결정론적 외란에 대한 적응제어 알고리즘의 연구

김용세*, G. Feng*, 이권순**
*Univ. of New South Wales, **동아대학교 전기과

Deterministic Disturbance Rejection for Model Reference Adaptive Control

Yong Sei Kim*, G. Feng*, and Kwon S. Lee**
*Univ. of New South Wales, **Dong-A University

Abstract

This paper presents the general MRAC algorithm design, it's real time implementation and investigates the effect of purely deterministic disturbances to adaptive control algorithm. The design of adaptive control algorithm to *reject the disturbances properly is also presented.* In real time application, adaptive control algorithm is considered to investigate its performance by using DC motor. Disturbance rejection algorithm is investigated in *simulation.*

1. Introduction

There are two methods to design adaptive controller in Model Reference Adaptive System. One is direct method and the other is indirect method. Both have been well studied in [1] and [2]. In this paper direct adaptive control method is used to design adaptive control algorithm. In direct method no effort is made to identify the plant parameters, but the control parameters are adjusted directly so as to minimize some measure of the error between outputs of the plant and reference model. The recursive least squares algorithm can be used to estimate controller parameters. DC servo motor is used to implement in real time. The aim of this practical experiment is to control DC motor speed to track the reference model output with unknown DC motor parameters. In indirect adaptive control design method, large computing time is needed for implementation of adaptive control due to on-line parameter estimation and underlying control design. However, the numerical calculation is simple in direct adaptive control method. A synchronous control program is used to control DC motor in real time.

When the deterministic disturbances are added to the systems, the system output cannot follow the reference model output properly. Therefore, it is needed to change adaptive control algorithm to reject the disturbances properly. This modified adaptive algorithm is proposed in [3] and also well studied in [4]. It is needed to make modelled disturbances to reject them. We have to reconstruct the plant model which includes modelled disturbances. It leads to a high order ARMA with common factor. In order to control new ARMA model, the controller is divided into two parts. One is to control the plant output to follow the reference model properly and the other is to cancel the disturbances. But these two parts are combined to control the systems.

2. Plant Modelling

The system is a physical object that has observed *output signal. It's output at time t is denoted by y(t).* The system also has a measurable input signal which denotes the input signal at time t by u(t). With above input and output signal, we can consider dynamic systems through *the linear difference equation.*

$$a_0y(t) + a_1y(t-1) + \cdots + a_ny(t-n) = b_1u(t-1) + \cdots + b_mu(t-m)$$

Let's define q^{-1} the delay operator. The above equation can be rewritten as

$$A(q^{-1})y(t) = B(q^{-1})u(t) = q^{-d}B'(q^{-1})u(t) \qquad (2\text{-}1)$$

where $A(q^{-1}) = (a_0 + a_1q^{-1} + \cdots\cdots + a_nq^{-n})$
$B(q^{-1}) = q^{-d}(b_0 + b_1q^{-1} + \cdots\cdots + b_mq^{-m})$
$B'(q^{-1}) = (b_0 + b_1q^{-1} + \cdots\cdots + b_mq^{-m})$

From above ARMA model, It is not needed to know the parameters of the system to develop adaptive prediction error control algorithm. But, the upper bound on the order of $A(q^{-1})$ and $B(q^{-1})$ are needed.

2-1 With Purely Deterministic Disturbance

When the purely deterministic disturbance is presented in the systems, the output is written as $y'(t) = y(t) + d(t)$. The modelled disturbance will be represented as a solution to the homogeneous difference equation.

$$D(q^{-1})d(t) = 0 \qquad (2\text{-}2)$$

Substituting (2-2) into (2-1),

$$A(q^{-1})y(t) = B(q^{-1})u(t) + A(q^{-1})d(t). \qquad (2\text{-}3)$$

Multiplying $D(q^{-1})$ on both side, we can get

$$D(q^{-1})A(q^{-1})y(t) = D(q^{-1})B(q^{-1})u(t) \qquad (2\text{-}4)$$

Equation (2-4) can be written as

$$A''(q^{-1})y(t) = B''(q^{-1})u(t) \qquad (2\text{-}5)$$

If the purely deterministic disturbance is presented, $A''(q^{-1})$ and $B''(q^{-1})$ must have common roots. $A''(q^{-1})$ and $B''(q^{-1})$ are relatively prime. It is shown that the purely determinstic disturbances appear to the system as a second order polynomial. These increase system order by two.

