Theoretical considerations of the minimum power consumption of Korean total artificial heart

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Introduction

To develop a totally implantable TAH, it is necessary to have three components in the body: driving system with pumping chamber, controller system, and power supply. The Korean TAH adapted the "moving-actuator" type energy converter (EC) with integrated variable volume space (VVS) to eliminate the occupied space of the fixed-actuator in the conventional electro-mechanical TAHs with pusher plate and their additional space of volume compensation chamber (VCC). Throughout several trail of anatomical fitting, it shows that the driving system and pumping chamber of the Korean TAH can be implanted inside human thoracic cavity. Since, total mass of the moving EC is so large, somewhat large energy loss is occurred in the Korean TAH. Minimum power consumption in implanted TAHs is very important for the development of transcutaneous energy transmission (TET) system and internal battery.

Materials and Methods

Bond-Graph modeling, optimal control technique and variational approach were used to simulate an optimal controller which minimizes power consumptions

In the dynamic model consisted of seven differential equations, a brushless DC motor, all of mechanical components, the pump system with integrated VVS and a simple circulatory system model were included. Two different sets of seven differential equations were separately developed for the left and right systolic states of the Korean TAH. Another two different sets of equations were derived to simulate two cases of the right blood sac attached or detached on the energy converter.

An optimal controller was developed with these mathematical models of the Korean TAH and the circulatory model. This controller drives the Korean TAH through a desired stroke angle while forcing the end stage velocity of the energy converter to zero. The values of the final time for the left and right systolic strokes is determined by the desired heart rate (H.R.). Optimal control theory was used to find a control law which minimizes electric power consumption in the Korean TAH. In order to obtain robustness of the controller, a closed loop control scheme was considered. Optimal velocity profiles were also simulated in each conditions.

Dynamic models of Koean TAH

The Korean TAH can be categorized into two types by attaching of outer membrane; LARD mode (left blood sac attached while right blood sac detached) and LARA mode (left and right blood sac attached).

Block diagram of the implanted Korean TAH in which there exists a brushless DC motor, gear trains including journal bearings, two blood sacs and integrated VVS with a simple circulatory system is shown in Figure 1. In this circulatory system model, bronchial circulation is also considered as shown in Figure 2. Bond-Graphs of

