3 shows an example of the simulation results, when the linear part of the system is of second-order with $\zeta = 0.5, \omega_n = 1.0$, where ζ is the damping ratio and ω_n is the natural angular frequency of the second order system. The linear impulse response is clearly seen for $\tau < 50$, and the 2nd Volterra kernal $g_2(\tau - 73, \tau - 75)$ is obtained from $50 < \tau < 100$ and also $g_2(\tau - 146, \tau - 150)$ is obtained from $100 < \tau < 200$.

In Fig.3 , ○ indicate the simulation result and solid line shows the theoretical result, showing a good agreement with each other.

4 Measurement of 3rd Volterra Kernal

Let the system to be identified be a nonlinear system having the first and third Volterra kernal. This system is realized as shown in Fig.4. The output $y(t)$ is written as follows

$$y(t) = \int_0^\infty g(\tau)u(t-\tau)\,d\tau + \{\int_0^\infty g(\tau)u(t-\tau)d\tau\}^3$$
$$= \int_0^\infty g(\tau)u(t-\tau)d\tau$$
$$+\int_0^\infty\int_0^\infty\int_0^\infty g(\tau_1)g(\tau_2)g(\tau_3)$$
$$\times u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)\,d\tau_1\,d\tau_2 d\tau_3 \quad (10)$$

Therefore Volterra kernals are

$$g_1(\tau_1) = g(\tau_1)$$
$$g_3(\tau_1,\tau_2,\tau_3) = g(\tau_1)g(\tau_2)g(\tau_3) \quad (11)$$

Fig.5 shows an example of the result of simulation when the system is composed of the first and third Volterra kernal.

5 Selection of M-sequence

We can assume $k_i < k_{i+1}$ without losing generality. Then in order to obtain $g_i(\tau_1,\tau_2,\cdots,\tau_i)$ from Eq. (6), each k_i^* satisfying Eq.(4) must be sufficiently apart from each other.

Here we assume that Volterra kernal $g_i(\tau_1,\tau_2,\cdots,\tau_i)$ is small enough when $\tau_1,\tau_2,\cdots > 50\Delta t$, where Δt is the time increment. Then it is enough that we choose those M-sequences whose k_i^*'s are apart more than $50\Delta t$.

We have searched all known primitive polynomials over GF(2) up to 34 degrees(total 6178 polynomials) to find those M-sequences for $k_i^* < 300$.

Table 1 shows some of the obtained characteristic polynomials suitable for measurement of second Volterra kernal. The characteristic polynomials $f(x)$ of M-sequence are shown in Table 1 in octal notation with $(k_1,k_2)-d$, where d is the distance of k_i^* to the nearest neighbour k_j^*. For example, 53233(127,128)-109 in Table 1 means $f(x) = 53233$ in octal notation, $k_1 = 127, k_2 = 128$ and the nearest k_j^* is apart by 109. df in the table indicates the difference $k_2 - k_1$. The usable characteristic polynomials are obtained for df up to 99, only the first 50 being shown in Table 1.

* in the table denotes that there is no such peaks in the searching range of $k_i* < 300$.

We have obtained those M-sequences suitable for obtaining 2nd Volterra kernal of nonlinear system for $df = k_2 - k_1$ up to 99, among which first 50 are shown in Table 1.

In the same way, the usable characteristic polynomials suitable for obtaining third order Volterra kernal are searched and obtained for $d1 = k_3 - k_1, d2 = k_2 - k_1$ up to 99. Table 2 shows some of the usable characteristic polynomials for obtaining third order Volterra kernal of nonlinear system.

6 Conclusion

A new method for obtaining Volterra kernal of nonlinear system by use of pseudorandom M-sequence is proposed. A specially chosen M-sequence is applied to the nonlinear system to be indentified, and the crosscorrelation function between the input and the output gives us not only the linear impulse response of the linear portion of the system, but also some crosssections of the Volterra kernal $g_i(\tau_1,\tau_2,\cdots\tau_i)$ along some 45 degree lines in $(\tau_1,\tau_2,\cdots\tau_i)$ plane.

This method for obtaining Volterra kernal is simulated on the computer for nonlinear systems having up to second and third order Volterra kernals. The results of simulation show a good agreement with the theoretical considerations.

