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State Estimation based on Fuzzy Finite State Transition Model

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Abstract

In this paper, we attempt to estimate the state of a finite state system. In such system, we can observe time series data which has some significant behaviors corresponding to its system states. The behavior is characterized by feature parameters extracted from time series. Our thought is that the system output time series data is expressed as a sequence of behavior patterns which are represented by clusters in feature parameters space. An algorithm jointing fuzzy clustering to fuzzy finite state transition model is suggested.

1. Introduction

There are systems which have some different properties. For example, a system which has some structures and changes its structure according to its operating conditions is one of these systems. We can interpreted that such a system has some properties respective to structures. In this system, movements of output are similar to each other while the system is keeping a certain property, and different movements can be observed when the system changes its property.

The outline of our thought is as follows. The behavior in a short part of time series data is represented as a set of feature parameters extracted from that part of data . Thus similar behaviors make a cluster in feature parameter space. It would be ideal that the clusters mean the states of system respectively, but it cannot be expected because extracted feature parameters aren't sufficient to distinguish one state from the others. In addition, boundaries of the clusters aren't clear. Consequently , when the state for the part of data is estimated, we encounter the nonspecificity such that it is difficult to choose one state from candidates. To deal with this difficulty, a fuzzy finite state transition model is formed such that states of the model are corresponding to the system states respectively and its state transition keeps constraints about changes of actual system state , and state estimation is done based on a transition of the model.

The estimation method which is suggested in this paper consists of two procedures. In the first stage, using fuzzy clustering method for the feature parameters extracted from time series data obtained in advance, some fuzzy clusters are made. In the second stage, the state are estimated , comparing the feature parameters extracted from the part of the analyzed time series with the fuzzy clusters . We calculate the possibilities, the grades that each part of the time series belongs to the clusters in the feature parameters space and the fuzzy model state transition are done by these possibilities. As the fuzzy model is changing its state according to the possibilities belonging to the clusters, we get another possibilities that the processed part corresponds to any state . Based on the later possibilities , the candidates are given, the nonspecificity is evaluated, and the estimated state is judged.

In section 2, we review fuzzy algorithms used in our estimation method. Section 3 presents our estimation method and in chapter 4 we describe an application of this algorithm to a practical problem.

2. Fuzzy Algorithms

2.1 Fuzzy Finite State Transition Model

Fuzzy finite state transition model is defined as follows.

$\Sigma(U,S,f,h)$,

where $U=\{u_1,u_2,\cdots,u_m\}$ is the set of inputs,

$S=\{S_1,S_2,\cdots,S_n\}$ is the set of states,

$f : S \times U \times S \to [0,1]$,
$h : S \to [0,1]$.

$f(S_i, u_j, S_k)$ is a membership value which means a possibility such that the model will be in state S_k, if the model is in state S_i and receives input u_j. $h_t(S_i)$ denotes a possibility such that the model state is S_i at time t. Then state transition is given by

$$h_{t+1}(S_k) = \max_{j=1,\cdots,m} [\max_{i=1,\cdots,n} [f(S_i, u_j, S_k) \wedge h_t(S_i)]] \qquad (1)$$

2.2 Fuzzy Clustering[1)]

The fuzzy c-means clustering algorithm organizes clusters such that patterns within a cluster are more similar to each other than are patterns within different clusters by minimizing the following objective function with respect to fuzzy membership v_{ij} and cluster centroid w_i.

$$J(V,W) = \sum_{i=1}^{c}\sum_{j=1}^{M}(v_{ij})^p \|\theta^j - w_i\|^2 \qquad (2)$$

where $\{\theta^1, \theta^2, \cdots, \theta^M\}$ are date points, M is the number of data, c is the number of clusters, v_{ij} denotes the membership of data point θ^j belonging to i cluster, p $(1 \le p < \infty)$ is fuzzy index, and w_i is the cntroid vector of i cluster. v_{ij} ($i = 1,\cdots,c$, $j = 1,\cdots,M$) are rewritten by division matrix

$V = [v_{ij}]$,

where $\sum_{j=1}^{M} v_{ij} > 0$, $i \in \{1,2,\cdots,c\}$, $\sum_{i=1}^{c} v_{ij} = 1$, $j \in \{1,2,\cdots,M\}$.

