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Riccati Equation Approach to H_∞ Robust Performance Problem for Descriptor Form System

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Abstract: In this paper, we discuss H_∞ robust performance problem for uncertain system described in a descriptor form. We show that the method based on Riccati equation can be extended to solve this problem. First, such a sufficient condition is given that the system described in a descriptor form is quadratic stable and H_∞ norm of a specified transfer function is less than a given level. Using this result, a state feedback law which ensures H_∞ robust performance of closed loop system is derived based on a positive definite solution of a Riccati equation. This result shows that a solution of the problem can be also obtained by solving H_∞ standard problem for an extended plant. Finally, a design example and simulation results will be given.

1 Introduction

In the last few years, an algebraic Riccati equation approach was developed to solve H_∞ robust performance problem for the plant with parameter uncertainties[1]~[3]

$$\dot{x} = (A+E\Sigma F_a)x+(B_1 + E\Sigma F_1)w+(B_2+E\Sigma F_2)u \quad (1)$$
$$z = C_1x + D_{12}u \quad (2)$$

where Σ denotes an unknown matrix which belongs to a given bounded set $\Omega = \{\Sigma(t) \mid \Sigma^T(t)\Sigma(t) \le 1, \forall t\}$. In the literatures, it is shown that a closed loop system with static state feedback or strictly proper dynamic output feedback is H_∞ robust sub-optimal, if an extended system without uncertainty is H_∞ sub-optimal. In other words, such a controller that the closed loop system is quadratically stable and $\|T_{zw}\| \le 1$ can be obtained by applying the Riccati equation approach of H_∞ standard design problem, where T_{zw} denotes the closed loop transfer function.

However, many mechanical systems are described in the following descriptor form[4],[5]

$$(\Gamma+H\Sigma F_i)\dot{x} = (\Psi+H\Sigma F_a)x+(\Phi_1+H\Sigma F_1)w+(\Phi_2+H\Sigma F_2)u \quad (3)$$
$$z = C_1x + D_{12}u \quad (4)$$

With this expression tight estimation of uncertainty is possible.

In this paper, we will extend the Riccati approach to cover the plant (3),(4). We will show that the closed loop system described in descriptor form is quadratically stable and $\|z\|_2 < \|w\|_2$ for any $\Sigma \in \Omega$, if an extended system without uncertainty is H_∞ sub-optimal[1]~[3]. Hence, a controller ensuring H_∞ robust performance can be obtained by solving a Riccati equation. A design example of an inverted pendulum was given to illustrate the proposed approach.

Before going to the detail of the approach, we will introduce some notations. Let

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

then a linear fractional transformation associated with G and Σ is defined by

$$LFT(G,\Sigma) := G_{11} + G_{12}\Sigma(I - G_{22}\Sigma)^{-1}G_{21}$$

A transfer matrix $T(s)$ with a state space realization $D + C(sI - A)^{-1}B$ will be denoted by

$$T(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

2 Basic Result

Consider a linear system with parameter uncertainty described by descriptor form model

$$(\Gamma+H\Sigma F_i)\dot{x} = (\Psi+H\Sigma F_a)x+(\Phi_1+H\Sigma F_1)w \quad (5)$$
$$z = C_1x \quad (6)$$

where $x \in R^n$ denote state vector, $w \in R^p (w \in L_2[0,\infty))$ disturbance input, $z \in R^q$ controlled output, and $\Gamma, \Psi, \Phi_1, \Phi_2, C_1, D_{12}, H, F_i, F_a$ and F_1 are matrices with appropriate dimensions. $\Sigma(t) \in R^{h\times k}$ is unknown matrix which belong to given set Ω defined by

$$\Omega := \{\Sigma(t) \mid \Sigma^T(t)\Sigma(t) \le 1, \ \forall t\} \quad (7)$$