References

1. Fujii, K. and K.Nakao; Identification and Modelling of Nonlinear Dynamical Systems Using Volterra Functional Series, Trans. SICE, Vol.5, No.4, 368/377(1969)
2. Barker,H.A. *et al*; Performance of antisymmetric pseudorandom signal in the measurement of second-order Volterra kernals by crosscorrelation, Proc. IEE, Vol.119, No.3, 353/362 (1972)
3. Barker H.A. *et al*; Combined crosscorrelation method for the measurement of 2nd -order Volterra kernals, *ibid*,Vol.120, No.1 (1972)
4. Peterson, W.W. ; Error-correcting codes, MIT press, (1961)

Table 1: Usable characteristic polynomials for 2nd order Volterra kernal measurement

df	$f(x)(k_1,k_2)-d$
1	53233(127,128)*
2	56463(164,166)-83
3	71721(97,100)-100
4	34035(125,129)-127
5	24465(87,92)-92
6	37335(176,182)-91
7	31223(134,141)*
8	33705(133,141)*
9	130125(102,111)*
10	65513(111,121)-102
11	54635(86,97)-97
12	26617(99,111)-111
13	165033(111,124)*
14	70251(105,119)-93
15	32641(78,93)-93
16	55103(162,178)-89
17	34555(94,111)-111
18	136457(145,163)*
19	35315(88,107)-100
20	24165(95,115)-78
21	32445(100,121)-121
22	34767(125,147)*
23	20341(98,121)-121
24	170277(103,127)*
25	63227(86,111)-111
26	37445(79,105)-105
27	57107(86,113)-113
28	33441(132,160)-80
29	34047(95,124)-124
30	20761(95,125)-116
31	23231(59,90)-90
32	46305(81,113)-113
33	77031(80,113)-113
34	60253(107,141)*
35	74531(64,99)-99
36	77057(77,113)-113
37	141445(116,153)*
38	174467(63,101)-101
39	65277(58,97)-97
40	32751(69,109)-109
41	41625(59,100)-100
42	126433(93,135)-135
43	162241(79,122)-122
44	51243(111,155)*
45	60057(81,126)-103
46	101507(90,136)-68
47	64617(62,109)-109
48	150225(79,127)-127
49	110501(58,107)-107
50	103035(121,171)*

Table 2: Usable characteristic polynomials for 3rd order Volterra kernal measurement, R means reciplocal.

(d1,d2)	$f(x)(k_1,k_2,k_3)-d$
(2,1)	734703(160,161,162)*
(3,1)	R320055(151,153,154)-109
(3,2)	R335061(198,199,201)-132
(4,1)	772047(56,58,59)-62
(4,2)	R377645(180,182,184)-92
(4,3)	R277707(92,93,96)-96
(5,1)	337503(139,143,144)-69
(5,2)	R301021(122,125,127)-60
(5,3)	R311465(94,96,99)-99
(5,4)	316145(170,171,175)-95
(6,1)	201607(65,70,71)-71
(6,2)	R263641(112,116,118)-59
(6,3)	R124341((107,110,113)-41
(6,4)	R330233(127,129,133)
(6,5)	166541(83,84,89)
(7,1)	255505(84,90,91)-90
(7,2)	R255267(170175,177)-103
(7,3)	247743(102,106,109)*
(7,4)	40123(190,193,197)-36
(7,5)	250641(99,101,106)*
(7,6)	106677(62,63,69)-69
(8,1)	243703(141,148,149)*
(8,2)	772047(116,122,124)-62
(8,3)	R240631(54,59,620-62
(8,4)	370321(51,55,59)-59
(8,5)	206603(137,140,145)-100
(8,6)	R332017(195,197,203)*
(8,7)	276277(113,114,121)-60
(9,1)	153731(58,66,67)-57
(9,2)	401121(131,138,140)*
(9,3)	R217527(76,82,85)-85
(9,4)	R244377(185,190,194)-106
(9,5)	R145427(37,41,46)-46
(9,6)	R346173(93,96,102)-49
(9,7)	R137613(149,151,158)-59
(9,8)	356057(188,189,197)-93
(10,1)	125323(179,188,189)-63
(10,2)	R302021(75,83,85)-84
(10,3)	232561(94,101,104)*
(10,4)	236511(188,194,198)-99
(10,5)	663013(68,73,78)-78
(10,6)	R311465(188,192,198)-99
(10,7)	340115(80,83,90)-41
(10,8)	271317(195,197,205)*
(10,9)	R303657(186,187,196)-53
(11,1)	R261455(81,91,92)-80
(11,2)	R125403(98,107,109)-99
(11,3)	170543(182,190,193)-103
(11,4)	313071(143,150,154)*
(11,5)	332707(188,194,199)*
(11,6)	213727(141,146,152)-43
(11,7)	105413(101,105,112)-53
(11,8)	375715(165,168,176)-122
(11,9)	225073(160,162,171)-43
(11,10)	640635(116,117,127)*

