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Performance Analyses of RHLQG/FIRF Controller

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Abstract

In this paper we analyze the RHLQG/FIRF optimal control law presented in [4,5] in order to stabilizes a stochastic linear time varying systems with modeling uncertainty. It is shown by the frequency domain analysis that the RHC is robuster than the LQ control law. Explicit LTR procedures are given to improve the robust performance of RHLQG/FIRF control law. Using the mismatching function technique [8], we propose an LTR method which makes the RHLQG/FIRF controller recover the feedback properties of the RHC law. Also we compare the LTR performance of the RHLQG/FIRF via simulation with conventional LTR methods.

1 Introduction

Robust stabilization has been one of the hot issues in control system theory during the past decades and thus there exist many literatures on this problem[1,2]. Most existing method are,however, proposed for linear time-invariant systems based on infinite horizon optimization of a nonsingular cost functional. For linear time-varying systems, such methods require a fairly good information about the system parameter over all times, which is an impractical assumption and a major defect of such methods so far.

Recent results have shown that the formal mathematical synthesis procedures based on LQG with loop transfer recovery, the so-called LQG/LTR techniques, provide a broad flexibility in achieving the necessary loop transfer functions. However, in spite of its remarkable robustness, the LQG/LTR techniques may suffer from poor stability property for the systems with parameter variation, nonminimum phase zeros or time-varying elements. Also another remarkable results have been reported that the LQ control law may acquire far off unstable modes for small variations in the plant parameter [3]. These findings cast some doubts practically on the asymptotic recovery, even in the minimum phase case, since the recovered stability margins cannot guarantee the LQG design robustness for all possible quadratic performance criteria.

The above reason has provided a motivation for investigation new methods to robustly stabilize stochastic linear time-varying systems and the RHLQG/FIRF (Receding Horizon LQG with FIR filter) is presented by Kwon et al.[4] and Yoo et al.[5] as one method. It is noted that RHLQG/FIRF requires quantitative knowledge over a finite time interval around at the current time, which is a practical assumption. The closed-loop stability of the RHLQG/FIRF is analyzed in this paper and it is shown that the RHC law [6] has better robustness properties than the LQ control law, which can be proven by the FARE(Fake Algebraic Riccati Equation)[7]. Also the transfer function of the FIR filter is derived here to analyze its frequency domain characteristics and RHLQG/ FIRF robustness properties in frequency domain.

The LTR property of the RHLQG/ FIRF controller is analyzed here and it is shown that the loop gain transfer function for the RHLQG/FIRF designs recovers those of the RHC law as the data observation window T approaches to zero and thus it recovers the associated robustness properties of RHC. At this stage, for investigating the LTR performance of RHLQG/FIRF, the mismatching function method [8,9] will be applied, where it is a phase difference of the target and achieved loop transfer function. The LTR performance of the RHLQG/FIRF is compared with the conventional LTR procedure [10,11,12] and Chen's method[8] by checking the singular values of the mismatch function.

This paper is organized as follows: In Section 2 we propose the RHLQG/FIRF control algorithm. In Section 3 we analyze the robustness of the RHC in frequency domain. In Section 4 we propose the LTR procedure of the RHLQG/FIRF control law and verify the LTR performance of the RHLQG/FIRF using the mismatching function. In Section 5, a numerical example is given to exemplify the LTR performance of the RHLQG/FIRF compared to the Chen's and conventional LTR methods. In Section 6 conclusions will be given.

2 Receding Horizon LQG Controller with FIRF

The particular theory we shall be concerned with here is that of the so-called RHLQG/FIRF problem. Let us consider the continuous time-varying state-space model

$$\dot{x}(t) = A_t x(t) + G_t u(t) + B_t w(t) \tag{1}$$

$$y(t) = C_t x(t) + v(t), \tag{2}$$

where $x(\cdot)$, $u(\cdot)$ and $z(\cdot)$ are the state vector, the control input vector, and the observation vector, respectively. The initial state vector $x(0)$ is a random variable with $E[x(0)] = m_0$ and $Cov[x(0)] = P_0$ and the system noise $w(\cdot)$ and the observation noise $v(\cdot)$ are zero-mean white with covariances $E[w(t)w^T(s)] = Q_t\delta(t-s)$ and $E[v(t)v^T(s)] = I\delta(t-s)$, respectively. We also assume that $x(0)$, $w(\cdot)$, and $v(\cdot)$ are uncorrelated each other. Although all matrices A_t, B_t, C_t, G_t and Q_t are time-varying, we will delete the subscript t, which represents time-dependence, for the convenience of mathematical description.