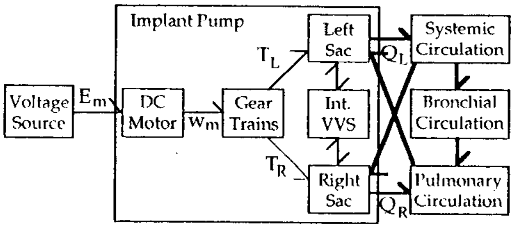

Fig 1. System Configuration of Korean TAH

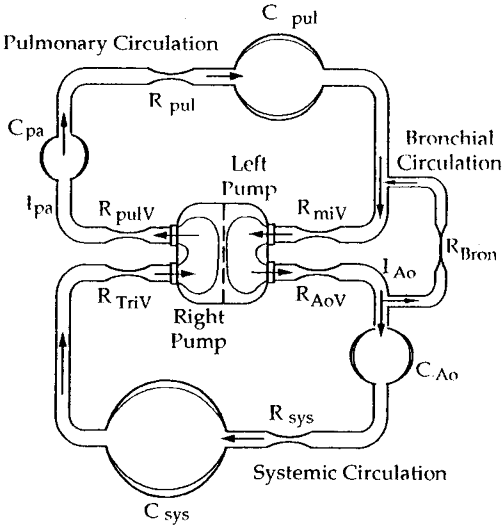

Fig. 2 Circulatory system model

Table 1. Parameter value of modeling of the Korean TAH

Rm	motor amature resistance	1.98 [ohms]
Rr	motor rotor gab resistance	2.956 [kg-mm2/s]
Im	motor amature moment of inertia	0.6398 [kg-mm2]
Ke	motor back emf constant	0.036 [V-s/rad]
Kt	motor torque constant	36238.75 [kg-mm2/A-s2]
Gr1	gear ratio of 1st PGT	0.2143
Gr2	gear ratio of 2nd PGT	0.2143
Gr3	gear ratio of 3rd PGT	0.2727
Grmf	gear ratio of hypocyclic GT	1.3333
eff1	efficiency of 1st PGT	0.9724
eff2	efficiency of 2nd PGT	0.9724
eff3	efficiency of 3rd PGT	0.9724
effmf	efficiency of Hypocyclic GT	0.986
I1	moment of inertia of 1st PGT and cage	1.148 [kg-mm2]
I2	moment of inertia of 2nd PGT and cage	1.5466 [kg-mm2]
I3	moment of inertia of 3rd PGT and cage	6.822 [kg-mm2]
Rj1	journal bearing loss of 1st PGT	0.0382 [kg-mm2/s]
Rj2	journal bearing loss of 2nd PGT	0.121 [kg-mm2/s]
Rj3	journal bearing loss of 3rd PGT	1.086 [kg-mm2/s]
r	motion radius of EC	28 [mm]
M	mass of EC's motion part	0.478 [kg]
Rsq	EC squeeze resistance	0.3853 [kg/s]
ApL	left pusher plate area	2274 [mm2]
ApR	right pusher plate area	1891 [mm2]
Cvar	compliance of VVS filled air	231.385 [mm3/mmHg]
IAo	aortic inertance	2.753e-6 [kg/mm4]
CAo	aortic compliance	1330 [mm3/mmHg]
RAoV	aortic valve resistance	5.e-5 [mmHg-s/mm3]
RmiV	mitral valve resistance	2.4e-5 [mmHg-s/mm3]
Rsys	systemic atrial and venous resistance	0.001 [mmHg-s/mm3]
Csys	systemic atrial and venous compliance	4850 [mm3/mmHg]
RBron	bronchial circulation resistance	0.01 [mmHg-s/mm3]
Ipa	pulmonary inertance	2.7e-7 [kg/mm4]
Cpa	pulmonary compliance	1000 [mm3/mmHg]
RpulV	pulmonary valve resistance	3.e-5 [mmHg-s/mm3]
RtirV	tricuspid valve resistance	2.4e-5 [mmHg-s/mm3]
Rpul	pulmonary atrial and venous resistance	1.e-4 [mmHg-s/mm3]
Cpul	pulmonary atrial and venous compliance	1000. [mm3/mmHg]

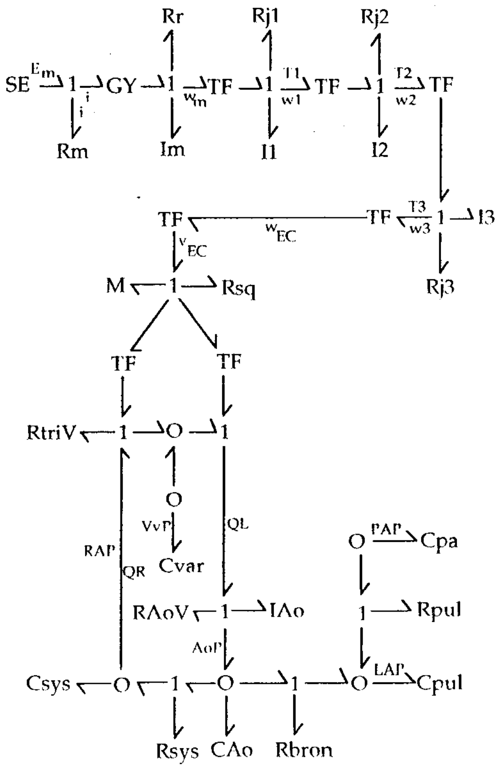

Fig. 3 The Bond-Graph of LARA mode of Korean TAH - left systolic period

these models are shown in Figure 3 for left systolic period in LARD mode.

Optimal Control Formulations

The optimal controller is designed to drive the Korean TAH through a desired stroke angle while minimizing electric power consumption, and forcing the end stage velocity of its energy converter to zero.

Recasting the system equations in standard form with state vector $X(t)$ and input $U(t)$ as follows:

$$X(t) = \left[\theta_{EC}(t) - \theta_d \quad \omega_{EC}(t) \quad AoP(t) \quad LAP(t) \quad PAP(t) \quad RAP(t) \quad VvP(t)\right]^T$$

$$U(t) = e_m(t)$$

The system state equations are:

$$\dot{X}(t) = A \cdot X(t) + B \cdot U(t)$$

The cost function which is total energy consumption during a one stroke can be expressed as follows:

$$J(t) = \frac{1}{2} \cdot X(t_f)^T \cdot H \cdot X(t_f) + \int_{t_0}^{t_f} \left[X(t)^T \cdot N \cdot U(t) + \frac{1}{2} \cdot U(t)^T \cdot R \cdot U(t)\right] dt$$

where H is the weighting matrix which is a trade-off between the power consumption and the accuracy of the final conditions.