We assume that the matrix Γ is nonsingular and (Ψ, Φ_2) is stabilizable. Then, taking $A = \Gamma^{-1}\Psi$, $B_1 = \Gamma^{-1}\Phi_1$ and $E = \Gamma^{-1}H$, the system (5),(6) can be reformulated by

$$(I+\Delta M)\dot{x} = (A+\Delta A)x+(B_1 + \Delta B_1)w \quad (8)$$
$$z = C_1x \quad (9)$$

$$[\Delta M \ \ \Delta A \ \ \Delta B_1] = E\Sigma(t)[F_i \ \ F_a \ \ F_1] \quad (10)$$

Our interest is to find a condition such that the system (8)~(10) is quadratically stable and $\|z\|_2 < \|w\|_2$, $\forall w \in L_2$ for any $\Sigma \in \Omega$. Next Lemma is a slight modification of the result given by [4], and which will be used to derive our main results.

Lemma 1. The system (8)∼(10) is quadratically stable if and only if A is a stable matrix and

$$\|F_i E + (F_i A - F_a)(sI - A)^{-1}E\|_\infty < 1 \tag{11}$$

Remark 1. With condition (11), $det(I + E\Sigma F_i) \neq 0 \ \forall \Sigma \in \Omega$ is assumed since $\bar{\sigma}(F_i E) < 1$. Thus, the system dose not contain impulse mode if the linear system (8)∼ (10) is quadratically stable.

Theorem 1. The system (8)∼(10) is quadratically stable and $\|z\|_2 < \|w\|_2$ for any $w \in L_2$, if there exits $\lambda > 0$ and $\epsilon > 0$ such that $I - \hat{D}^T\hat{D} > 0$ and

$$\begin{aligned} &A^TP + PA + \hat{C}^T\hat{C} + \epsilon I \\ &+ \ (P\hat{B} + \hat{C}^T\hat{D})(I - \hat{D}^T\hat{D})^{-1}(\hat{D}^T\hat{C} + \hat{B}^TP) = 0 \end{aligned} \tag{12}$$

has a positive definite solution $P > 0$, where

$$\hat{B} = [B_1 \ \ \lambda E], \ \ \hat{C} = \begin{bmatrix} C_1 \\ \frac{1}{\lambda}(F_a - F_i A) \end{bmatrix},$$

$$\hat{D} = \begin{bmatrix} 0 & 0 \\ \frac{1}{\lambda}(F_1 - F_i B_1) & -\frac{1}{\lambda}F_i E \end{bmatrix}$$

Proof. Since (8) can be rewritten as

$$\dot{x} = Ax + E\Sigma(t)\{-F_i\dot{x} + F_a x + F_1 w\} + B_1 w$$

the system (8)∼(10) becomes to

$$\dot{x} = Ax + \hat{B}\begin{bmatrix} w \\ w_\sigma \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} z \\ z_\sigma \end{bmatrix} = \hat{C}x + \hat{D}\begin{bmatrix} w \\ w_\sigma \end{bmatrix} \tag{14}$$

$$w_\sigma = \Sigma(t)z_\sigma \tag{15}$$

where auxiliary signals w_σ and z_σ are defined by

$$z_\sigma = \frac{1}{\lambda}\{-F_i\dot{x} + F_a x + F_1 w\} \tag{16}$$

$$w_\sigma = \Sigma(t)z_\sigma \tag{17}$$

with a scaling parameter $\lambda > 0$.

From Lemma 2.2 in [6], if there exist $\lambda > 0$ and $\epsilon > 0$ such that the Riccati equation (12) has a positive definite solution, then A is stable matrix and

$$\|\hat{D} + \hat{C}(sI - A)^{-1}\hat{B}\|_\infty < 1 \tag{18}$$

holds. However, it is easy to show that

$$\hat{D} + \hat{C}(sI - A)^{-1}\hat{B} = \begin{bmatrix} * & * \\ * & F_i E + (F_i A - F_a)(sI - A)^{-1}E \end{bmatrix} \tag{19}$$

where $*$ denote a appropriate matrix. Thus, from (18) we have that

$$\|F_i E + (F_i A - F_a)(sI - A)^{-1}E\|_\infty < 1 \tag{20}$$

which implies the system (8)∼(10) is quadratically stable by Lemma 1.

