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Implicit Self Tuning Controller with Pole Restriction

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Abstract

In this paper, a design method of controller which incorporates pole restriction into implicit self tuning algorithm is proposed. The idea behind pole restriction is that the closed loop poles of the system are restricted to a user-chosen circle in the region to meet maximum percentage overshoot and settling time specification. Most algorithms based on pole restriction are explicit schemes involving a parameter estimation and synthesis stage to obtain controller parameters. The object of this paper is to have an algorithm that has the idea of pole restriction and the simplicity of the implicit approach.

1. Introduction

One of important classes of adaptive controllers is the self-tuning controller proposed by Astrom and Wittenmark[1] and extended by Clarke and Gawthrop[2] as the generalized minimum variance controller. These controllers based on modern control theory and include a selection of the weighting polynomials associated with the performance criteria. The selection of the weighting polynomials in performance criteria is not easy for the engineers. The system performance is usually specified in qualitative terms and it is not obvious how the polynomials should be chosen to achieve the desired performane.

To choose the appropriate weighting polynomials $P(z^{-1})$ and $Q(z^{-1})$, a trial and error procedure may be used. Alternatively, the pole placement procedure has been presented in Allidina and Hughes[3]. This method is a suitable design approach for processes with nonminimum phase behavior and with maximum system response rate constraints. However, placing closed-loop poles to fixed and prescribed locations in the z-plane may not always result in the desired performance when the process dynamics are varying. Instead of using pole placement, Lim *et al.*[4] proposed the use of pole restriction. The pole restriction method is not to fix the closed-loop poles at fixed location but to restrict the poles to be within a certain permissible region defined by engineering specifications, such as maximum percentage overshoot, settling time etc[5].

In self tuning control there are two basic schemes, 'explicit' or 'implicit'. An explicit scheme is one where the parameters being estimated are the system parameters which are then used to synthesize a set of controller parameters. An implicit scheme is the one where the controller parameters are estimated directly. With the implicit schemes, there is an attraction benefit that the design calculations are simplified considerably[6]. Most algorithms based on pole restriction are explicit schemes involving a parameter estimation and synthesis stage to obtain controller parameters.

This paper presents a design method of controller which is combined implicit schemes with the pole restriction procedure based on the algorithm of [5]. The proposed algorithm has the advantages of the implicit approch in which the design calculations are simplified considerably.

2. Self-tuning controller design.

Single-input/sigle-output randomly disturbed system is represented by the discrete-time parametric model.

$$Ay(t) = q^{-k}Bu(t) + C\xi(t) \tag{1}$$

where A ,B and C are polynominals in q^{-1} , with $a_0=c_0=1$, and k represents the system time delay in sample instants. Variables y and u are the system output and input, with ξ an uncorrelated random sequence of zero mean. The argument of the variables corresponds to the sampling instant.

The control laws for self tuning was developed by Clarke and Gawthrop[2] has as its target control objective the minimisation of the variance of the plant output y(t). By introducing a cost function incorporating system input, output and set point variations, the facility for control input weighting and set point following is provided in addition to the capability of dealing with nonminimum phase systems. The control law is based on prediction and the control objective can be stated as minimizing the variance of an auxiliary function $\Phi(t)$, defined as:

$$\Phi(t)=Py(t)+Qu(t-k)-Rr(t-k) \qquad (2)$$

where P,Q and R are weighting polynomials in q^{-1} , r(t) is the reference set point. The function $\Phi(t)$ can be considered to be the output of the auxiliary system shown in Fig. 1. The object of the self tuning controller becomes that of minimizing

$$J=E[\Phi^2(t+k)] \qquad (3)$$

where E[·] is the expectation operator.

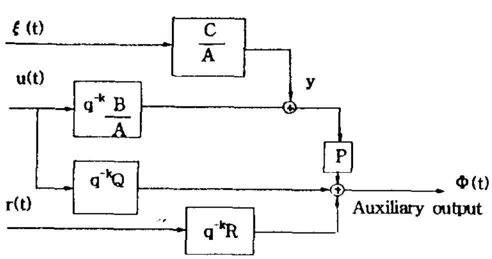


Fig. 1. Block diagram of the generalized minimum-variance controller

It can be shown that (2) and (1) can be combined to give

$$\Phi(t+k)=\frac{1}{C}(Hu(t)+Gy(t)+Er(t))+F\xi(t) \qquad (4)$$

where

$$H = BF + QC , \quad E =-CR \qquad (5)$$

and the order of F is (k-1). F and G are defined according to

$$\frac{PC}{A}=F+q^{-k}\frac{G}{A} \qquad (6)$$

or equivalently

$$PC=AF+q^{-k}G \qquad (7)$$

The minimum variance of the auxiliary output signal is obtained when the following control law is used

$$Hu(t)+Gy(t)+Er(t)=0 \qquad (8)$$

The orders of the controller polynomials H,G and E are set to

$$n_H = (n_Q + n_C) \text{ or } (n_B + k - 1)$$
$$n_G = (n_A - 1) \text{ or } (n_P + n_C -1)$$
$$n_E = (n_C + n_R)$$

where n_P, n_Q, n_R are the orders of the auxiliary function polynomials P , Q and R respectively.