The necessary conditions to minimize object function are

$$w_i = \sum_{j=1}^{M}(v_{ij})^p \theta^j \Big/ \sum_{j=1}^{M}(v_{ij})^p \in R^r,\ i = 1,2,\cdots,c \qquad (3)$$

$$v_{ij} = \frac{1}{\sum_{k=1}^{c}\left(\frac{\|\theta^j - w_i\|^2}{\|\theta^j - w_k\|^2}\right)^{1/(p-1)}} \quad i = 1,\cdots,c,\ j = 1,\cdots,M. \qquad (4)$$

Fuzzy clustering is executed in the following steps.

1) Give c, p , and initial values of division matrix $V^{(0)}$.
2) Compute centroid vector $W^{(\ell)} = \{w_1^{(\ell)}, w_2^{(\ell)}, \cdots, w_c^{(\ell)}\}$ by (3) , where ℓ denotes iteration number.
3) Update the division matrix $V^{(\ell)}$.
   If $\theta^j \ne w_i^{(\ell)}$, compute $V^{(\ell+1)}$ by (4),
   otherwise $v_{ij}^{(\ell+1)} = \begin{cases} 1 & j = i \\ 0 & j \ne i \end{cases}$.
4) Repeat 2) and 3) until $\|V^{(\ell+1)} - V^{(\ell)}\| \le \varepsilon$ for ε.

2.3 Nonspecificity[3)]

It is assumed that only one of n events cm occur at a time. When possibilities of events occurring, $1 \ge \rho_1 \ge \rho_2 \ge \cdots \ge \rho_n \ge \rho_{n+1} = 0$, are given , the difficulty to specify the occurring event is evaluated by

$$Un(\rho) = \sum_{i=1}^{n}(\rho_i - \rho_{i+1}) log_2 i \ . \qquad (5)$$

It is called nonspecificity.

3. Estimation Algorithm using Fuzzy Finite State Transition Model

3.1 Inputs of Model

In advance , data including outputs for the various system states are obtained through preliminary experiments and primary clusters are made by clustering for those data. The clusters which have been made in this way are interpreted as inputs of the fuzzy finite state transition model , and converging division matrix is written as V_c . It is assumed that the number of data points is M and the number of clusters is m. The memberships of feature parameter vectors $\{\theta^1, \theta^2, \cdots, \theta^N\}$ belonging to each cluster are computed by fuzzy clustering remarked in section 2.2 with initial division matrix $V^{(0)} = [V_C \vdots V_0]$, which is extended for analyzed data. Thus we get memberships of θ^t belonging to input u_i by

$\mu_{u_i}(\theta^t) = v_{i(M+t)}$ $\quad i = 1,\cdots,m,\ t = 1,\cdots,N$

where $V = [v_{ik}]$ $\quad (i = 1,\cdots,m,\ k = 1,\cdots,M, M+1,\cdots,M+N)$.

3.2 State Estimation Algorithm

The state is estimated based on the transition on fuzzy finite state transition model , evaluating the nonspecificity at each step.

State Estimation by fuzzy finite state transition model is executed as following steps.

1) Give $h_t(S_i)$, $i \in \{1,2,\cdots,n\}$ initial values.
2) Compute possibilities $h_t(S_i), i \in \{1,2,\cdots,n\}$ by (1).
3) Judge estimated state based on nonspecificity.

If nonspecificity is less , than given δ, then estimated state is the state whose possibility is largest. Otherwise the transition is executed in next step without decision of estimation and validity is evaluated by possibilities of these 2 steps. The estimation algorithm is shown in Fig.1.