In order to show that $\|z\|_2 < \|w\|_2$ for any $\Sigma \in \Omega$, define $v(t) = x^TPx$ and consider differentiation of the $v(t)$ along the solution of the equation (8),

$$\begin{aligned} \frac{d}{dt}v(t) &= \dot{x}^TPx + x^TP\dot{x} \\ &= x^T(A_k^TP + PA_k)x + (w^TB_1^T + \lambda w_\sigma^TE^T)Px \\ &\quad + x^TP(B_1 w + \lambda E w_\sigma^T) \end{aligned} \tag{21}$$

Substitution of the Riccati equation (12) into (21) gives

$$\begin{aligned} \frac{d}{dt}v(t) &= -z^Tz + w^Tw - \xi^T(I - \hat{D}^T\hat{D})^{-1}\xi - z_\sigma^Tz_\sigma + w_\sigma^Tw_\sigma - \epsilon I \\ &< -z^Tz + w^Tw \\ &\quad - \xi^T(I - \hat{D}^T\hat{D})^{-1}\xi - z_\sigma^T(I - \Sigma^T\Sigma)z_\sigma \end{aligned} \tag{22}$$

where

$$\xi = \hat{B}^TPx + \hat{D}\begin{bmatrix} z \\ z_\sigma \end{bmatrix} - \begin{bmatrix} w \\ w_\sigma \end{bmatrix}$$

Integrating (22) from 0 to ∞, we get

$$\begin{aligned} 0 &< -\|z\|_2^2 + \|w\|_2^2 - \int_0^\infty \xi^T(I - \hat{D}^T\hat{D})^{-1}\xi dt \\ &\quad - \int_0^\infty z_\sigma^T(I - \Sigma^T\Sigma)z_\sigma dt, \ \ \forall \Sigma \in \Omega \end{aligned} \tag{23}$$

Therefore, $\|z\|_2 < \|w\|_2$, $w \in \Omega$ hold for any $\Sigma \in \Omega$.

Remark 2. Let

$$G = \left[\begin{array}{c|cc} A & B_1 & \lambda E \\ \hline C_1 & 0 & 0 \\ \frac{1}{\lambda}(F_a - F_i A) & \frac{1}{\lambda}(F_1 - F_i B_1) & -\frac{1}{\lambda}F_i E \end{array}\right] \tag{24}$$

Theorem 1 implies that if A is a stable matrix and $\|G\|_\infty < 1$, then the system is quadratically stable and $\|LFT(G, \Sigma)\|_\infty < 1$ for any $\Sigma \in \Omega$.

Remark 3. If $\Delta M = 0$, then Theorem 1 is equivalent to the result given by [2]. And the case when $\Delta M = \Delta B = 0$ is equivalent to the result given by [1].

3 State Feedback Law

Consider a linear plant with parameter perturbation given by

$$(I + \Delta M)\dot{x} = (A + \Delta A)x + (B_1 + \Delta B_1)w + (B_2 + \Delta B_2)u \tag{25}$$

$$z = C_1 x + D_{12}u \tag{26}$$

$$[\Delta M \ \ \Delta A \ \ \Delta B_1 \ \ \Delta B_2] = E\Sigma(t)[\ F_i \ \ F_a \ \ F_1 \ \ F_2\ ] \tag{27}$$

In this section, we derive a state feedback law $u = Kx$ such that the closed loop system of the plant with the state feedback is quadratically stable and $\|z\|_2 < \|w\|_2$, $\forall w \in L_2$ for any $\Sigma \in \Omega$.