In self tuning, the parameters of H,G and E can be identified from (4), using recursive least squares, and then these estimated parameters can be employed in the control law of (8).

3. Proposed algorithm

3.1 The closed loop system

The self tuning controller described in section 2 gives the closed-loop equations

$$y(t)=\frac{BR}{BP+AQ}r(t-k)+\frac{H}{BP+AQ}\xi(t) \qquad (9)$$

Often the control weighting polynomial Q is written in the λQ. Then (9) becomes

$$y(t)=\frac{BR}{BP+\lambda AQ}r(t-k)+\frac{H}{BP+\lambda AQ}\xi(t) \qquad (10)$$

The characteristic equation of the closed loop system is

$$BP+\lambda AQ'=0 \tag{11}$$

For (11) to have a solution, the order of the polynomials P and Q' must be:

$$n_P = n_A - 1 \ , n_{Q'} = n_B - 1$$

When self tuning, the system parameters A and B are unknown and hence (11) cannot be solved directly to obtain the appropriate P and Q of the performance cost function. However, multiplying (11) throughout by F gives

$$BPF+\lambda AFQ'=0$$

Substituting for AF from (7) results in

$$BPF+\lambda PCQ'-q^{-K}\lambda GQ'=0$$

Further use of (5) reduces this to

$$PH-q^{-K}\lambda GQ'=0 \tag{12}$$

or equivalently

$$I+\lambda T=0 \tag{13}$$

where I = PH and T = $-q^{-k}$ GQ' , λ can be interpreted as the root-locus parameter that moves the closed-loop poles from the roots of I to those of T.

3.2 Pole restriction

The design of a control system involves the changing of system parameters and/or the addition of compensators to achieve certain desired system characteristics. With a known plant, lead and/or lag compensators can be designed for the system to meet output performance specifications such as maximum percentage overshoot and settling time. The classical control theory can be used to design the control weighting polynomials P and 1/Q' as lead and/or lag compensators to meet output performance specifications of maximum percentage overshoot and settling time.

In all systems, the transient response can be characterised by the location of the closed-loop poles. From the performance specifications, a certain desired region can be specified in which the closed-loop poles will reside[5].

In classical control root-locus design, a value of the root-locus parameter is determined offline to bring the closed-loop poles to desired locations to meet performance specifications. Similarly, this algorithm calculates online a value for the root-locus parameter λ to bring the closed poles of the system, as defined in (13), into the defined circle to meet the maximum percentage overshooot and settling time specifications.

The solution of the pole restriction problem involves the transformation of the desired circle in the z-plane to a new ω-plane . As in Wittenmark *et al.*[7], employed is the bilinear transformation which is shown as

$$z=\frac{1+\omega}{\beta(1-\omega)}+\alpha \tag{14}$$

where α and β are constants and z is equivalent to q. It transforms a circle in the z-plane into the left half ω-plane (See Fig. 2).

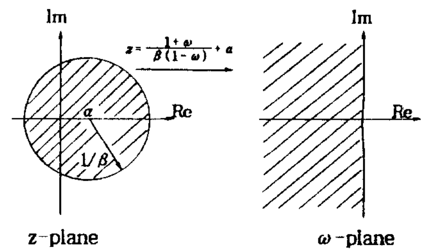


Fig. 2 Bilinear transformation

From (13), the characteristic equation of the closed-loop system may be described by the following equation.

$$\sum_{i=0}^{n}(I_i+\lambda T_i)z^{-i}=0 \tag{15}$$

where n is the order of the equation, and I_i and T_i are the coefficients of the polynomials I and T respectively. Applying the bilinear transformation (14) to (15), we obtain

$$\sum_{i=0}^{n}\nu_i\omega^{n-i}=0 \tag{16}$$

where ν_i is a function of α, β, λ as well as coefficients of polynomials I and T.

Restricting the closed-loop poles to the desired circle is equivalent to confining the roots of (16) to the left-half plane. This implies that there must exist a λ such that the characteristic polynomial in (16) is Hurwitz. It is obvious that the resultant closed-loop poles are stable. The value of λ can be determined by directly applying the Routh-Hurwitz criterion. A detailed description of finding the value λ is be in [5].

3.3 Algorithm

In this subsection, an algorithm to incorporate pole restriction into the implicit self-tuning controller can be summarised as follows:

Step 1: Choose weighting polynomials P,Q and R and form

$$\Phi(t) = Py(t) + Qu(t-k) - Rr(t-k)$$

Step 2: Estimate parameters of the polynomials H,G and E from

$$\Phi(t) = \frac{1}{C}(Hu(t-k) + Gy(t-k) + Er(t-k)) + e(t),$$

$$(e(t) = F\xi(t))$$

Step 3: Apply control u(t) according to the control law

$$Hu(t) + Gy(t) + Er(t) = 0$$

Step 4: Assign to λ, the lower bound of the solution to the inequalities given in the Appendix of [5].