3. Parameter Estimation

The role of the parameter estimation is to estimate the controller parameters by using tracking error which is the difference of output between system and reference model. A mathematical model can be expressed as

$$y(t) = \theta\phi^T(t) \qquad (3\text{-}1)$$

where $\theta = [a_1, \cdots, a_{n-1}, b_1, \cdots, b_m]$
a, b are unknown parameters,
$\phi(t) = [-y(t-1), \cdots, -y(t-n-1), u(t-1), \cdots, u(t-m)]$
y and u are known variables. The model is indexed by time t. The parameter vector θ is to be estimated from the measurements of y(t) and $\phi(t)$.

From [4] and matrix inversion lemma, the estimated parameter can be written as

$$\theta(t) = \theta(t-1) - P(t)\phi(t)(y(t) - \phi(t)^T\theta(t-1)) = \theta(t-1) + K(t)e(t) \quad (3\text{-}2)$$

where $K(t) = P(t-1)\phi(t-1)(I + \phi(t)^T P(t-1)\phi(t))^{-1}$.

4. Adaptive Control Law

In the MRAC system, the aim is to choose the optimal controller parameters. Then, it is to calculate controller output which will reduce the tracking error of output between system and reference model. The aim of the adaptive control algorithm is to achieve

$$\lim_{t\to\infty} e(t) = y(t) - y_m(t) = 0$$

where $y_m(t)$ is reference model output.

4-1 Without Disturbance.

The objective of the adaptive controller can be achieved by using

$$E(q^{-1})y_m(t+d) = q^{-d}gH(q^{-1})r(t). \quad (4\text{-}1)$$

For the closed loop system is stable, the following conditions should be satisfied:

1) The zeros of the system lie inside or on the unit circle.
2) The poles of the system lie strictly inside the unit circle.
3) Any mode of the system on the unit circle have Jordan block size of one.

From the previous plant modelling, the system represented in ARMA model can be expressed d-step ahead predictor form as

$$E(q^{-1})y(t+d) = \alpha(q^{-1})y(t) + \beta(q^{-1})u(t) \quad (4\text{-}2)$$

where $\alpha(q^{-1}) = G(q^{-1})$,
$\beta(q^{-1}) = F(q^{-1})\,B(q^{-1})$,
and $F(q^{-1})$, $G(q^{-1})$ are the unique polynomials of order d-1, n-1 respectively, which satisfy

$$E(q^{-1}) = F(q^{-1})A(q^{-1}) + q^{-d}G(q^{-1}). \quad (4\text{-}3)$$

And, it is clear that the design objective can be achieved by determining u(t) according to

$$\alpha(q^{-1})y(t) + \beta(q^{-1})u(t) = gH(q^{-1})r(t) \quad (4\text{-}4)$$

Thereby, giving the closed loop system

$$E(q^{-1})y(t+d) = gH(q^{-1})r(t) = E(q^{-1})y_m(t+d) \quad (4\text{-}5)$$

$$E(q^{-1})y(t+d) = [\alpha_0 + \alpha_1 q^{-1} + \cdots + \alpha_{n-1}q^{-(n-1)}]y(t) + [\beta_0 + \beta_1 q^{-1} + \cdots + \beta_{m+d-1}q^{-(m+d-1)}]u(t)$$

