Adaptive Control of Multiplexed Closed Circuit Anesthesia

Gyu-In Jee* and Rob Roy**

*Department of Electronics Engineering
Konkuk University
Seoul 133-701, KOREA
**Biomedical Engineering Department
Rensselaer Polytechnic Institute
Troy, NY 12180, U.S.A.

Abstract

This paper describes the design of an adaptive closed circuit anesthesia controller based on a multiplexed mass spectrometer system. The controller deals with measurement deterioration caused by measurement delay and rise time through a long catheter as well as long sampling times due to the multiplexed measurements. Measurement data is extrapolated between sampling periods to increase the estimation convergence rate. A multiple-step-ahead predictive control algorithm is used to calculate intermediate control inputs between sampling intervals. Simulations are used to validate the designed controller.

1 Introduction

With the advent of reliable operating room mass spectrometers the monitoring required for an anesthetic is available, and closed circuit anesthesia can be administered in a safe, reliable manner. The couping of low cost computers and increasing sophisticated monitoring has brought closed circuit anesthetic regimes into operating room.

A recent publication by Vishnoi and Roy [1] address the couping of computer and mass spectrometer to provide adaptive control of closed circuit anesthesia(CCA). By considering flow and mass balances around the closed circuit, bilinear state equations are obtained for alveolar oxygen, circuit volume, and alveolar halothane concentration. A recursive least square algorithm is used to estimate oxygen consumption, nitrous oxide uptake, and halothane uptake from the four measured quantities; alveolar oxygen, halothane, nitrous oxide concentration, and circuit volume. Since the state equations for the CCA system are nonlinear, a predictive control system is used.

This paper examines the problems raised when the control system of [1] is used with a multiplexed mass spectrometer. Since a single mass spectrometer in each operating room is too costly, the usual arrangement is to have a central mass spectrometer service a number of operating rooms, typically ten. In this situation the gas profiled from the patients at ten current sites are stored in long sampling catheters and directed to the centrally located mass spectrometer [2]. The mass spectrometer switches between the catheters, providing end tidal and inspired gas concentrations which are displayed at each site. Since the sampling catheters are long (typically 30m) the gas concentration measurements are degraded [3]. It has been shown [4] that the gas delivery characteristics of the catheter can be modeled as a first order system with a transport delay. The process of multiplexing increases the sampling interval of an individual site. A mass spectrometer dedicated to a single site will have a sampling interval of six seconds. If the mass spectrometer services ten locations then each location will be sampled every 60 seconds. If one of these locations is using the adaptive closed circuit anesthesia delivery system of [1], then a modification of the control law must be made to account for the increased sampling interval, rise time, and transportation delay.

2 Model and Controller

2.1 Model

The model of the closed circuit can be derived by setting up the gas mass flow rate equations across compouent of the circuit [1]. The resultant model is a SISO systems for the halothane concentration and a MIMO system for oxygen and circuit volume with oxygen and nitrous oxide as inputs. The effect of halothane on the MIMO system is assumed to be negligible.

By considering the total inflows to and outflows from the circuit, the state equation for the circuit volume is obtained as,

$$\dot{V}_S = U_{O_2} + U_{N_2O} - V_{O_2} - V_{N_2O} - V_L \quad (1)$$

Considering the mass balance for oxygen at the ventilator and completing the circuit we obtain a bilinear state equation for the alveolar oxygen concentration as follows:

$$F_{AO_2} = [F_{AO_2}(V_{O_2} + V_{N_2O}) - F_{AO_2}(U_{O_2} + U_{N_2O}) + U_{O_2} - V_{O_2}]/V_F \quad (2)$$

Thus we obtain a MIMO nonlinear(bilinear) state space model for the alveolar oxygen concentration and circuit volume.

Following the same procedure, we can obtain a similar equation for alveolar halothane concentration.

$$F_{AH} = [F_{AH}(V_{O_2} + V_{N_2O}) - F_{AH}(U_{O_2} + U_{N_2O} + U_H) + U_H - V_H]/V_F \quad (3)$$

In the derivation it is assumed that the inspired and alveolar concentrations of oxygen and halothane are equal. This assumption, though not valid for open-circuit, generally holds in the closed-circuit in steady state.