The RHLQG/FIRF controller combines the receding horizon controller [6] with the optimal FIR filter developed in [4] to stabilize stochastic linear time-varying systems. The RHLQG/FIRF problem is, under the system description of (1) and (2), to design a feedback control law which minimizes the control cost function

$$J_c = E\{x^T(t+T_c)F_c x(t+T_c) + \int_t^{t+T_c}[y^T(\tau)Q_y y(\tau) + u^T(\tau)R_u u(\tau)]d\tau\}, \quad (3)$$

where $y(t) := Cx(t)$, T_c is the width of the control cost interval and F_c, Q_y and R_u are weighting matrices with

$$F_c \geq 0, \quad Q_y \geq 0, \quad R_u > 0.$$

We assume that available information at present time t are F_c, Q_y and R_u over the interval $[t, t+T_c]$ and Q, R and the observation data z over the past interval $[t-T, t]$.

The solution to the RHLQG/FIRF problem is achieved by two step procedure. The first step is to determine the optimal FIR filter as follows:

$$\hat{x}(t \mid T) = \int_{t-T}^{t} H(t,\tau;T)z(\tau)d\tau + \int_{t-T}^{t} H_u(t,\tau;T)u(\tau)d\tau, \quad (4)$$

where the impulse responses $H(t,\cdot;T)$ and $H_u(t,\cdot;T)$ are calculated by

$$H(t,s;T) = S^{-1}(t,T)L(t,s;T), \quad t-T \leq s \leq t \quad (5)$$

$$\frac{\partial}{\partial\sigma}L(t,s;\sigma) = -[A^T + S(t,\sigma)BQB^T]L(t,s;\sigma), \quad 0 \leq T-t+s < \sigma \leq T \quad (6)$$

$$L(t,s;T-t+s) = C^T$$

$$\frac{\partial}{\partial\sigma}S(t,\sigma) = -S(t,\sigma)A - A^TS(t,\sigma) + C^TC - S(t,\sigma)BQB^TS(t,\sigma) \quad (7)$$

$$S(t,0) = 0, \quad 0 < \sigma \leq T$$

$$H_u(t,s;T) = \int_{t-T}^{s} H(t,\tau;T)C\Phi(\tau,s)Gd\tau, \quad t-T \leq s \leq t. \quad (8)$$

The second step is to find the control law which will minimize the cost function (3). The control law is determined by RHC method[6], using the FIR filter $\hat{x}(t \mid T)$ as the state estimator as shown below:

$$u(t) = -R_u^{-1}G^TK(t,t+T_c)\hat{x}(t \mid T) = -K_{RHC}(t,T_c)\hat{x}(t \mid T), \quad (9)$$

where $K_{RHC}(t,T_c) := R_u^{-1}G^TK(t,t+T_c)$ is the receding horizon control gain matrix and $K(t,t+T_c)$ satisfies the Riccati differential equation

$$-\frac{\partial}{\partial\sigma}K(\sigma,t+T_c) = A^TK(\sigma,t+T_c) + K(\sigma,t+T_c)A + C^TQ_yC - K(\sigma,t+T_c)GR_u^{-1}G^TK(\sigma,t+T_c) \quad (10)$$

$$K(t+T_c,t+T_c) = F_c, \quad t \leq \sigma < t+T_c.$$

The RHLQG/FIRF control law guarantees the closed-loop stability under some condition as follows:

THEOREM 1 *[4] If the system of (1) and (2) is uniformly completely controllable and observable and if G, B, Q, Q_y, and R_u are uniformly bounded, then the RHLQG/FIRF (9) stabilizes the system asymptotically with $F_c = \infty I$, $T \geq \ell_o$ and $T_c \geq \ell_c$, where ℓ_o and ℓ_c are the controllability and observability index, respectively.*

In case of linear time-invariant systems, the RHLQG/FIRF has very simple forms with the time-invariant state estimator and the constant gain feedback control as follows:

$$\hat{x}(t \mid T) = \int_{t-T}^{t} H(t-\tau;T)z(\tau)d\tau + \int_{t-T}^{t} H_u(t-\tau;T)u(\tau)d\tau, \quad (11)$$

$$u(t) = -K_{RHC}T_c\hat{x}(t \mid T) = -R_u^{-1}G^TK(T_c)\hat{x}(t \mid T), \quad (12)$$

where $K(T_c)$ is the constant solution of (10) on the interval $[0, T_c]$ and the impulse responses $H(\cdot;T)$ and $H_u(\cdot;T)$ are calculated by

$$H(t,T) = S^{-1}(t,T)L(t,T), \quad 0 \leq t \leq T \quad (13)$$

$$\frac{\partial}{\partial\sigma}L(t,\sigma) = -[A^T + S(\sigma)BQB^T]L(t,\sigma), \quad 0 \leq T-t < \sigma \leq T \quad (14)$$

$$L(t,T-t) = C^T$$

$$\frac{\partial}{\partial\sigma}S(\sigma) = -S(\sigma)A - A^TS(\sigma) + C^TC - S(\sigma)BQB^TS(\sigma) \quad (15)$$

$$S(t,0) = 0, \quad 0 < \sigma \leq T$$

$$H_u(t,T) = \int_t^T H(\tau,T)C\Phi(t-\tau)Gd\tau, \quad 0 \leq \sigma \leq T. \quad (16)$$

In this case, we can take T and T_c as any positive finite values.