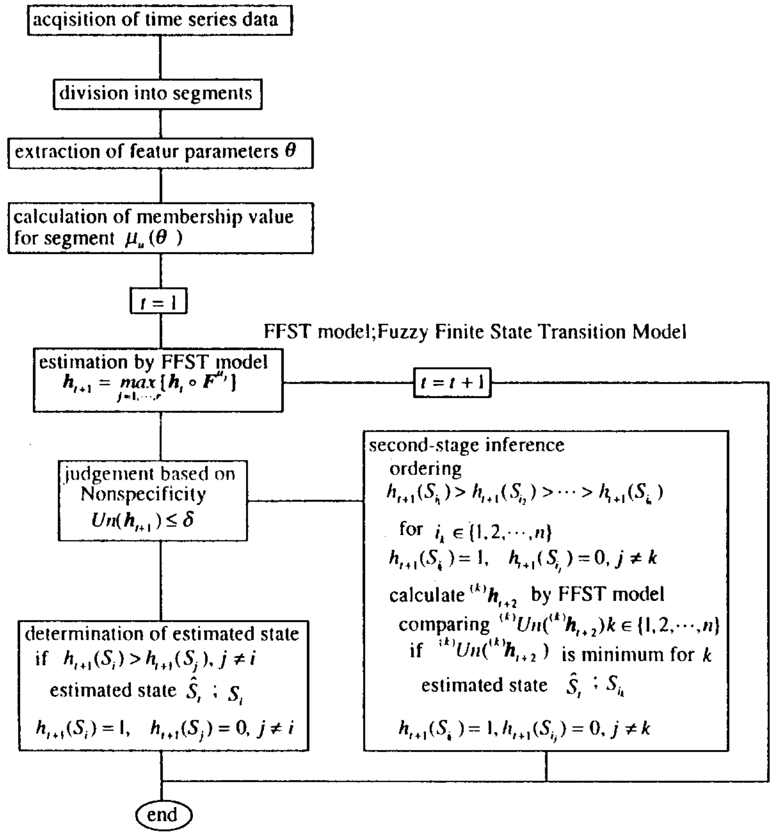

Fig. 1 State Estimation Algorithm

4. Application to Card Counter

In this section, an application of this estimation algorithm to the machine which counts number of magnetic cards, called 'Card Counter', is described. In Fig.2, a structure of Card Counter is shown. Cards are set in order on the card counter and an optical censor runs under the edges of cards and gets the light reflected by the edgas in the machine. Magnetic card has two layer, one made of plastic and the other made of magnetic material. Reflecting from magnetic layer is weaker than that from plastic layer, because magnetic layer is daker than plastic layer.

One cycle of sinelike wave shows one card to be counted, in just usage. This fact permit us to count number of cycles instead of number of cards. This card counter sometimes shows wrong number or can't count. These cases are caused by illegal way to set cards, cards with paint and crack in their edges, or cards with different structure from ordinary card's. In these cases, we get signals which include extraordinary behaviors corresponding to illegal cards. Present algorithm for counting fails to process these kinds of signals. To improve this machine, we should specify the part showing unusual behaviors , and estimate number of cards corresponding to the part of signal .

We can observe two kinds of behaviors in the signal. One is periodic and is such as sine wave. The signal with this kind of behavior corresponds to cards normally set on counter. The other might arise from illegal cards and various kinds of movements are observed in it. Also such movement begins without any sign. Our purpose is to distinguish the latter part from the whole signal.

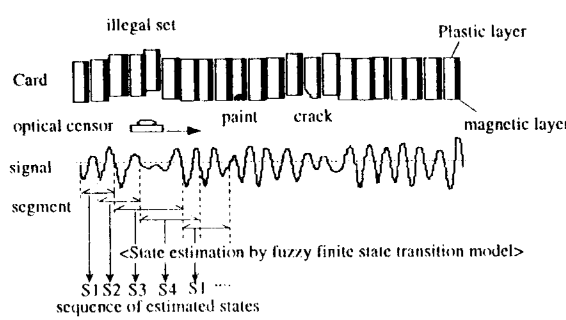

Fig. 2 Card Counter

Patterns of Behavior

The signal is divided into the short parts which are composed of two cycles of wave, that is, signal from first zero-crossing point and fifth zero-crossing point. These parts are formed in the way that the second wave of a part is the first wave of the next part (See Fig.2). There are two kinds of movements, the movement which is close to sine wave and has almost constant amplitude and period, and the others. Let first movement be called normal movement and others be called abnormal movements. We prepare four states for the divided part of signal. Since the divided part has two waves in it, the divided part is in state S_1, if both waves show normal movements. In state S_2, first wave is seen normal and second is seen abnormal. Both are abnormal in S_3 and first wave is abnormal and second is normal in S_4.

Feature Parameters

The feature parameters shown in Fig. 3 are taken . These feature parameters are selected from 11 feature parameters to distinguish the states as well as possible. Fuzzy clustering are executed for 200 data points and 3 clusters are made according to the criterion of the number of clusters shown in Fig.4. This criterion is given by

$$Q=\sum_{j=1}^{M}\sum_{i=1}^{c} v_{ij}{}^{p}\left(\|\theta^{j}-w_i\|^2-\|w_i-\Theta\|^2\right)/Var(\theta)$$

where $\Theta=\sum_{j=1}^{M}\theta^{j}/M$, $Var(\theta)=\sum_{j=1}^{M}\|\theta^{j}\|^2/M$.