In order to solve above optimal control problems, calculus of variations technique was used. Applying Pontryagin's minimum principle to the above problem, optimal control equations can be rewritten as follows:

$$\begin{bmatrix} \dot{X}(t)^* \\ \dot{p}(t)^* \end{bmatrix} = \begin{bmatrix} (A - B \cdot R^{-1} \cdot N^T) & -B \cdot R^{-1} \cdot B^T \\ N \cdot R^{-1} \cdot N^T & (N \cdot R^{-1} \cdot B^T - A^T) \end{bmatrix} \cdot \begin{bmatrix} X(t)^* \\ p(t)^* \end{bmatrix}$$

where $p(t)$ is the costate vector.

The solution of these matrix equation has the form of:

$$\begin{bmatrix} X(t_f)^* \\ p(t_f)^* \end{bmatrix} = \begin{bmatrix} \Phi_{11}(t_f,t) & \Phi_{12}(t_f,t) \\ \Phi_{21}(t_f,t) & \Phi_{22}(t_f,t) \end{bmatrix} \cdot \begin{bmatrix} X(t)^* \\ p(t)^* \end{bmatrix}$$

where $\Phi(t,t_0)$ is the transition matrix of control problem.

As a results, optimal control equations can be obtained as follows:

$$U(t)^* = -R^{-1} \cdot \left[N^T + B^T \cdot K(t)\right] \cdot X(t)^*$$

where the matrix $K(t)$ satisfies the matrix Riccati equations.

The feedback gain matrix of optimal regulator controller is:

$$F(t) = -R^{-1} \cdot \left[N^T + B^T \cdot K(t)\right]$$

At last, the optimal state trajectories are calculated as follows:

$$\dot{X}(t)^* = \left[A + B \cdot F(t)\right] \cdot X(t)^*$$

The matrix Riccati equation is independent of the system state value which enables one to calculate it off-line.

Results

In the previous study, it was shown that the parabolic velocity profile is one of the best profiles

Table 2. Comparison of energy consumption between parabolic velocity profile and optimal profile

Simulation Mode		Energy Consumption (Joules)		Reduction of Energy (%)
		Parabolic Profile	Optimal Profile	
LARD	LS Period	1.720	1.501	12.7
	RS Period	0.483	0.313	35.2
LARA	LS Period	2.385	2.058	13.7
	RS Period	0.362	0.173	52.2