So,

$$u(t) = -\alpha'(q^{-1})y(t) - \beta'(q^{-1})u(t) + 1/\beta_0[y^*(t+d)] \quad (4\text{-}6)$$

where $\alpha'(q^{-1}) = 1/\beta_0\,[\alpha_0 + \alpha_1 q^{-1} + \cdots + \alpha_{n-1}q^{-n+1}]$
$\beta'(q^{-1}) = 1/\beta_0\,[\beta_1 q^{-1} + \cdots + \beta_{m+d-1}q^{-m-d+1}]$, $\beta_0 \neq 0$
and $y^*(t+d) = E(q^{-1})y(t+d)$. We can use recursive least squares algorithm to calculate $\hat{\theta}$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\phi(t-d)[u(t-d) - \phi(t-d)^T\hat{\theta}(t-1)]}{1 + \phi(t-d)^T\phi(t-d)} \quad (4\text{-}7)$$

where $\phi(t)^T = [-y(t),\cdots, -y(t-n+1), u(t-1),\cdots, u(t-d-m+1)\ y^*(t+d)]$.

Using the estimated $\hat{\theta}(t)$, The controller output is generated from

$$u(t) = \phi'(t)^T\hat{\theta}(t) \quad (4\text{-}8)$$

where $\phi'(t) = [-y(t),\cdots, -y(t-n+1), u(t-1),\cdots, u(t-d-m+1)\ r^*(t)]$,
$r^*(t) = gH(q^{-1})r(t)$.

4-2 With Purely Deterministic Disturbance

With a little modification to the adaptive control law without any disturbances, we can obtain the adaptive control law which has purely determinstic disturbances. The main purpose is to find u(t) satisfying

$$E(q^{-1})B(q^{-1})u(t) = E(q^{-1})A(q^{-1})y_m(t) \quad (4\text{-}9)$$

All procedure to get the adaptive control algorithm is the same as the without disturbances. We need the assumption which satisfy following conditions:

1) An upper bound is known for the order of the polynomials in above equation.
2) The system time dely d is known.
3) The system is "minimum phase" in the special sense described in privious condition.
4) The sign and lower bound of $1/\beta_0$ are known. (This is not always needed.)

The idea is keep the estimate of $1/\beta_0$ away from zero using standard constrained estimator.

If $\hat{\theta}_{n+m+d}(t)\,\mathrm{sign}(1/\beta_0) > \mathrm{Min}\ |1/\beta_0|$

then
$$\hat{\theta}'(t) = \hat{\theta}(t) \quad (4\text{-}10)$$

Otherwise,

$$\hat{\theta}'(t) = \hat{\theta}(t), \quad \text{for } i = 1, 2, \cdots, n+m+d-1$$

$$\hat{\theta}'_{(n+m+d)}(t) = \mathrm{Min}\ |1/\beta_0|\,\mathrm{sign}(1/\beta_0), \quad i = n+m+d \quad (4\text{-}11)$$

So, the adaptive controller output is generated from

$$u(t) = \phi'(t)^T\hat{\theta}'(t) \quad (4\text{-}12)$$

From (4-11) and (4-12), it is ensured that

$$\hat{\theta}'_{(n+m+d)}(t) \geq \mathrm{Min}\ |1/\beta_0| > 0 \quad (4\text{-}13)$$

This is the key point in the convergence for the this adaptive control algorithm.

5. Simulation

The convergence of the estimated controller parameters and disturbance rejection were investigated through the simulation. When the disturbance rejection algorithm is used, we can obtain the good performances as shown below. The reference model, input, and system for the simulation is selected as:

Model
$$\frac{y_m(t)}{r(t)} = \frac{0.5Z^{-1}}{1 - 0.5Z^{-1}} \quad (5\text{-}1)$$

Input $r(t) = \pm 1$ (for every 100 sampling time)

System
$$G(z) = \frac{1}{1 - a\,z^{-1} - b\,z^{-2}} \quad (5\text{-}2)$$

a = -0.6 b = -0.08

Initial parameters = (1, 7, -2, 0.9, -0.5, 1, 1.2)