2.2 Estimator

The estimated parameters are oxygen consumption, nitrous oxide and halothane uptakes. The controlled variables are alveolar oxygen(F_{AO_2}), alveolar halothane concentration(F_{AH}) and circuit volume(V_S). This allows the estimator to be decoupled into the following three equations.

$$V_{O_2} = U_{O_2} - V_F F_{AO_2} - V_S F_{AO_2} - V_S F_{IO_2} - V_L F_{AO_2} \quad (4)$$

$$V_{N_2O} = U_{N_2O} - V_F F_{AN_2O} - V_S F_{AN_2O} - V_L F_{AN_2O} - V_S F_{IN_2O} \quad (5)$$

$$V_H = U_H - V_F F_{AH} - V_S F_{AH} - V_L F_{AH} - V_S F_{IH} \quad (6)$$

The estimation of these parameters is performed by using recursive least square estimation. It is assumed that the circuit leak(V_L) and the lung functional residual capacity(V_F) are known.

2.3 Controller-Predictive Control

To control the bilinear system a predictive control algorithm [5] is used. It has a advantage of being able to utilized the true nonlinear model of the process. Thus it may be better able to capture the intrinsic features of the control problem than a control law based on a linear approximation. Predictive control is based on an assumed model of the process and on an assumed scenario for the future control inputs. This gives a sequence of control inputs. Only the first one is applied to the process, and a new sequence of control signals is calculated at next sampling time. This strategy is called a receding horizon control [6]. One common assumption on the future inputs is that the control input will remain constant. This constant future control input assumption is used here. The k-step-ahead predictive controller is designed to minimized the quadratic cost function consisting of the control effort and predicted output error variance from the desired output. The incorporation of the estimator with this controller results in an indirect adaptive predictive control scheme [1].

3 Extension to Multiplexed System

3.1 Design Consideration

The manner in which the gases are drawn has a direct bearing on the design of the control system. A 20 seconds profile of airway gas concentration is stored in each sampling catheter. This profile is sent down a long(30 meter), thin(1.07 mm int. dia.) catheter to the central mass spectrometer. The gas flow rate is low due to a small inlet(760 torr) pressure gradient. This results in a transient time of approximately 21 seconds. When a particular catheter is switched to the mass spectrometer for gas analysis the outlet pressure drops to 80 torr, and the gas profile to be analyzed in approximately 6 seconds. The analysis provides for a single breath to be validated. If the same catheter remains connected (single room analysis) the transport delay is reduced to 6.5 seconds. Consequently the range of transport delay is from 6.5 seconds to 21 seconds. This sampling, compression, and expansion of the airways provides a single analysis for each of 10 operating rooms every 60 seconds even though the data form each operating room may have undergone a 20 second transport delay. Furthermore since the sampling catheters have a the gas delivery characteristic which can be modeled by the previously discussed transport delay. Therefore the current measurement vector from the mass spectrometer $Z(n)$ is assumed equal to the true value $Y(n)$ of the gas concentration N_d samples in the past.

$$Z(n) = Y(n - N_d) \tag{7}$$

The problems caused by the increased sampling time and transport lag lie in the controller design and the estimator equations. The original state equations of the closed circuit systems are in continuous time. The conversion to a sampled data system involves calculating the derivatives as backward differences. As the sampling interval increases these approximations to the derivatives becomes less accurate. One of the assumptions used in the derivation of the state equations is that alveolar and inspired concentrations are equal. This is certainly not true during transient disturbances, and as the sampling interval increase becomes less true during transients, causing large errors.

The state equations are bilinear, requiring a k-step-ahead predictive control. The bilinearity becomes more pronounced as the sampling interval lengthens, increasing the difficulty of providing smooth control.

Although all of the preceding problems with increase sampling interval cause a degradation of system performance the principle problem is with the recursive least square estimator. The estimates of oxygen uptake, nitrous oxide uptake, and halothane uptake are necessary to provide the proper fresh gas flow to maintain the circuit volume and the proper concentrations. The number of samples required for convergence is the same regardless of the sampling interval. If the sampling interval is increased by a factor of ten, then the estimator takes ten times longer for the parameter values to convergence. This is clearly too long for adequate control, and simulations have shown that instabilities can arise with erroneous estimation. Faster convergence is achieved by using extrapolated data between samples. The extrapolation used is based on the assumption that uptake is proportional to the inverse of the square root of time. With this type of extrapolation the estimator is reinitialized at each true data point. This method produces a smooth and rapid convergence.