3 Robustness of the Receding Horizon Control

In this section we consider robustness property of the RHC law. The LQ control law has been known to have excellent gain and phase margin in time-invariant systems. However, it appears to have poor performance when applied to the parameter variation and time varying systems. Hence we here compare the robustness performance of RHC with that of the LQ, which shows that RHC has better robustness property than the robustness property of LQ control law.

First of all, we introduce the FARE(Fake Algebraic Riccati Equation) [7] for the frequency domain equality of (10).

LEMMA 1 *Consider the FARE*

$$\bar{Q}(\sigma) = K(\sigma,t+T_c)GR_u^{-1}G^TK(\sigma,t+T_c) - A^TK(\sigma,t+T_c) - K(\sigma,t+T_c)A \quad (17)$$

and assume that
i) $[A, B]$ is a controllable pair,
ii) $\bar{Q}(\sigma) \geq 0$ and $[A, \bar{Q}(\sigma)^{1/2}]$ is a stabilizable pair.
Then the RHC law (9) asymptotically stabilizes the plant.

Proof: If we define $\bar{Q}(\sigma) = C^TQC + \dot{K}(\sigma,t+T_c)$, then (17) comes from (10). The connection between monotonicity of $K(\sigma,t+T_c)$ and stabilizability of $[A, \bar{Q}(\sigma)^{1/2}]$ then emerges [7].

Note that it is the stabilizability of the pair $[A, \bar{Q}(\sigma)^{1/2}]$ (when $\bar{Q}(\sigma) \geq 0$) that determines the asymptotic stability of (17), since then $K(\sigma, t+T_c)$ satisfies a legitimate ARE. □□□

Let us investigate the frequency domain characteristics of RHC for the time-invariant system as shown in the following theorem:

THEOREM 2 *The return difference matrix $F_{RHC}(s)$ of the system with the RHC satisfies the following relation in the frequency domain:*

$$F_{RHC}^T(-j\omega)R_u F_{RHC}(j\omega) = R_u + G^T\Phi^T(-j\omega)\bar{Q}\Phi(j\omega)G, \quad (18)$$

where $\Phi(s) := (sI - A)^{-1}$ and

$$\bar{Q} := C^T Q_y C + \frac{\partial}{\partial \tau}K(\tau, t+T_c)|_{\tau=t} \quad (19)$$

□□□

The above result comes from the same procedure as that of the LQ control law [13] using the definition (19). It is noted that $\bar{Q}$ of (19) is constant in case of the time-invariant system. From Theorem 2 we intuitively recognize that it is possible to show the robustness of the RHC compare to that of the LQ control law. Note that in RHC problem $K(\cdot, t+T_c)$ is a monotone increasing function and that $\frac{\partial}{\partial\tau}K(\tau, t+T_c)|_{\tau=t} \geq 0$ for all t. Thus we have $\bar{Q} \geq C^T Q_y C$ and $\sigma_{max}\{F_{RHC}^{-1}(j\omega)\} \leq \sigma_{max}\{F_{LQ}^{-1}(j\omega)\}$ where $F_{LQ}(s)$ is the return difference matrix of the LQ regulator. We have then

$$\sigma_{max}\{I - F_{RHC}^{-1}(j\omega)\} \leq \sigma_{max}\{I - F_{LQ}^{-1}(j\omega)\}, \quad (20)$$

which implies that the RHC is robuster in stability than the LQ control law since the smaller the upper bound of the maximum singular value of the closed-loop transfer function $\sigma_{max}\{I - F^{-1}(j\omega)\}$ is, the robuster is the feedback control system.

4 LTR of RHLQG/FIRF

In this section we consider three factors of the RHLQG/FIRF control law. First, using the results in Section 3, we present the structure of RHLQG/FIRF controller which is determined by the frequency domain transfer function of the RHL QG/FIRF. Second, for enhancing the robustness of the RHLQG /FIRF control law, we introduce a new LTR method which makes the loop transfer function of the RHLQG/FIRF control law recover that of the RHC law. Finally, we compare the LTR performance of the RHLQG/FIRF control law with that of Chen's [8] method and show that the former has the better LTR performance than the latter.

4.1 Loop Transfer Function of RHLQG/FIRF Controller

Let us derive the loop transfer function for the time-invariant system with the RHLQG/FIRF controller as shown in Fig.4.1. It is constructed by the gain of the RHC and FIR filter transfer function.