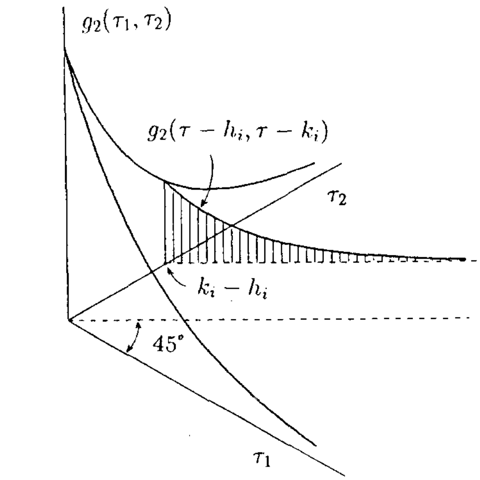

Fig.1 Crosssections of Volterra kernal are obtained.

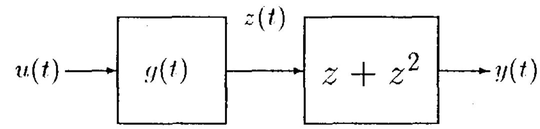

Fig.2 A nonlinear system with 2nd order Volterra kernal.

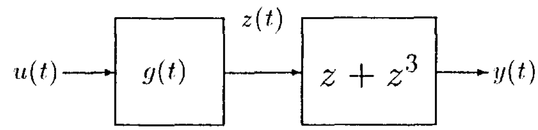

Fig.4 A nonlinear system with 3rd order Volterra kernal.

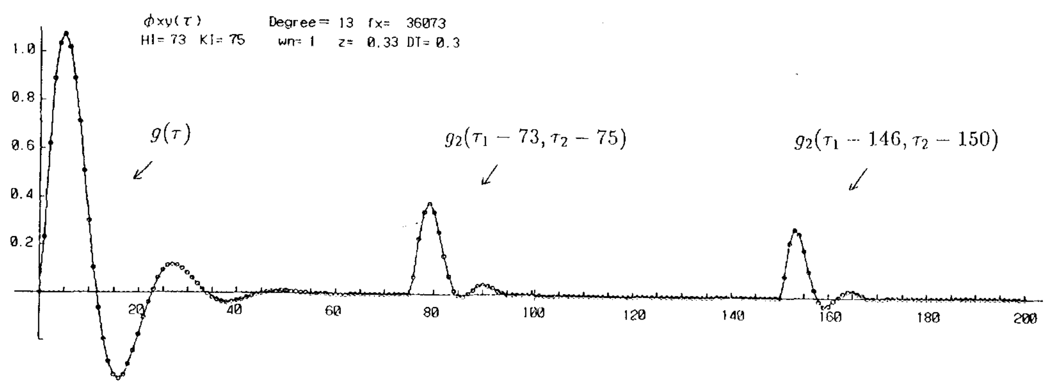

Fig.3 Simulation result on 2nd order Volterra kernal measurement

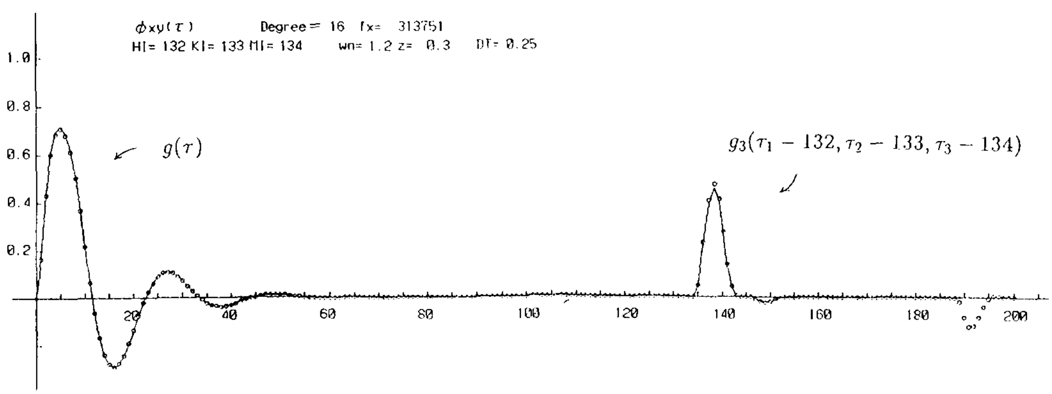

Fig.5 Simulation result on 3rd order Volterra kernal measurement