Theorem 2. Assume that D_{12} is of full column rank. If there exists $\lambda > 0$ and $\epsilon > 0$ such that $I - \hat{D}^T\hat{D} > 0$ and Riccati equation

$$\hat{A}^T P + P\hat{A} + +\hat{C}^T R^{-1}\hat{C} + P\hat{B}_1 S^{-1}\hat{B}_1 P$$
$$-(P\hat{B}_2+\hat{C}^T R^{-1}\hat{D}_{12})Q^{-1}(\hat{B}_2^T P+\hat{D}_{12}^T R^{-1}\hat{C})+\epsilon I=0 \quad (28)$$

has a positive definite solution $P > 0$, then a desired state feedback law is given by

$$K = -Q^{-1}(\hat{B}_2^T P + \hat{D}_{12}^T R^{-1}\hat{C}) \quad (29)$$

where

$$\begin{aligned}
\hat{A} &= A + \hat{B}S^{-1}\hat{D}^T\hat{C}_1, \\
\hat{B}_1 &= \hat{B}, \ \hat{B}_2 = B_2 + \hat{B}S^{-1}\hat{D}^T\hat{D}_{12}, \\
\hat{C} &= \begin{bmatrix} C_1 \\ \frac{1}{\lambda}(F_a - F_i A)\end{bmatrix}, \ \hat{D}_{12} = \begin{bmatrix} D_{12} \\ \frac{1}{\lambda}(F_1 - F_i B_2)\end{bmatrix}, \\
Q &= \hat{D}_{12}^T R^{-1}\hat{D}_{12}, \ S = I - \hat{D}^T\hat{D}, \ R = I - \hat{D}\hat{D}^T
\end{aligned}$$

Proof. The closed loop system of the plant (25)~(27) with the state feedback law is given by

$$\begin{aligned}
(I+E\Sigma F_i)\dot{x} &= (A_k+E\Sigma F_k)x + (B_1+E\Sigma F_1)w \quad (30) \\
z &= C_k x \quad (31)
\end{aligned}$$

where $A_k = A + B_2 K, \ F_k = F_a + F_2 K, \ C_k = C_1 + D_{12}K$. From Theorem 1, if there exist a $\lambda > 0$ and $\epsilon > 0$ such that Riccati equation

$$A_k^T P + PA_k + \hat{C}_k^T\hat{C}_k + \epsilon I$$
$$+ \ (P\hat{B} + \hat{C}_k^T\hat{D})(I - \hat{D}^T\hat{D})^{-1}(\hat{D}^T\hat{C}_k + \hat{B}^T P) = 0 \quad (32)$$

has a positive definite solution, then the closed loop system is quadratically stable and $||z||_2 < ||w||_2$ for any $\Sigma \in \Omega$. However, it is easy to show that solution of the Riccati equation (28) is a solution of the Riccati equation (32) when K is given by (29).

Remark 4. From Theorem 1 and 2, it is easy to prove that the K given by Theorem 1 is a solution of H_∞ standard problem with plant as

$$P_\lambda = \left[\begin{array}{c|cc} A & [B_1 \ \ \lambda E] & B_2 \\ \hline \begin{bmatrix} C_1 \\ \frac{1}{\lambda}(F_a - F_i A)\end{bmatrix} & \begin{bmatrix} 0 \\ [\frac{1}{\lambda}(F_1 - F_i B_1) \ \ -F_i E]\end{bmatrix} & \begin{bmatrix} D_{12} \\ \frac{1}{\lambda}F_2 \end{bmatrix} \\ I & 0 & 0 \end{array}\right] \quad (33)$$

with a scaling parameter $\lambda > 0$. It implies that a solution of H_∞ robust performance design problem for plant (25)~(27) can be obtained by solving the H_∞ standard problem.

Remark 5. When $\Delta M = 0$, the state feedback law (29) is equivalent to the controller given [2], and when $\Delta M = \Delta B_1 = 0$ the K is equivalent to the controller given by [1].

4 Design Example

Now, we use Theorem 2 to design a robust controller for an inverted pendulum. Consider an inverted pendulum as

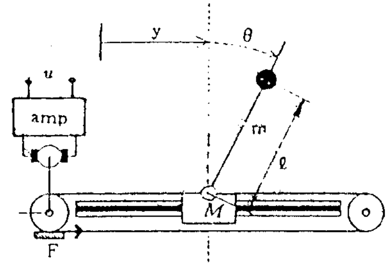


Figure 1: Inverted pendulum with time-varying length

shown in Fig.1. The dynamics of the pendulum is described by the following non-linear differential equations.