The transfer function of the time-invariant FIR filter is obtained from (11) and (12) as follows:

$$\begin{aligned}\hat{X}(s) &= H(s;T)Y(s) + H_u(s;T)U(s) \\ &= H(s;T)Y(s) - H_u(s;T)K_{RHC}\hat{X}(s) \quad (21)\end{aligned}$$

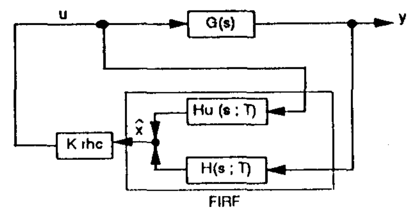

Fig. 4.1 The structure of the RHLQG/FIRF controller

Then (21) yields

$$\hat{X}(s) = [I + H_u(s;T)K_{RHC}]^{-1}H(s;T)Y(s) \quad (22)$$

and we have

$$\begin{aligned}U(s) &= -K_{RHC}\hat{X}(s) \\ &= -K_{RHC}[I + H_u(s;T)K_{RHC}]^{-1}H(s;T)Y(s) \\ &= -K_{R/F}(s)Y(s) \quad (23)\end{aligned}$$

where $K_{R/F}(s) = K_{RHC}[I + H_u(s;T)K_{RHC}]^{-1}H(s;T)$.

Thus, we can obtain the following loop transfer function of the RHLQG/FIRF control law:

$$\begin{aligned}T_{R/F} &:= K_{R/F}(s)P(s) \\ &= K_{RHC}[I + H_u(s;T)K_{RHC}]^{-1}H(s;T)C\Phi(s)G \quad (24)\end{aligned}$$

where $\Phi(s) = (sI - A)^{-1}$. We can derive the relationship between $H_u(s;T)$ and $H(s;T)$ as follows:

LEMMA 2 *The transfer function $H_u(s;T)$ is represented by*

$$H_u(s;T) = [I - H(s;T)C]\Phi(s)G \quad (25)$$

Proof: Applying the Laplace transformation to (16), we have

$$\begin{aligned}H_u(s;T) &= \int_0^\infty [\int_t^T H(\tau;T)C\Phi(t-\tau)Gd\tau]e^{-st}dt \\ &= \int_0^T [\int_t^T H(\tau;T)C\Phi(t-\tau)d\tau]e^{-st}dtG \\ &= \int_0^T H(\tau;T)C[\int_0^\tau \Phi(t-\tau)e^{-st}dt]d\tau G \\ &= \int_0^T H(\tau;T)C[e^{-A\tau} - e^{-s\tau}I]d\tau\Phi(s)G \quad (26)\end{aligned}$$

Here we can show that [4]

$$\int_0^T H(\tau;T)Ce^{-A\tau}d\tau = I. \quad (27)$$

Substitution of (27) into (26) gives (25) □□□

4.2 LTR Procedure of the RHLQG/FIRF

In order to enhance the robustness and performance of the RHL QG/FIRF control law, we shall apply in this subsection the LTR method to it. We will show that, as T approaches to zero, the loop transfer function of RHLQG/ FIRF is able to recover that of the RHC.

LEMMA 3 *The loop transfer function of RHLQG/FIRF controller recovers that of RHC as T approaches to zero, i.e.,*

$$\begin{aligned} T_{R/F}(s) &= K_{RHC}[I + H_u(s;T)K_{RHC}]^{-1}H(s;T)C\Phi(s)G \\ &\rightarrow K_{RHC}(sI - A)^{-1}G = T_{RHC}(s) \end{aligned} \quad (28)$$

as $T \rightarrow 0$.

Proof: From (13)-(16) we can obtain

$$\begin{aligned} H(s;T)C &= S^{-1}L(s;T)C \\ &= [\int_0^T \Psi(\tau,T)C^TR^{-1}C\Phi(\tau,T)d\tau]^{-1}\int_0^T \Psi(T-t,T)C^TR^{-1}Ce^{-st}dt \\ &= [\int_0^T \Psi(\tau,T)C^TR^{-1}C\Phi(\tau,T)d\tau]^{-1}\int_0^T \Psi(\tau,T)C^TR^{-1}Ce^{-s(T-\tau)}d\tau \\ &\rightarrow I \end{aligned} \quad (29)$$

as $T \rightarrow 0$, where $\Psi(\cdot,\cdot)$ is the transition matrix of $[A^T + S(\sigma)BQB^T]$. We have then from (29)

$$H_u(s;T) = [I - H(s;T)](sI - A)^{-1}G \rightarrow 0 \quad (30)$$

as $T \rightarrow 0$. Thus we can obtain the LTR property (24) of RHLQG/FIRF control law. □□□

To examplify the LTR procedure of the RHLQG/ FIRF let us examine the scalar system.