But the clusters made by fuzzy clustering are overlapped and aren't corresponding to the states respectively.

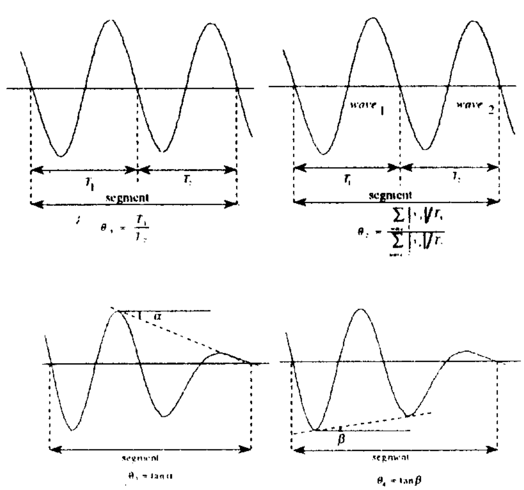

Fig. 3 Featur Parameters

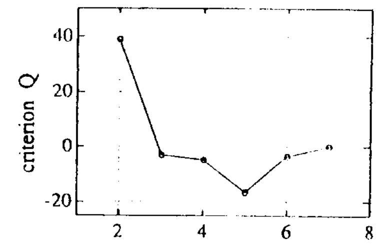

Fig.4 Criterion for number of cluster

Estimation by Fuzzy Finite State Transition Model

Since S_i (i=1,2,3,4) is taken as mentioned above, some constraints about transition of the fuzzy finite state transition model arise. For instance, the transition from S_1 to S_4, from S_1 to S_3, or from S_2 to S_1 are impossible. These constraints are written by state connection matrices as follows.

$$A^{u_1}=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&1&0\\1&0&0&0\end{bmatrix},\quad A^{u_2}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&1&0\\0&1&0&0\end{bmatrix}$$

$$A^{u_3}=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{bmatrix}$$

where i-j element of A^{u}, a_{ij}=1 means that transition from S_i to S_j is possible and a_{ij}=0 means that the transition is impossible. This state transition diagram is shown in Fig. 5.

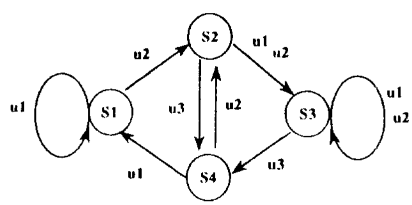

Fig. 5 State Transition Diagram

Usinng the state conection matrices and memberships μ_{u_i} (1) is written by

$$h_{t+1}=\max_{j=1\cdots4}[h_t\circ F^{u_j}]$$

where $F^{u_j}=[f(S_i,u_j,S_k)],\ i,j=1,2,\cdots,4,$

$F^{u_j}(\theta^t)=\mu_{u_j}(\theta^t)A^{u_j},i,j=1,2,\cdots,4,$

$h_t=[h_t(S_1),h_t(S_2),\cdots,h_t(S_n)].$

Memberships values belonging to input set with respect to each feature parameter are shown in Fig.6.