$$\begin{aligned}
&(M+m)\frac{d^2y}{dt^2} \\
&= -F\frac{dy}{dt} - m\frac{d^2E_x}{dt^2} + au \quad (34) \\
&mE_y\frac{d^2y}{dt^2} + J\frac{d^2\theta}{dt^2} \\
&= -f\frac{d\theta}{dt} + m\{(g + \frac{d^2E_y}{dt^2})E_x - \frac{d^2E_x}{dt^2}E_y\} \quad (35) \\
&E_x(t) = l(t)\sin(\theta(t)) \quad (36) \\
&E_y(t) = l(t)\cos(\theta(t)) \quad (37)
\end{aligned}$$

where

$u(t)$:	input voltage to amplifier
$Y(t)$:	the position of the cart
$\theta(t)$:	the angle of the pendulum
M:	the mass of the cart
m:	the mass of the pendulum
F:	friction constant of the cart
f:	friction constant of the pendulum
J:	moment of inertia of the pendulum
a:	gain constant
$l(t)$	the length of the pendulum

we make the following assumptions about the uncertain parameter $l(t)$.

$(A1)$ $l(t) = l_0 + \Delta l(t)$, l_0 is constant and $|\Delta(t)| < \delta_1$.

$(A2)$ $|\frac{dl}{dt}| < \delta_2$.

$(A3)$ $\frac{d^2l}{dt^2} \simeq 0$.

As the state vector of this system, let

$$x^T = [y \ \ \frac{dy}{dt} \ \ \theta \ \ \frac{d\theta}{dt}]$$

and assume the state is measurable. The design specification is given by

(S1) the closed loop system is locally robust stable for all $l(t)$ satisfying the assumptions $A1 \sim A3$.

(S2) robust performance of disturbance attenuation.

The specification (S2) is given in (49). The system (34)~(37) is linearized at the origin $x = 0$ as follows:

$$\frac{d^2y}{dt^2} + h_3 l \frac{d^2\theta}{dt^2} = -h_2 \frac{dy}{dt} - 2h_3 \frac{dl}{dt}\frac{d\theta}{dt} + h_1 u + h_3 w_1 \qquad (38)$$

$$\frac{d^2y}{dt^2} + (l + \frac{J}{ml})\frac{d^2\theta}{dt^2} = -(2\frac{dl}{dt} + \frac{f}{ml})\frac{d\theta}{dt} + g\theta + w_2 \qquad (39)$$

where $w_1(t)$ and $w_2(t)$ are disturbance input signals which denote the high order model errors.

Define w and $h_i (i = 1, \sim 3)$ by $w^T = [w_1 \ \ w_2]$ and let

$$h_1 = \frac{a}{M+m} \qquad (40)$$

$$h_2 = \frac{F}{M+m} \qquad (41)$$

$$h_3 = \frac{m}{M+m} \qquad (42)$$

Then, (38),(39) can be presented in the descriptor form

$$(M + \Delta M(t))\dot{x} = (A + \Delta A(t))x + B_1 w + B_2 u \qquad (43)$$

with matrices $M, A, B_1, B_2, \Delta M(t)$ and $\Delta A(t)$ given by

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & h_3 l_0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & l_0 + \frac{J}{m} n_0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -h_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g & -\frac{f}{m} n_0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ h_3 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ h_1 \\ 0 \\ 0 \end{bmatrix},$$

$$\Delta M(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_3 \Delta l(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta l(t) + \frac{J}{m}\Delta n(t) \end{bmatrix},$$

$$\Delta A(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2h_3 \frac{dl(t)}{dt} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\frac{dl(t)}{dt} - \frac{f}{m}\Delta n(t) \end{bmatrix}$$

where $\Delta = \frac{1}{l} - \frac{1}{l_0}, n_0 = \frac{1}{l_0}$. Let

$$\delta_3 = \max_{|\Delta l| \le \delta_1} |\Delta n|$$

and define matrices

$$E = \begin{bmatrix} 0 & 0 & 0 \\ h_2\delta_1 & -2h_3\delta_2 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & \frac{J}{m}\delta_3 \end{bmatrix} \qquad (44)$$

$$F_i = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (45)$$

$$F_a = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{f}{J} \end{bmatrix} \qquad (46)$$

$$\Sigma(t) = \begin{bmatrix} \frac{\Delta l(t)}{\delta_1} & 0 & 0 \\ 0 & \frac{dl(t)}{dt}/\delta_2 & 0 \\ 0 & 0 & \frac{\Delta n}{\delta_3} \end{bmatrix} \qquad (47)$$