In the scalar system we can show some limiting properties with respect to $H(s;T)$ and $H_u(s;T)$ as follows:

Example:

$$\begin{aligned} H(s;T) &= \frac{l^2}{c(e^{\gamma T} - e^{-\gamma T})}\{\frac{\alpha e^{-\gamma T}}{(s-\gamma)}[1 - e^{-(s-\gamma)T}] - \frac{\beta e^{\gamma T}}{(s+\gamma)}[1 - e^{-(s+\gamma)T}]\} \\ &\rightarrow \frac{l^2}{2c\gamma}\{\frac{\alpha}{(s-\gamma)}(s-\gamma) - \frac{\beta}{(s+\gamma)}(s+\gamma)\} \\ &= \frac{l^2}{2c\gamma}(\alpha - \beta) \\ &= \frac{1}{c} \quad as \quad T \rightarrow 0, \end{aligned} \quad (31)$$

where $l^2 = b^2q$, $\alpha = \frac{-a+\gamma}{l^2}$, $\beta = \frac{-a-\gamma}{l^2}$, $\gamma = \sqrt{a^2 + l^2c^2r^{-1}}$, $\alpha - \beta = \frac{2\gamma}{l^2}$. Hence we have $H(s;T)c \rightarrow 1$ *as* $T \rightarrow 0$. Also we have

$$\begin{aligned} H_u(s;T) &= \frac{g}{c(e^{\gamma T} - e^{-\gamma T})}\{\frac{e^{\gamma T}}{(s+\gamma)}[1 - e^{-(s+\gamma)T}] - \frac{e^{-\gamma T}}{(s-\gamma)}[1 - e^{-(s-\gamma)T}]\} \\ &\rightarrow \frac{g}{2\gamma}\{\frac{1}{(s+\gamma)}(s+\gamma) - \frac{1}{(s-\gamma)}(s-\gamma)\} \\ &= 0 \quad as \quad T \rightarrow 0. \end{aligned} \quad (32)$$

Thus we can obtain the LTR procedure in scalar case as follows:

$$\begin{aligned} T_{R/F} &= K_{RHC}[I + H_u(s;T)K_{RHC}]^{-1}H(s;T)c(sI - a)^{-1}g \\ &\rightarrow K_{RHC}(sI - a)^{-1}g \\ &= T_{RHC} \quad as \quad T \rightarrow 0. \end{aligned}$$

Note this LTR results implies that the LTR procedure is performed by diminishing T only and the LTR can be performed independently of the RHC gain K_{RHC}.

4.3 LTR performance of the RHLQG/FIRF

The purpose of this subsection is that the LTR performance of the RHLQG/FIRF control law is examined by using the mismatching function proposed by Chen et. al [8]. Most often in the literature, the maximum and minimum singular value graphs of the target and achieved loop transfer matrices are drawn with respect to ω and are then compared. These graphs could be misleading. Although the singular values of target and achieved loop transfer matrices may match perfectly, the difference or mismatch between them. It is the best way is to check the singular values of the mismatch function between the target and achieved loop transfer function. Let us consider the target loop transfer function of the RHLQG /FIRF is given $L(s) = K_{RHC}\Phi(s)G$. The transfer function of the observer based controller, i.e. the transfer function from the output y of the plant to $\hat{u}$ is

$$C_{R/F}(s) = K_{RHC}[I + H_u(s;T)K_{RHC}]^{-1}H(s;T) \quad (33)$$

where $C_{R/F}(s)$ is the controller of the RHLQG/FIRF and the open-loop transfer function when the loop is broken at the input point of the plant is $L_o = C_{R/F}(s)P(s)$ where $P(s) = C\Phi G$. Thus the error or mismatch between the target loop transfer function $L(s)$ and that realized by the FIR observer is

$$E_{R/F} = L(s) - L_o(s) \quad (34)$$

where $E_{R/F}$ is the error function of the RHLQG/FIRF. The following two lemma represents the result of the mismatching error conditions for analysis of LTR performance, which can be shown by [8].

LEMMA 4 *The error between the target loop transfer function L(s) and that realized by the RHLQG/FIRF controller is given by*

$$\begin{aligned} E_{R/F} &= M_{R/F}\{(K_{RHC}^{-1} + H_u(s;T))M_{R/F}\}^{-1} \\ &\quad \times\{(I + H_u(s;T)K_{RHC}) - H(s;T)C\}\Phi(s)G \end{aligned} \quad (35)$$

where

$$M_{R/F}(s) = K_{RHC}(I + H(s;T)C)^{-1} \quad (36)$$

$M_{R/F}(s)$ is the mismatching function of the RHLQG/FIRF. □□□

LEMMA 5 *Perfect matching condition of LTR is given as follows:*

$$E_{R/F} = 0 \quad iff \quad M(j\omega) = 0 \quad for \quad all \quad \omega \in \Omega \quad (37)$$

where Ω is the set of all $0 \leq \omega < \infty$ for which $L_o(j\omega)$ and $L(j\omega)$ are well defined(i.e. all required inverses exist).