Disturbance : Mag = 1 , Frequency = 6.2 Hz

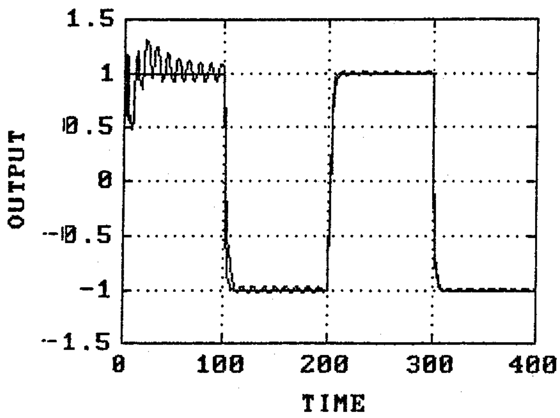

Fig. 2

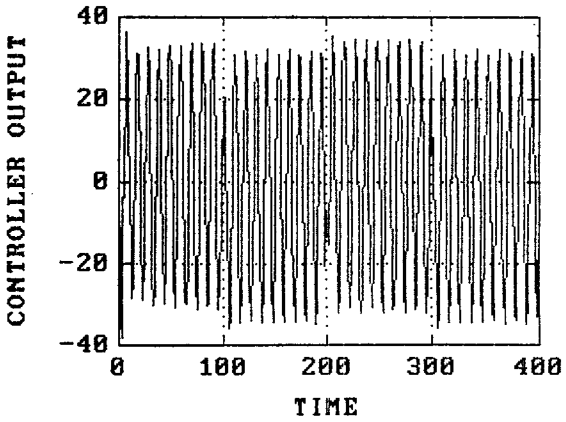

Fig. 3

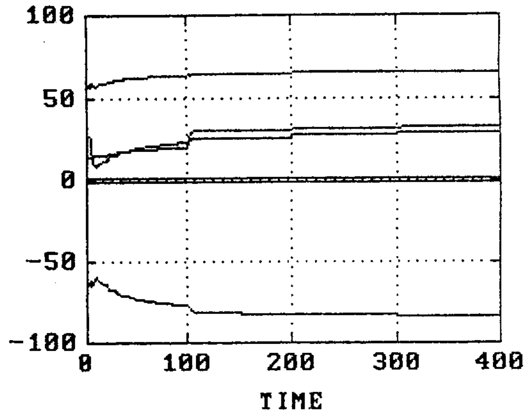

Fig. 4

6. Real Time Implementation

The above has mainly described the theoretical aspects of direct adaptive control algorithm. In here, I have used DC motor to control it's speed by using above adaptive control algorithm. The overall design of this practical implementation is shown in Fig. 1. The calculation function is not complicated in the direct adaptive control algorithm. It does not need multi-rate sampling. Therefore, it is possible to use a synchonous program

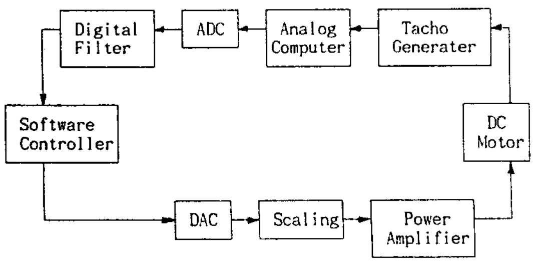

Fig. 1 Block Diagram for Control Systems.

6-1 System Modelling

The transfer function relating speed to voltage can be represented as

$$\frac{w(s)}{v(s)} = \frac{K(t)}{Ra\ F + Kt\ Kw}\ \frac{1}{1 + s\tau_m} \qquad (6\text{-}1)$$

where $\tau_m = \dfrac{J\ Ra}{Ra\ F + Kt\ Kw}$: Time constant

From (6-1), we can know the system order.

6-2 Reference model selection

The saturation is a big problem in real time application for adaptive control. To overcome this saturation problem, the reference model is selected to respond to the reference input slowly. The reference model is chosen as

$$G(s) = \frac{2.6}{s + 2.6} \qquad (6\text{-}2)$$

6-3 Sampling Time Calculation

The selection of sampling interval depends on the design algorithm and desired closed-loop performances. So, the sampling rate is an essential design issue. In order to get proper sampling time, I work out the time constant of DC motor. From DC motor operational parameters, the time constant can be obtained as τ_m = 364 m sec.