3.2 Modification of Control Algorithm

The previous changes are incorporated in the design of multiplexed adaptive controller. Predictor, estimator, and controller are modified to compensate the delay in the measurements.

After compensation the equation for halothane concentration can be given by

$$\bar{F}_{AH}(n+1|n) \triangleq A_H(n|n)\bar{F}_{AH}(n) + B_H(n|n)U_H(n - N_d) + E_H(n|n) \tag{8}$$

where

$$A_H(n|n) \triangleq 1 + \frac{h}{V_F}(V_{O_2}(n|n) + V_{N_2O}(n|n) \tag{9}$$

$$-U_{O_2}(n) - U_{N_2O}(n)) \tag{10}$$

$$B_H(n|n) \triangleq \frac{h}{V_F}(1 - \bar{F}_{AH}(n|n)) \tag{11}$$

$$E_H(n|n) \triangleq -\frac{h}{V_F}V_H(n|n) \tag{12}$$

Note that the above equation is a time-varying bilinear equation (*i.e.*, $A_H(n|n)$, $B_H(n|n)$ and $E_H(n|n)$ is time- varying).

Now we consider prediction of Halothane concentration. Note that $A_H(n+1|n), A_H(n+2|n), \ldots, E_H(n+1|n), E_H(n+2|n), \ldots$ are unknown future variables at time n. We will consider them as constant during the derivation of the prediction equation. Then N_d-step-ahead prediction of Halothane concentration at time n, $\bar{F}_{AH}(n+N_d|n)$ which corresponds to $F_{AH}(n|n)$ can be obtained by

$$\bar{F}_{AH}(n+N_d|n) = A_H^{N_d}(n|n)\bar{F}_{AH}(n) + \sum_{i=0}^{N_d-1} A_H^{N_d-1-i}(n|n)G_H(n+i|n)U_H(n+i-N_d) + \sum_{i=0}^{N_d} A_H^i(n|n)E_H(n|n) \tag{13}$$

Note that this a known quantity at time n. The $k+N_d$-step-ahead prediction of halothane concentration is calculated based on current available variables. $\bar{F}_{AH}(n+N_d|n)$ which corresponds to $F_{AH}(n|n)$ can be given by

$$\bar{F}_{AH}(n+N_d+k|n) = F_H(n)\bar{F}_{AH}(n+N_d|n) + G_H(n)U_H(n) + E_H(n) \tag{14}$$

where

$$F_H(n) \triangleq A_H^k(n|n) \tag{15}$$

$$G_H(n) \triangleq (\sum_{i=0}^{k-1} A_H^i(n|n))B_H(n+N_d|n) \quad (16)$$

$$E_H(n) \triangleq (\sum_{i=0}^{k-1} A_H^i(n|n))E_H(n|n) \quad (17)$$

Using the same reasoning the coupled O_2 and V_S prediction can be given by

$$\begin{bmatrix} \bar{F}_{AO_2}(n+N_d+k|n) \\ V_S(n+k|n) \end{bmatrix} = F(n) \begin{bmatrix} \bar{F}_{AO_2}(n+N_d|n) \\ V_S(n) \end{bmatrix} + G(n) \begin{bmatrix} U_{O_2}(n) \\ U_{N_2O}(n) \end{bmatrix} + E(n) \quad (18)$$

where

$$F(n) \triangleq \begin{bmatrix} A_{O_2}^k(n|n) & 0 \\ 0 & 1 \end{bmatrix} \quad (19)$$

$$G(n) \triangleq \begin{bmatrix} \sum_{i=0}^{k-1} A_{O_2}^i(n|n)B_{1O_2}(n+N_d|n) \\ kh \\ \sum_{i=0}^{k-1} A_{O_2}^i(n|n)B_{2O_2}(n+N_d|n) \\ kh \end{bmatrix} \quad (20)$$

$$E(n) \triangleq \begin{bmatrix} \sum_{i=0}^{k-1} A_{O_2}^i(n|n)E_{O_2}(n|n) \\ kE_{V_S}(n|n) \end{bmatrix} \quad (21)$$