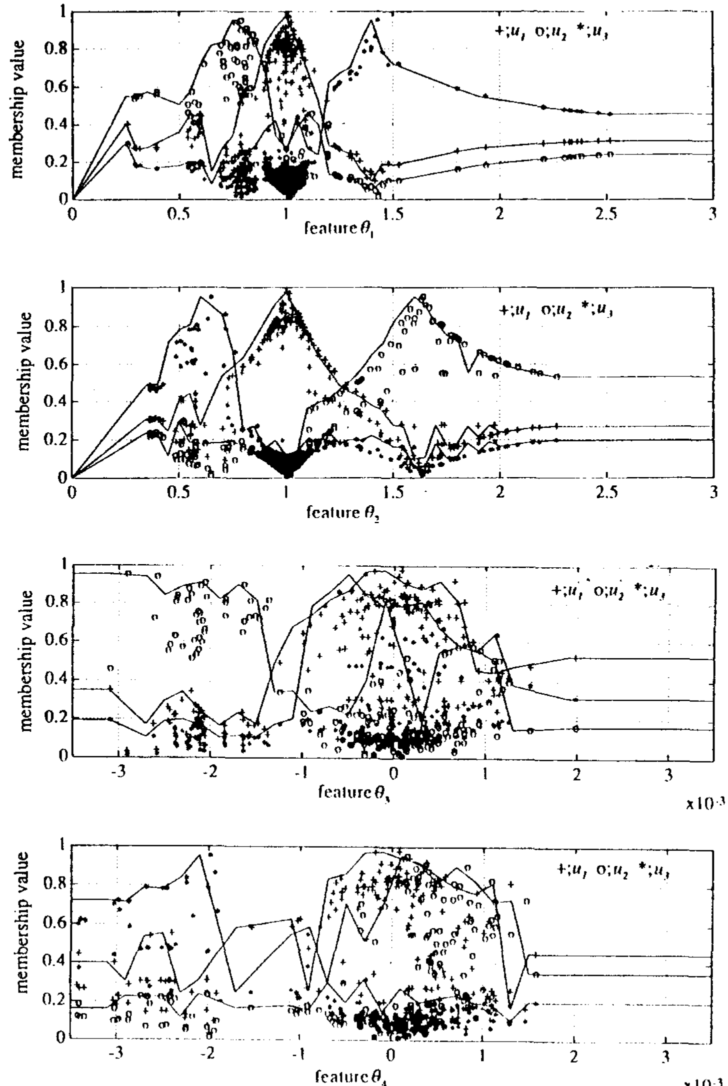

Fig. 6 Membership value belonging to input set

Results of estimation by this method for the signal in Fig. 7 is shown in Table 1. In this experiment, 50 cards are to be counted actually. According to the number of zero-crossing points of the signal, there are 48 cycles of wave in it. But we cannot declare that 48 cards exist. Our estimation algorithm tell us that the signal includes abnormal behaviors, from 18th part to 19th part, from 23rd part to 24th part, and from 44th part to 46 part. In addition, comparing these abnormal parts with normal parts, we know that 50 cards exist in total.

5. Summary

In this paper, we attempted to estimate the state of a finite state system which has some finite states and changes its state from one to another, based on the some feature parameters extracted from observed time series signal . An estimation algorithm, joining fuzzy clustering to fuzzy finite state transition model , was suggested. We adapted this algorithm to practical problem. We dealt with a kind of counter, in which there are some states depending on the machine working conditions. We estimated the machine's state and improved its performance.

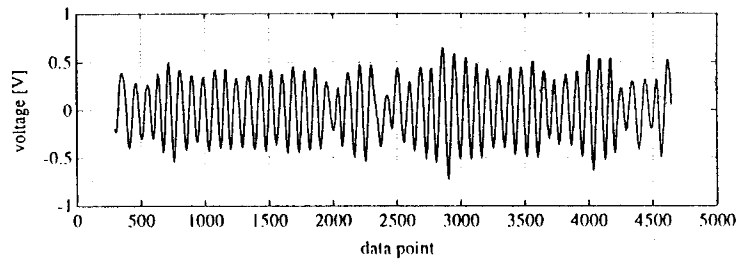

Fig. 7 Time series data

Table 1 Estimated Results

segment	1	2	3	4	5	6	7	8	9	10
estimation	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1
Un(h)	0	0.3282	0.2741	0.3356	0.2392	0.4986	0.2213	0.2636	0.2357	0.4041
segment	11	12	13	14	15	16	17	18*	19*	20
estimation	S1	S1	S1	S1	S1	S1	S1	S2	S4	S1
Un(h)	0.2219	0.0933	0.2276	0.4131	0.2417	0.0912	0.0717	0.9493	0.6129	0.4756
segment	21	22	23	24	25	26	27	28	29	30
estimation	S1	S1	S2	S4	S1	S1	S1	S1	S1	S1
Un(h)	0.2747	0.326	0.3273	0.1605	0.2238	0.3664	0.222	0.3583	0.1066	0.4852
segment	31	32	33	34	35	36	37	38*	39	40
estimation	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1
Un(h)	0.1406	0.3151	0.3236	0.1042	0.193	0.1403	0.2272	0.8052	0.4065	0.2146
segment	41	42	43	44	45	46	47			
estimation	S1	S1	S1	S2	S3	S4	S1			
Un(h)	0.3469	0.1852	0.337	0.164	0.3952	0.8141	0.8141			

References

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