시변 슬라이딩 평면을 이용한 로봇 제어기의 설계

이주장 [0]

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Design of Robot Controller using Time-Varying Sliding Surface

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Abstract

In this paper, a variable structure controller with time-varying sliding surface is proposed for robot manipulators. The proposed time-varying sliding surface ensures the existence of sliding mode from an initial state, while the conventional sliding surface cannot achieve the robust performance against parameter variations and disturbances before the sliding mode occurs. Therefore, error transient can be fully prescribed in advance for all time. Furthermore, it is shown that the overall system is globally exponentially stable.

The efficiency of the proposed method for the trajectory tracking has been demonstrated by simulations.

1 Introduction

In the conventional controller design for robotic manipulator, the control algorithm is based on nonlinear compensations of the plant. This approach requires a detail model of the manipulator and an exact load forecast [2]. In order to avoid these requirement, several control algorithms using the theory of variable structure systems (VSS) have been developed [3],[4],[6],[7].

The VSS is a special class of the nonlinear systems characterized by a discontinuous control action which changes a structure upon reaching a set of sliding surfaces. A fundamental property of VSS is the sliding motion of the state vector on the intersection of the sliding surfaces. In the sliding mode the system has invariance properties, yielding the motion which is independent of parameter uncertainties and external disturbances, and the system behaves like a linear system [1],[5].

In the design of variable structure controller (VSC) with conventional sliding surfaces, the sliding mode is attained when the system state reaches and remains in the intersection of the all sliding surfaces. Thus there is a reaching phase in which the trajectories starting from a given initial state off the sliding surface tend towards the sliding surfaces. In other words, the trajectories are sensitive to parameter variations and disturbances before the sliding mode occurs.

This paper presents a variable structure controller with time-varying sliding surface designed to guarantee the sliding mode occurrence from a given initial state. For the proposed control law, the sliding condition is always guaranteed. Hence, the system is always confined to be in the sliding mode for all time. In addition, the proposed sliding surface comprises a set of decoupled linear differential equations. As a result, the highly coupled nonlinear system is completely decoupled and linearized. Most significantly, when the initial conditions are given, the system's behavior can be fully predicted and has nothing to do with parameter variations and external disturbances.

The existence of sliding modes on these time-varying sliding surfaces are verified by Lyapunov second method. The effectiveness of the proposed time-varying sliding surfaces is demonstrated through the digital simulations for a two degrees-of-freedom robot manipulator.

2 Modeling of Robotic Manipulator

The dynamic equation of an n degrees-of-freedom robot manipulator can be derived using Lagrangian formulation as

$$M\ddot{q} + B\dot{q} + h = u + d \tag{1}$$

where

$q, \dot{q}, \ddot{q}$	:	$n \times 1$ position, velocity and acceleration vectors, respectively,
M	:	$M(q)$, $n \times n$ symmetric and positive-definite inertial matrix,
B	:	$B(q,\dot{q})$, $n \times n$ matrix corresponding to Coriolis and centrifugal factors,
h	:	$h(q)$, $n \times 1$ vector caused by gravitational force,
u	:	$n \times 1$ control input vector,
d	:	$n \times 1$ bounded disturbance vector.

Let

$$\begin{aligned} M &= M^{o} + \Delta M, \\ B &= B^{o} + \Delta B, \\ h &= h^{o} + \Delta h, \end{aligned}$$

where "o" denotes the mean value and "Δ" denotes the estimation error. Assume that the $\Delta M_{ij}, \Delta B_{ij}$ and Δh_i are bounded by M_{ij}^m, B_{ij}^m and h_i^m as follows

$$\begin{aligned} |\Delta M_{ij}| &\le M_{ij}^m, \\ |\Delta B_{ij}| &\le B_{ij}^m, \end{aligned}$$

$$|\Delta h_i| \leq h_i^m,$$

where "m" denotes the maximal absolute estimation error of each element. At the same time, we assume

$$|d_i| \leq d_i^m.$$

3 Design of Control System

Define the tracking error as

$$e(t) = q(t) - q_d(t),$$

where $q_d(t)$ represents the desired trajectory. We choose a time-varying sliding surface as

$$s_i(t) = \dot{e}_i(t) + \lambda_i e_i(t) - (\dot{e}_i(t_0) + \lambda_i e_i(t_0))e^{-k_i(t-t_0)},$$

that is,

$$s(t) = \dot{e}(t) + \Lambda e(t) - N(t), \tag{2}$$

where $s \in \Re^n$, $s = [s_1, s_2, \cdots, s_n]^T$, $\Lambda \in \Re^{n\times n}$, $N \in \Re^n$, and