□□□

The robust stability and nominal performance of a system are directly reflected in the singular values of sensitivity and complementary sensitivity functions whereas the level of recovery(i.e. the size of $E_{R/F}(j\omega)$) is directly dependent on the singular values of $M(j\omega)$. With this point of view, [12] derive some analytical expressions for the discrepancy between the desired and the achieved sensitivity and complementary sensitivity functions. The results of this things are given in the following lemma.

LEMMA 6 *[8] Consider the configuration of Fig.4.1 we have the following bounds on all singular values i = 1 to m of $S_{R/F}$ and $T_{R/F}$:*

$$\frac{|\sigma_i[S_{R/F}(j\omega)] - \sigma_i[S_F(j\omega)]|}{\sigma_{max}[S_F(j\omega)]} \leq \sigma_{max}[M_{R/F}(j\omega)], \qquad (38)$$

and

$$\frac{|\sigma_i[T_{R/F}(j\omega)] - \sigma_i[T_F(j\omega)]|}{\sigma_{max}[S_F(j\omega)]} \leq \sigma_{max}[M_{R/F}(j\omega)], \qquad (39)$$

where $S_{R/F}$ is the sensitivity function of the RHLQG/FIRF and $T_{R/F}$ is the complementary sensitivity function of the RHLQ G/FIRF.

Proof: Appendix E in [8]. □□□

The expressions given above can be used to analyze the trade-off between good recovery as indicated by $\sigma_{max}[M_{R/F}(j\omega)]$ and robustness and performance as reflected in the sensitivity and complementary sensitivity functions.

The following theorem shows that the LTR performance of RHLQG/ FIRF achieves better degree of recovery than the Chen's method and conventional observer based controller.

THEOREM 3 *Let us assume that*

$$\sigma_{min}[L(j\omega)] = \sigma_{min}[K_{RHC}((j\omega) - A)^{-1}G] \geq \alpha \ \ for\ all \ \ \omega \in D. \quad (40)$$

where α is arbitrary constant , $\alpha \leq 1$ for some frequency region of interest, D. Then for all $\omega \in D$, the mismatch between the target loop transfer function and the RHLQG/FIRF is less than the Chen's method and conventional LQG/LTR. More specifically, we have

$$\sigma_{max}[E_{LQG/LTR}(j\omega)] \geq \sigma_{max}[E_{CHEN}(j\omega)] \geq \sigma_{max}[E_{R/F}(j\omega)] \qquad for\ all \ \ \omega \in D, \quad (41)$$

where $E_{R/F}(s)$ is as in (35), $E_{LQG/LTR}(s)$ and $E_{CHEN}(s)$ are as in [8].

Proof: Recalling the expression for $E_{LQG/LTR}(j\omega)$, we have

$$\begin{aligned} \sigma_{max} \ & \{E_{LQG/LTR}(j\omega)\} \\ = \ & \sigma_{max}\{M(j\omega)[I + M(j\omega)]^{-1}(I + K_{LQ}\Phi(j\omega)G)\} \\ \geq \ & \sigma_{max}[M(j\omega)]\sigma_{min}\{[I + M(j\omega)]^{-1}\} \\ & \times\sigma_{min}[I + K_{LQ}\Phi(j\omega)]^{-1}\} \\ = \ & \frac{\sigma_{max}[M(j\omega)]\sigma_{min}[I + K_{LQ}\Phi(j\omega)G]}{\sigma_{max}[I + M(j\omega)]} \\ \geq \ & \sigma_{max}[E_{CHEN}(j\omega)]\alpha(j\omega) \qquad (42) \end{aligned}$$

where

$$\alpha(j\omega) = \frac{\sigma_{min}[K_{LQ}\Phi(j\omega)G]}{\sigma_{max}[I + M(j\omega)]}.$$

and K_{LQ} is a LQ gain. Now by our assumption, $\sigma_{max}[M(j\omega)]$ is $\leq \alpha$ and σ_{min} $[K_{LQ}\Phi(j\omega)G]$ is ≥ 1 for all $\omega \in$ D and hence $\alpha(j\omega)$ is ≥ 1 for all $\omega \in$ D. Thus

$$\sigma_{max}[E_{LQG/LTR}(j\omega)] \geq \sigma_{max}[E_{CHEN}(j\omega)]. \qquad (43)$$