Equation (14) and (18) can be expressed as

$$Z(n+k+N_d|n) = F(n)Z(n) + G(n)U(n) + E(n) \quad (22)$$

A k-step cost function can be written as

$$\begin{aligned} I = &[Z(n+k+N_d|n) - Y^*(n+k)]^T \\ &\cdot Q_y[Z(n+k+N_d|n) - Y^*(n+k)] \\ &+[U(n) - U(n-1)]^T Q_u[U(n) - U(n-1)] \end{aligned} \quad (23)$$

By setting $\partial I/\partial U(n) = 0$ the control law is obtained as

$$\begin{aligned} U(n) = &-\left[G^T(n)Q_yG(n) + Q_u\right]^{-1} \\ &\cdot\left[G^T(n)Q_y\left(F(n)Z(n) + E(n) - Y^*(n+k)\right) - Q_uU(n-1)\right] \end{aligned} \quad (24)$$

Note that control law is function of known measurement $Z(n), Z(n-1), \ldots$ and the past inputs $U(n-1), U(n-1), \ldots$.

Due to the multiplexed operation of mass spectrometer, the measurement $Z(n)$ is not available at every step n (in case of 10-room operation, once per 10 step). Define N as sampling time T between each data point. To resolve this problem we calculate the control input $U(n)$ at the moment of sampling only, and use this value until the next sampling time n+N.

$$U(n+i) = U(n), i = 1, \ldots, N-1 \quad (25)$$

4 Simulations and Discussions

The adaptive control algorithms described in the previous sections are tested on a seven compartment model for a 25 kilogram patient. This allows the simulation results to be compared with experiment results from a 25 kilogram dog. Simulation program is written in ASYST. We set BPM equal to 10 and h equal to 6 second. Sampling time T ranging from 6 to 60 second and time delay T_d ranging from 0 to 24 second are considered in simulations.

Simulation results show that one-step prediction horizon $k = 1$ results in a faster response but larger overshoot and oscillation in all cases. This is mainly due to the nonlinearity of the system. So multi-step prediction horizons ($k = 2, 3, 4$) are used and result in better output responses but they get slower as k increases.

Control performance (reference tracking performance) is degraded due to the delay in sampling and long sampling time. To see how the time delay T_d affect output response and how the direct delay compensation works, we first set sampling time $T = 6$. Note that three control methods are all equivalent in this case. When $k = 2$, output responses show larger oscillation as T_d increases. But by estimating T_d properly we can compensate the delay effect. Since controller uses output prediction to compensate time delay , large value of $\hat{T}_d$ results in longer step output prediction so output prediction gets inaccurate as $\hat{T}_d$ increases.

5 Conclusions

The feasibility of extending the single patient adaptive CCA controller to a multipatient mass spectrometer set-up has been demonstrated. Worst-case simulations show that the modified controllers perform satisfactorily when the sampling period is increases tenfold. Animal experiment is currently being performed to validate the designed controller.

Further development of the multiplexed controller involves modeling and simulation of the multiplexed spectrometer system incorporating catheter delays and sample compression. The circuit volume measurement needs to be improved. The on-line measurement of functional residual capacity during nitrogen washout is required. A further application of the adaptive controller would be to use the continuous estimate of oxygen uptake as a basis for continuous monitoring of cardiac output.

References

[1] Vishnoi, R. (1988). "An Adaptive Bilinear Controller for Closed Circuit Anesthesia," Ph.D. Thesis, Rensselaer Polytechnic Institute, Troy, New York.

[2] Ozanne, G.M., *et. al.* (1981). "Multipatient Anesthetic Mass Spectrometry: Rapid Analysis of Data Stored in Long Catheters." *Anesthesiology*, vol. 55, 62-70.

[3] Davies, N.J.H. and D.M. Denison (1979). "The Use of Long Sampling Probes in Respiratory Mass Spectrometry", *Resp. Physiology*, vol 37, 335-346.

[4] Carlon, G.C., *et. al.* (1990). "Frequency Response of the Peripheral Sampling Sites of a Clinical Mass Spectrometer", *Anesthesiology*, vol 72, 187-190.

[5] Åstrom, K.J., and B. Wittenmark (1989). *Adaptive Control*, Adison-Wesley.

[6] Ydstie, B.E. (1984). "Extended Horizon Adaptive Control," *Proc. of IFAC, 9th Triennial World Congress*, Budapest, 911-915.