One rule of thumb, that is helpful for deterministic design method, is to choose the sampling interval Ts such that

$$\omega_0 Ts \approx 0.1 \sim 0.5$$

where ω_0 is the natural frequency of the dominating poles of the closed-loop systems. If the dominating pole is real, we can use

$$Ts/\tau_m \approx 0.1 \sim 0.5.$$

This rule implies that it will be about 5 ~ 20 samples in a step response of the closed-loop systems. In this practices, the system is first order and the proper sampling time is

$$Ts = 36.4 \sim 182 \text{ m sec.}$$

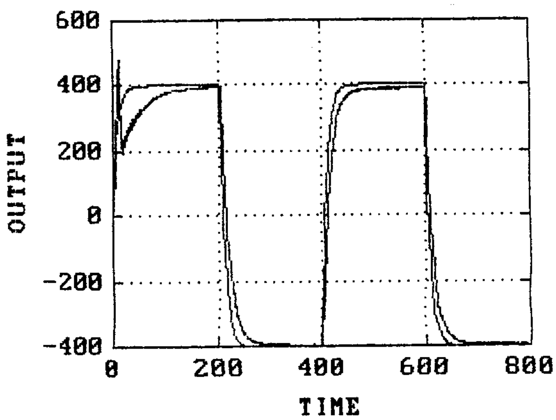

Fig. 5

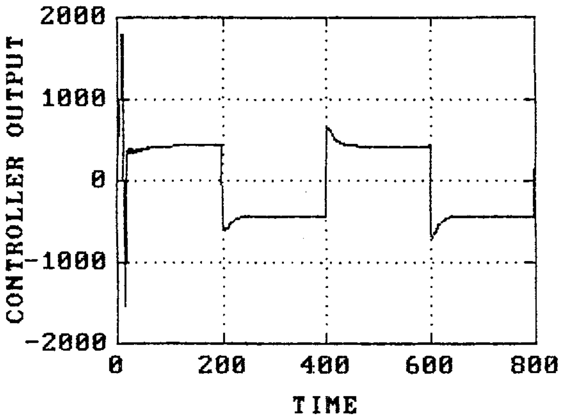

Fig. 6

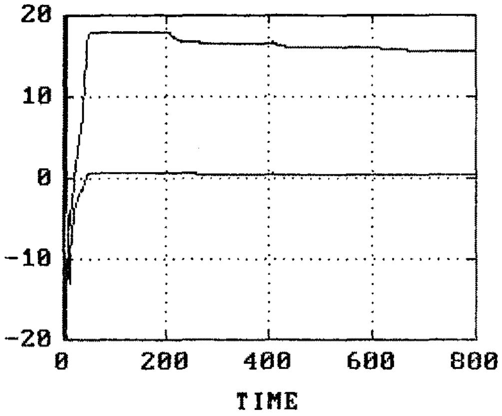

Fig. 7

7. Conclusion

We investigated the effect of purely deterministic disturbances for adaptive control algorithm to reject the disturbances properly. From the simulation results, the successful performance of disturbance rejection algorithm is demonstrated with unknown parameters and purely deterministic disturbances. Through the real time application results, the adaptive control algorithm operates very well with unknown parameters of DC motor. DC motor speed is successfully controlled to track the reference model output.

References

[1] G. C. Goodwin and K. S. Sin, Adaptive Filtering Prediction and Control, Prentice-Hall, Englewood Cliffs, NJ., 1984.

[2] K. J. Astrom and B. Wittenmark, Adaptive Control, Addison-Wesely Publishing Co., NY., 1989.

[3] G. C. Goodwin and S. W. Chan, Model Reference Adaptive Control of System Having Purely Deterministic Disturbance, IEEE Trans. Auto. Cont. Vol. AC-28, pp. 855-858. 1983.

[4] K. S. Narendra and A. M. Annaswamy, Stable Adaptive Systems, Prentice-Hall, England Cliffs, NJ., 1989.