Since we have the following RHLQG/FIRF controller related by the first line of (42)

$$\begin{aligned} & \sigma_{max}[M_{R/F}]\sigma_{min}[K_{RHC}^{-1} + H_u(s;T)M_{R/F}]^{-1} \\ & \times[I + H_u(s;T)K_{RHC} - H(s;T)C\Phi G]. \qquad (44) \end{aligned}$$

By the monotonicity of the Riccati equation, we can compare the magnitude of the RHC gain K_{RHC} with the LQ gain K_{LQ}. Hence we can obtain the following relation $K_{RHC} \geq K_{LQ}$ since the monotone increasing property of the finite interval Riccati equation. Therefore following is satisfied

$$\sigma_{max}[M_{R/F}] \leq \sigma_{max}[M_C(j\omega)] \qquad (45)$$

$$\sigma_{max}[(K_{RHC}^{-1} + H_u(s;T))M_{R/F}]^{-1} \geq \sigma_{max}[I + M_C(j\omega)]^{-1} \qquad (46)$$

$$\sigma_{max}[(I + H_u(s;T)K_{RHC})] \leq \sigma_{max}[I + K_{LQ}\Phi G] \qquad (47)$$

Hence the following relation is obtainded

$$\begin{aligned} & \sigma_{max}\{M_C(j\omega)[I + M_C(j\omega)]^{-1}(I + K_{LQ}\Phi(j\omega)G]\} \\ & \geq \sigma_{max}\{M_{R/F}[K_{RHC}^{-1} + H_u(s;T)M_{R/F}]^{-1} \\ & \times[(I + H_u(s;T)K_{RHC}) - H(s;T)C]\Phi G\}. \qquad (48) \end{aligned}$$

Thus we have

$$\sigma_{max}[E_{CHEN}(j\omega)] \geq \sigma_{max}[E_{R/F}(j\omega)]. \qquad (49)$$

and we have the result (41) from (43) and (49)

5 Examples

In this section we presents two examples to compare the LTR performance of the RHLQG/FIRF with Chen's method and conventional observer based LQG/LTR approach. Performances of all methods are here represented by the maximum singular value of the LTR error function. Therefore the LTR performance of the each control methods is quantified by mismatching function proposed by the [8].

To show the performance of the RHLQG/FIRF we shall now consider two kinds of systems with minimum phase zero and nonminimum phase zeros.

Case 1. The simulation is performed on minimum phase systems given by Doyle [11]

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 35 \\ -61 \end{bmatrix} w \qquad (50)$$

$$y = [2 \quad 1]x + v \qquad (51)$$

with $E(w) = E(v) = 0$ and $E[w(t)w(\tau)] = E[v(t)v(\tau)] = \delta(t-\tau)$. An LTR procedure of the RHLQG/FIRF control for minimum phase system is given in Fig. 5.1 The mismatching errors and maximum error value is given in Fig. 5.2, Fig. 5.3, respectively. This results show that the LTR performance of the RHLQG/FIRF has better performance than Chen's and LQG/LTR method when the LTR procedure is applied to systems with minimum phase systems to input break point.

Case 2. The simulation is performed on nonminimum phase systems is given by Saeki [14] by some modification of Doyle's plant.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 35 \\ -61 \end{bmatrix} w \qquad (52)$$

$$y = [2 \quad -0.1]x + v \tag{53}$$

with $E(w) = E(v) = 0$ and $E[w(t)w(\tau)] = E[v(t)v(\tau)] = \delta(t-\tau)$. Each LTR procedure is compared in Fig. 5.5 and the mismatching errors and maximum error values are given in Fig. 5.6, Fig. 5.7, respectively. This results also show that the LTR performance of the RHLQG/FIRF has better performance than Chen's and LQG/LTR method when the LTR procedure is applied to systems with nonminimum phase system to input break point. Clearly all the examples support the theoretical improvement given earlier and demonstrate that the RHLQG/FIRF approach is much better than Chen's method and LQG/ LTR method.

6 Conclusions

In this paper we have presented the robust performance of the RHLQG/FIRF. The robustness of the RHC is analyzed in frequency domain comparing the LQ control given in Theorem 2. The robustness of the FIR filter is analyzed here compared with kalman filter by its transfer function of the FIR filter. These results are given in Lemma 3. For enhancing the robustness of the RHLQG/FIRF we proposed here the LTR approach which is given in Section 3. It is noted that the RHLQG/FIRF always recovers its robust performance because of remarkable robust property of the RHC which is given by the Section 4. To conform the LTR performance here we applied Chen's method [8] to RHLQG/FIRF in Section 5. The examples showed that the RHLQG/FIRF has better LTR performance than Chen's method and LQG/LTR approach in terms of smaller values of the mismatching function.