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n), \quad \lambda_i > 0,$$

$$N(t) = \begin{bmatrix} (\dot{e}_1(t_0) + \lambda_1 e_1(t_0))e^{-k_1(t-t_0)} \\ (\dot{e}_2(t_0) + \lambda_2 e_2(t_0))e^{-k_2(t-t_0)} \\ \vdots \\ (\dot{e}_n(t_0) + \lambda_n e_n(t_0))e^{-k_n(t-t_0)} \end{bmatrix}, k_i > 0,$$

and $i = 1, 2, \cdots, n$. We consider the following positive-definite function as a Lyapunov function candidate

$$V = \frac{1}{2}s^T M s. \tag{3}$$

Differentiating Equation (3) with respect to time and adopting the relation between $\dot{M}(q)$ and $B(q, \dot{q})$ [4], we have

$$\begin{aligned} \dot{V} &= s^T M \dot{s} + s^T B s \\ &= s^T (M\dot{s} + Bs) \\ &= s^T (M\ddot{q} - M\ddot{q}_d + M\Lambda\dot{e} + MKN + Bs) \\ &= s^T (u + d - B\dot{q} - h + M(\Lambda\dot{e} + KN - \ddot{q}_d) + Bs) \\ &= s^T (u + d - h + M(\Lambda\dot{e} + KN - \ddot{q}_d) \\ &\quad + B(s - \dot{q})) \end{aligned} \tag{4}$$

where $K = \mathrm{diag}(k_1, k_2, \cdots, k_n)$. Therefore, the equivalent control input is

$$u_{eq} = -M^{\circ}(\Lambda\dot{e} + KN - \ddot{q}_d) - B^{\circ}(s - \dot{q}) + h^{\circ}. \tag{5}$$

Now, we introduce the control input such as

$$u = u_{eq} - F \bullet sgn(s) \tag{6}$$

where "$\bullet$" means the element-by-element multiplication of two vectors, and

$$\begin{aligned} F &= M^m |\Lambda\dot{e} + KN - \ddot{q}_d| + B^m |s - \dot{q}| + h^m \\ &\quad + d^m + \eta, \\ \eta &= [\,\eta_1, \eta_2, \cdots, \eta_n\,]^T, \qquad \eta_i > 0, \\ sgn(s) &= [\,sgn(s_1), sgn(s_2), \cdots, sgn(s_n)\,]^T, \\ sgn(s_i) &= \begin{cases} 1 & \text{if } s_i > 0 \\ 0 & \text{if } s_i = 0 \\ -1 & \text{if } s_i < 0 \end{cases}, \quad i = 1, 2, \cdots, n, \end{aligned}$$

and the absolute of a vector denotes the vector whose element has its absolute value, i.e., $|x| = [\,|x_1|, |x_2|, \cdots, |x_n|\,]^T$.

Lemma 1 *For the robot manipulator (1) and the control law (6), the sliding mode exists from a given initial state.*

Proof
Inserting Equation (6) in Equation (4), we obtain

$$\begin{aligned} \dot{V} &= s^T \{ -M^{\circ}(\Lambda\dot{e} + KN - \ddot{q}_d) - B^{\circ}(s - \dot{q}) + h^{\circ} \\ &\quad - (M^m |\Lambda\dot{e} + KN - \ddot{q}_d| + B^m |s - \dot{q}| + h^m \\ &\quad + d^m + \eta) \bullet sgn(s) + d - h \\ &\quad + M(\Lambda\dot{e} + KN - \ddot{q}_d) + B(s - \dot{q}) \} \\ &= s^T \{ (M - M^{\circ})(\Lambda\dot{e} + KN - \ddot{q}_d) \\ &\quad + (B - B^{\circ})(s - \dot{q}) - M^m |\Lambda\dot{e} + KN - \ddot{q}_d| \\ &\quad \bullet sgn(s) - B^m |s - \dot{q}| \bullet sgn(s) + (h^{\circ} - h) \\ &\quad - h^m \bullet sgn(s) + d - d^m \bullet sgn(s) - \eta \bullet sgn(s) \} \\ &\leq -\sum_{i=1}^{n} \eta_i |s_i|. \end{aligned}$$

Therefore, V is really a Lyapunov function. Since $\dot{V} \leq 0$, $\dot{V} = 0$ is true only for $s = 0$, and $V(t_0) = 0$, the Lyapunov function $V(t)$ is equal to zero for all time. This also implies that

$$s = 0 \qquad \forall t \geq t_0. \tag{7}$$

Thus, the system is forced to stay in the sliding mode from a given initial state.

□

Lemma 2 *The evolution of the i-th joint's tracking error, $e_i(t)$, can be predicted as*

$$e_i(t) = \begin{cases} \frac{1}{k_i - \lambda_i}\left[(\dot{e}_i(t_0) + k_i e_i(t_0))e^{-\lambda_i(t-t_0)} - (\dot{e}_i(t_0) \right. \\ \left. + \lambda_i e_i(t_0))e^{-k_i(t-t_0)}\right] \quad \text{if } k_i \neq \lambda_i, \\ e_i(t_0)e^{-\lambda_i(t-t_0)} + (\dot{e}_i(t_0) + \lambda_i e_i(t_0))(t - t_0) \\ e^{-\lambda_i(t-t_0)} \quad \text{if } k_i = \lambda_i, \end{cases} \tag{8}$$

for all time $t \geq t_0$.