Step 5: Repeat step 2 through step 4.

4. Simulation

The result of the implicit generalized self tuning controller with pole restriction algorithm will be given in terms of simulated example.

Example

Consider the nonminimum phase system defined by

$$(1-q^{-1})y(t) = q^{-2}(1+0.5q^{-1})u(t) + (1-0.2q^{-1})\xi(t)$$

where ξ is Gaussian white noise with variance 0.015. This system could arise by sampling the continuous time system given by

$$G(s) = 25\exp(-0.16s) * \frac{1}{s}$$

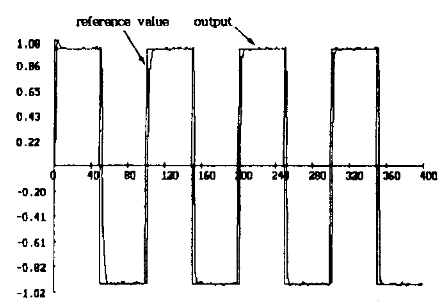
at a sampling interval of 100 ms. The output performance specifications for set point response are 20% overshoot and 2% settling time within 30 samples. These specifications are met if the closed-loop poles in a z-plane circle of center 0.3 and radius 0.5.

The orders of the controller and the weighting polynomial P and Q' are

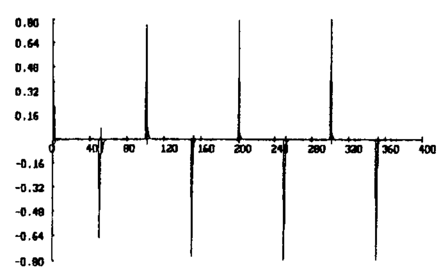
deg H = deg B + k - 1 =2
deg G = deg A - 1 =0
deg E = deg C + deg R = 1
deg P = deg A - 1 =0
deg Q' = deg B - 1 =0

The polynomials are choosen as P = Q' =1 and the reference following achieved by settling R =P. The forgetting factor of the least squres estimator is fixed at 0.995.

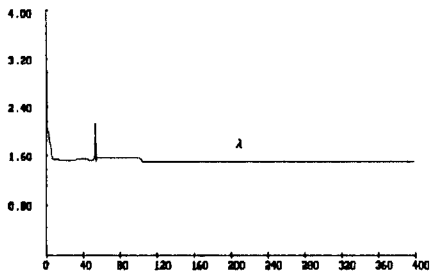
Fig. 3 shows the simulation results.



(a) Output



(b) Control input



(c) The control weighting

Fig. 3. Response of the system by the proposed method

In Fig. 3, the control weighting parameter λ is shown that there is the flexibility to define weighting polynomials P and Q' . to achieve certain control objectives.

5. Conclusion

This paper has discussed the implicit self tuning controller with pole restriction procedure. The closed-loop poles are restricted to a region determined from output settling time and maximum percentage overshoot. The pole restriction design has useful properties when the plant dynamics are varying due to changes in operation condition. The imlpicit scheme does not have disadvantages normally associated with explicit pole restriction concerning the solution of the diophantine equation in terms of computational requirements and the problem of having common factors in this equation. The proposed algorithm has good transient performance due to pole restriction and the simplicity of the implicit approach.

6. References

[1] K. J. Astrom, and B. Wittenmark, "On self-tuning regulators", *Automatica*, vol. 9, (2), pp. 185-199, 1973.

[2] D. W. Clarke, and P. Gawthrop, "A self - tuning controller" *Proc. IEE*, vol. 122, (9), pp. 929-934, 1975.

[3] A. Y. Allidina, and F. M. Hughes, "Generalized self-tuning controller with pole assignment", *IEE Proc. D, Control Theory & Appl.*, vol. 127, (1), pp. 13-18, 1980.

[4] C. C. Lim, C. C. Hang, and K. Y. Leong, "Adaptation of cost function parameters for an explicit self-tuner", *Control-Theory and Adv. Tech.*, vol. 2, (4), pp. 577-591, 1986.

[5] C. C. Hang, K. W. Lim, and W. K. Ho, "Generalised minimum-variance stochastic self-tuning controller with pole restriction", *Proc. IEE*, vol. 138, (1), pp. 25-32, 1991.

[6] A. . Allidina, B. Khosrowshahi, and F. M. Hughes, "Implicit pole-assignment self-tuning algorithm", *Int. J. Contr.*, vol. 51, (1), pp. 51-70, 1990.

[7] B. Wittenmark, R. J. Evans, and Y. C. Soh, "Constrained pole-placement using transformation and LQ design", *Automatica*, vol. 23, (6), pp. 767-769, 1987.