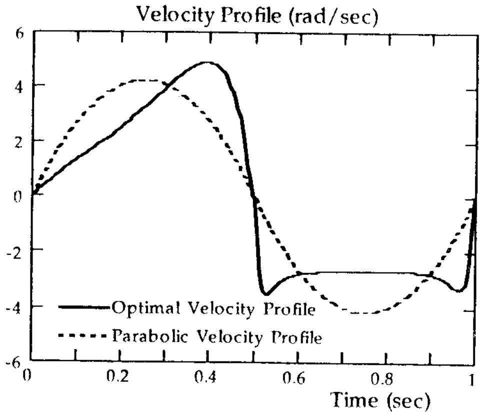

Fig. 4 Comparison of velocity profiles

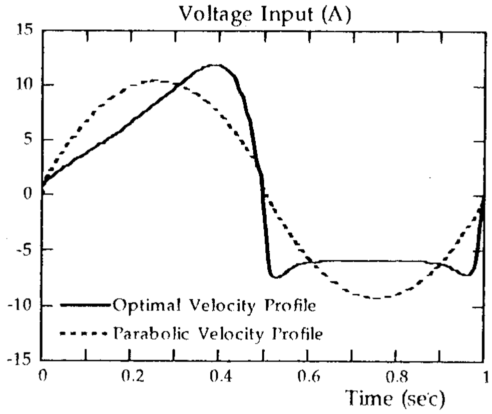

Fig. 5 Comparison of voltage input

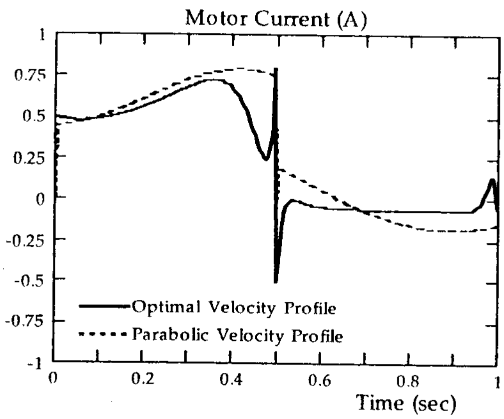

Fig. 6 Comparison of motor current

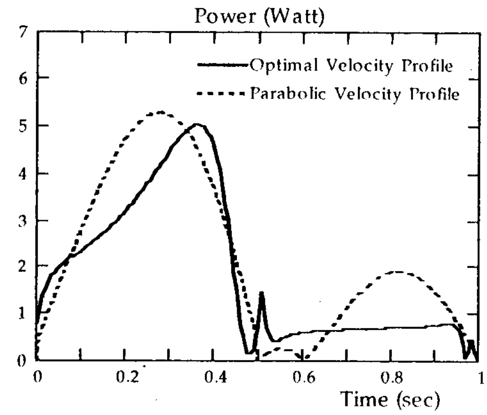

Fig. 7 Comparison of power consumption

with regard to minimizing inertial rocking and power consumption among several velocity profiles. Comparison in energy consumption for one stroke between parabolic velocity profile and new optimal velocity profile control was shown in Table 2. From Figure 4 to 7 we can show the comparisons in in LARA mode.

Conclusions

To minimize power consumption of the Korean TAH, the optimal controller with these mathematical models of the Korean TAH and the circulatory model has been developed. The optimal controller minimize the Korean TAH's power consumption, and drives the end stage velocity of the energy converter to zero. In order to obtain robustness of the controller, a closed loop controller was developed. Optimal velocity profile and optimal voltage input profile were also calculated in each conditions.

The optimal gains can be precalculated by off-line. It can be implemented optimally for any initial states. Comparisons in energy consumption for one stroke between the old parabolic velocity profile and the new optimal velocity profile control were simulated. 12 % ~ 52 % of energy consumption was saved by the optimal velocity profile compared to that of the old parabolic velocity profile.

References

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Appendix

A and B matrix of the state equations for left systolic period are:

$$A_L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha_L}{\beta_L} & -\frac{\frac{Gr}{Eff} r A_{pL}}{\beta_L} & 0 & 0 & 0 & \frac{\frac{Gr}{Eff} r A_{pL}}{\beta_L} \\ 0 & \frac{r A_{pL}}{C_{Ao}} & \frac{-R_{Bron} - R_{sys}}{C_{Ao} R_{Bron} R_{sys}} & \frac{1}{R_{Bron} C_{Ao}} & 0 & \frac{1}{R_{sys} C_{Ao}} & 0 \\ 0 & 0 & \frac{1}{R_{Bron} C_{pul}} & \frac{-R_{pul} - R_{Bron}}{C_{pul} R_{pul} R_{Bron}} & \frac{1}{R_{pul} C_{pul}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_{pul} C_{pa}} & \frac{-1}{R_{pul} C_{pa}} & 0 & 0 \\ 0 & 0 & \frac{1}{R_{sys} C_{sys}} & 0 & 0 & \frac{-R_{triV} - R_{sys}}{C_{sys} R_{triV} R_{sys}} & \frac{1}{R_{triV} C_{sys}} \\ 0 & -\frac{r A_{pL}}{C_{var}} & 0 & 0 & 0 & \frac{1}{R_{triV} C_{var}} & \frac{-1}{R_{triV} C_{var}} \end{bmatrix}$$

$$\alpha_L = -\frac{K_t K_e}{R_m Gr} - \frac{R_r}{Gr} - \frac{Gr}{eff} r^2 \left(R_{sq} + R_{AoV} A_{pL}^2 \right) - \frac{Gr_1 R_{j1}}{eff_1 \, Gr_{23mf}} - \frac{Gr_{12} R_{j2}}{eff_{12} \, Gr_{3mf}} - \frac{Gr_{123} R_{j3}}{eff_{123} \, Gr_{mf}}$$

$$\beta_L = \frac{I_m}{Gr} + \frac{Gr}{eff} r^2 \left(M + I_{Ao} A_{pL}^2 \right) + \frac{Gr_1 I_1}{eff_1 \, Gr_{23mf}} + \frac{Gr_{12} I_2}{eff_{12} \, Gr_{3mf}} + \frac{Gr_{123} I_3}{eff_{123} \, Gr_{mf}}$$

$$B_L = \begin{bmatrix} 0 & \frac{K_t}{\beta_L \cdot R_m} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$