Then, $\Delta M(t)$ and $\Delta A(t)$ can be formulated as shown in (27) with the matrices E, F_i, F_a and $\Sigma(t)$.

Assume that $w(t) \in L_2$. For describing the disturbance attenuation performance, define a controlled output by

$$z = C_1 x + D_{12} u \qquad (48)$$

$$C_1 = \begin{bmatrix} Q_x^{\frac{1}{2}} \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ R_u^{\frac{1}{2}} \end{bmatrix}$$

where Q_x and R_u are given positive definite matrices.

Consider the following robust performance that

$$\|z\|_2 \le \|w\|_2, \ \forall w, \ \forall l \qquad (49)$$

It is well known that (21) implies that the state of closed loop system x and the control input u satisfy

$$\int_0^\infty \{x^T Q_x x + u^T R_u u\} dt \le \|w\|_2 \qquad (50)$$

for all $l(t)$ satisfying the assumptions $A1 \sim A3$.

Fig.2 shows a simulation results with state feedback law

$$K = [0.1257 \ \ 1.2707 \ \ 4.5733 \ \ 0.6995]$$

which obtained by application of Theorem 2 with parameters as

$$\begin{aligned} M &= 5.383[kg], \ m = 0.1[kg] \\ F &= 23.73[kg/s], \ f = 1.761 \times 10^{-3}[kgm^2/s] \\ a &= 25.0[N/V], \ J = 1.526 \times 10^{-3}[kgm^2] \\ l_0 &= 0.115[m] \end{aligned}$$

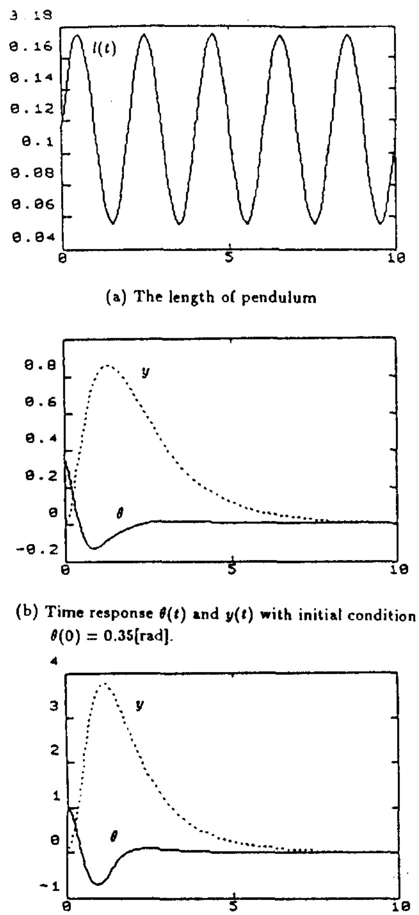
and assumption

$$|\Delta l(t)| \le \delta_1 = 0.05[m] \qquad (51)$$

$$|\frac{dl(t)}{dt}| \le \delta_2 = 0.1[m]/s \qquad (52)$$

5 Conclusion

H_∞ robust performance problem is discussed for linear system described in descriptor form with parameter perturbation. A sufficient condition is given such that the system is quadratically stable and H_∞ sub-optimal. Using the condition, a state feedback law is derived based on Riccati equation. The result is a extention of the conventional Riccati equation approach.



(a) The length of pendulum

(b) Time response $\theta(t)$ and $y(t)$ with initial condition $\theta(0) = 0.35$[rad].

(c) $\theta(0) = 1.00$[rad].

Figure 2: Simulation results

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