Acknowledgement

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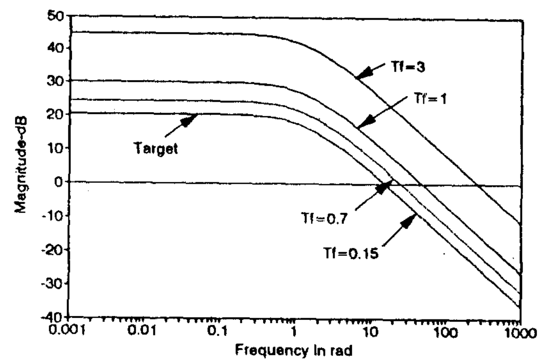

Fig. 5.1 LTR procedure of RHLQG/FIRF for minimum phase system

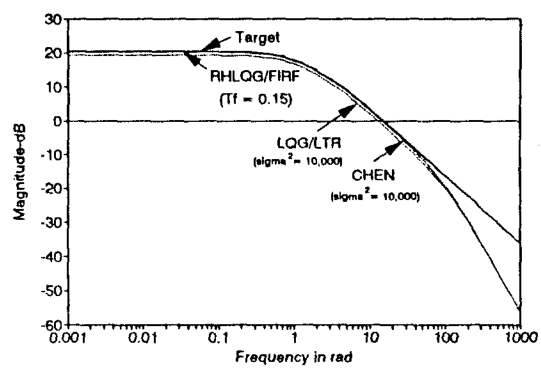

Fig. 5.2 Comparison of each LTR procedure of minimum phase system

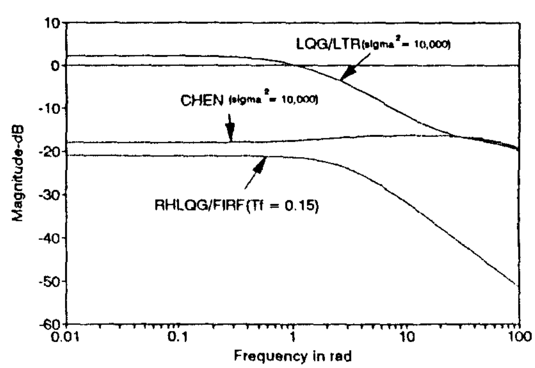

Fig. 5.3 Mismatching error for minimum phase system

<table>
<tr><th></th><th>$\sigma_{max}[E_{R/F}(j\omega)]$</th><th>$\sigma_{max}[E_{CHEN}(j\omega)]$</th><th>$\sigma_{max}[E_{LQG/LTR}(j\omega)]$</th></tr>
<tr><td>T_f = 3</td><td>166.3810</td><td rowspan="4">0.1524
(σ^2 = 10,000)</td><td rowspan="4">1.3010
(σ^2 = 10,000)</td></tr>
<tr><td>T_f = 1</td><td>42.7013</td></tr>
<tr><td>T_f = 0.7</td><td>27.3303</td></tr>
<tr><td>T_f = 0.15</td><td>0.0887</td></tr>
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Fig. 5.4 Maximum error value for minimum phase system

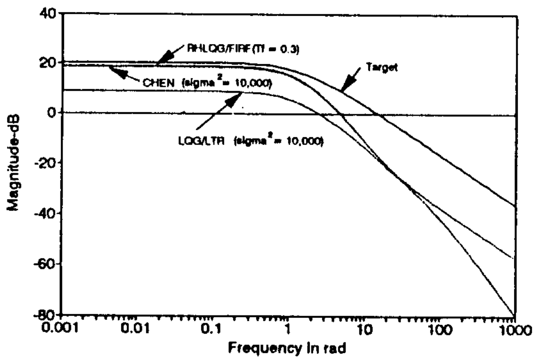

Fig. 5.5 Comparison of each LTR procedure of nonminimum phase system

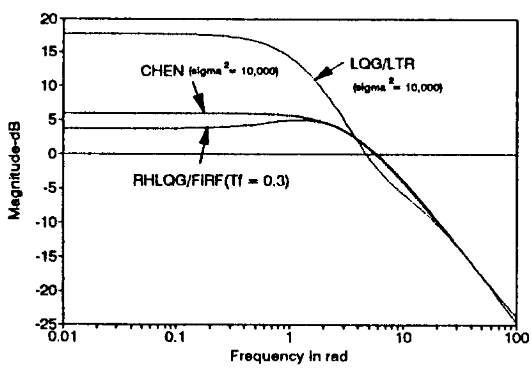

Fig. 5.6 Mismatching error for nonminimum phase system

	$\sigma_{max}[E_{R/F}(j\omega)]$	$\sigma_{max}[E_{CHEN}(j\omega)]$	$\sigma_{max}[E_{LQG/LTR}(j\omega)]$
T_f = 0.3	1.7858	2.0057 (σ^2 = 10,000)	7.7376 (σ^2 = 10,000)

Fig. 5.7 Maximum error value for nonminimum phase system