Proof
Using Equation (7), we can obtain the following equation.

$$s_i(t) = \dot{e}_i(t) + \lambda_i e_i(t) - (\dot{e}_i(t_0) + \lambda_i e_i(t_0))e^{-k_i(t-t_0)} = 0 \quad \forall t \geq t_0,$$

where $i = 1, 2, \cdots, n$. By solving the above differential equation, we can easily conclude that the tracking error is given by Equation (8). Since the detail derivation is somewhat tedious, we omit the details.

□

From the above lemma, we can conclude that the time history of the tracking error for each joint can be predicted completely for all time and they are decoupled each other. Therefore, we can derive the following theorem.

Theorem 1 *For the robot manipulator (1) and the control law (6), the overall system is globally exponentially stable.*

Proof
We choose the constants A_i, B_i as follows

$$\begin{aligned} A_i &= \frac{1}{|k_i - \lambda_i|}\left[|\dot{e}_i(t_0) + k_i e_i(t_0)| \right. \\ &\quad \left. + |\dot{e}_i(t_0) + \lambda_i e_i(t_0)|\right] \\ B_i &= \min\{\lambda_i, k_i\} \end{aligned}$$

where $i = 1, 2, \cdots, n$. Then from Equation (8), it is obvious that the following inequality is guaranteed for all i,

$$|e_i(t)| \leq A_i e^{-B_i(t-t_0)}, \qquad \forall t \geq t_0.$$

Therefore, the overall system is globally exponentially stable.

□

4 Simulation Results

Figure 1 shows a two degrees-of-freedom robot manipulator model used by Young [6]. The dynamic equation is given by

$$M(q)\ddot{q} + B(q,\dot{q})\dot{q} + h(q) = u + d$$

where $q = [q_1\ q_2]^T$,

$$\begin{aligned} M_{11} &= (m_1+m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2\cos q_2 + J_1 \\ M_{12} &= M_{21} = m_2r_2^2 + m_2r_1r_2\cos q_2 \\ M_{22} &= m_2r_2^2 + J_2 \\ B_{11} &= -2m_2r_1r_2\dot{q}_2\sin q_2 \\ B_{12} &= -m_2r_1r_2\dot{q}_2\sin q_2 \\ B_{21} &= m_2r_1r_2\dot{q}_1\sin q_2 \\ B_{22} &= 0 \\ h_1 &= \{(m_1+m_2)r_1\cos q_1 + m_2r_2\cos(q_1+q_2)\}\, g \\ h_2 &= m_2r_2g\cos(q_1+q_2). \end{aligned}$$

Parameter values are the same as those of [6].

Figure 2 shows the actual error transient of joint 1 and the predicted error transient that was given by Equation (8) in the lemma 2. From this figure, we can find that there is no difference between the predicted error transient and actual error transient. Therefore, we can completely prescribe the actual error transient in advance. The sliding surface, $s(t)$, of the proposed method and the conventional one are shown in Figure 3. For the proposed method, this figure shows that $s(t) = 0\ \forall t \geq 0$, while, for the conventional case, that condition cannot be guaranteed. So, the overall system using proposed method is robust against parameter uncertainties and external disturbances for all time.

5 Conclusions

In this paper, a time-varying sliding surface is proposed in order to remove the reaching phase. The proposed control system guarantees that the system states are in the sliding mode from a given initial state and so the error transient can be fully prescribed in advance and the system has robust performance for all time. Therefore overall system is robust from an initial time against parameter variations and external disturbances, and load forecast is not needed.

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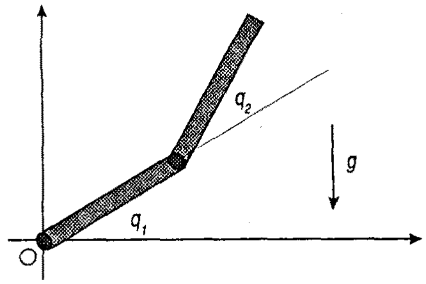

Figure 1. Two degrees-of-freedom robot manipulator.

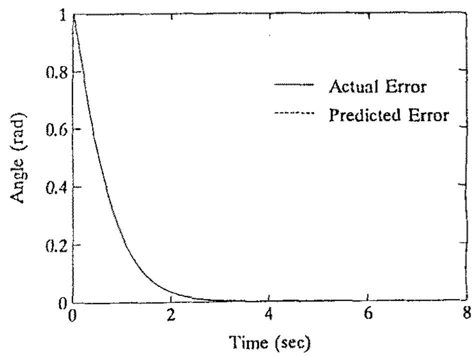

Figure 2. Actual and predicted error transient.

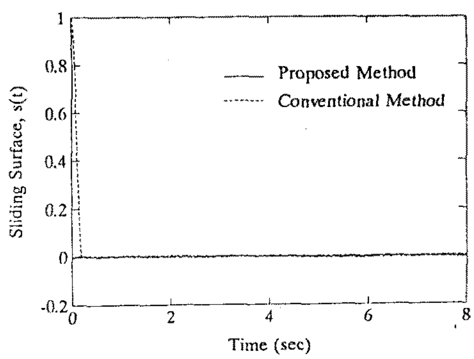

Figure 3. Sliding surface, $s(t)$.