A Bed Schematized Two-dimensional Sediment Transport Model

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1. Introduction

Bottom sediments, in various types of sediment transport models, have been usually assumed to be horizontally and/or vertically homogeneous. The assumption may be appropriate in well-sorted sedimentary environments including sand beaches and high turbid regions of fine grained cohesive sediments. In most of coastal sedimentary environments, however, bed characteristics are heterogeneous due to the differences of the external driving forces, the distances from the sources, the consolidation times, and the water contents *etc*. Therefore, even in well-sorted cohesive bed, it is necessary to consider the vertical profile of shear strength and consolidation for the reliable estimations of bed processes. Furthermore, the transport mechanism of poorly-sorted mixtures of cohesive and non-cohesive sediments is so complex that there are few studies.

As a step to improve the situations, a horizontally two-dimensional sediment transport model named BESST(BEd Schematized Sediment Transport) is proposed. BESST schematizes the sediment bed in several layers according to shear strengths as well as estimates the transport of heterogeneous sediment mixtures. Unfortunately, there are few experiments or fields data to quantitatively verify BESST. Although it has successfully simulated the sedimentation patterns around Busan Harbour in comparison with cores, it is still required to accomplish laboratory experiments and intensive field measurements in order to confirm the validity of this model.

2. Governing Equations

BESST estimates sediment transports separately in two modes depending on grain diameter. That is, one is suspended load for fine grains below 62.5 μm, and the other is bed load above this sand boundary. This approach means that the behavior of silts, transition sizes between cohesive and non-cohesive grains, is equivalent of clay.

Firstly, the depth-averaged two-dimensional mass balance for suspended load can be expressed by the following advection/diffusion equation with source/sink term,

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(uC) + \frac{\partial}{\partial y}(vC) = \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial C}{\partial x} + D_{xy}\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial y}\left(D_{xy}\frac{\partial C}{\partial x} + D_{yy}\frac{\partial C}{\partial y}\right) + \frac{S}{H} \quad (1)$$

where, C is depth-averaged concentration of sediment, u,v are depth-averaged flow velocities in x,y direction respectively, D_{xx}, D_{xy}, D_{yy} are turbulent-diffusion/dispersion coefficients (Elder,1959), S is source/sink term($= E + D$), E, D are the rates of erosion and deposition respectively, H is water depth, and t is time.

The rate of deposition, D, can be written by following relation,

$$D = -w_s C\left(1 - \frac{\tau}{\tau_d}\right), \qquad \tau < \tau_d \quad (2)$$

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where, τ is bed shear stress, τ_d is critical deposition shear stress ranging from 0.08 to 0.15 N/m^2, and w_s is the settling velocity of suspended floc.

There are two formulae for the rate of erosion depending on bed conditions. When the bed properties are relatively uniform over depth, the rate is given by,

$$E = M\left(\frac{\tau}{\tau_e} - 1\right), \qquad \tau > \tau_e \tag{3}$$

where, M is an empirical rate constant ranging about 0.00001 – 0.0004 kg/sm^2, and τ_e is cohesive shear strength with respect to erosion(critical erosion shear stress). For a stationary suspension and a consolidating bed, E can be written as following equation(Parchure and Mehta, 1985),

$$E(z) = E_o\exp\left[\alpha\{\tau - \tau_e(z)\}^{0.5}\right], \qquad \tau > \tau_e(z) \tag{4}$$

where E_o and α are rate determining coefficients. The shear strength, τ_e, in this case increases with depth(z) below the bed surface.

It is still difficult to effectively measure the natural critical shear stress, τ_e, in fields or flumes even though some attempts have been tried out. Migniot(1968) proposed a following recommendable formula which relates the dry density of sediment to the shear strength.

$$\tau_e = \zeta\rho_d^{\delta} \tag{5}$$

where, ρ_d is the dry density of sediment, and ζ, δ are empirical coefficients. Thorn and Parsons(1980), using three different natural muds, found $\zeta = 5.42 \times 10^{-6}$, $\delta = 2.28$.

To schematize the bed according to the shear strengths, Hayter(1983) induced the following formula through regression analyses with several experimental data of others.

$$\frac{\rho_d(z)}{\bar{\rho}_d} = A\left(\frac{h - z}{h}\right)^B \tag{6}$$

where, $\bar{\rho}_d$ is mean dry density, h is the thickness of considering bed, z is downward depth from the bed surface, and A, B are time dependent functions. The shear strength of the bed surface, being contact with overlying water, does not varies with time(Parchure and Mehta,1985).

There are a great many formulae to predict non-cohesive sediment transport, but none have been developed for use in the sea under oscillatory tidal currents. In this sense, the results of Heathershaw(1981) are very helpful in the choice of suitable equation. He compared measured rates with predictions computed by five modified equations which have been used frequently. And of all the predictions, only Gadd *et al.*'s(1978) fell consistently within limits. Their equation of a Bagnold's type is written as,

$$q_{sb} = \beta(u_{100} - u_{100cr})^3 \tag{7}$$

where, q_{sb} is the rate of bed load transport, u_{100}, u_{100cr} are current and critical current for grain movement 100 cm above sea-bed respectively, and β is a coefficient of proportionality depending on grain size.

3. Bed Process Algorithm

Sea bottom in BESST consists of maximum eight layers of which shear strengths increase with depth(z). Upper three layers belong to a stationary suspension, four layers of middle part organize a consolidating bed, and lower only one layer represents a consolidated bed. The thickness of each layer is fixed. In real fields, however, fixing of the thicknesses is unreasonable. For example, when four layers are eroded out after a certain period of time, the shear strength of the new top layer shouldn't be its initial value, that of layer five before the erosion. As prescribed, the shear strength of the bed surface is constant without dependence on consolidation times, which means that this phenomenon should happen in case of erosion, too. That is, the more layers are eroded out, the steeper the gradient of the shear strength profile should become. Such process can be explained by the liquefaction on which any experiment hasn't been carried out.

Deposition and consolidation processes of Hayter(1983) seem quite reasonable, but his approaches may cause a discontinuity in the shear strength profile. When deposition happens after the erosion of the previous paragraph, he fills layers downward from the top. And then, the shear strengths and the thicknesses of filled layers change with time. In this approach, there should be a gap in the shear strength profile until layer four is filled. Furthermore, his erosion should be underestimated without considering of the liquefaction.

It is, however, too difficult to consider the liquefaction in numerical models. When or if one layer is eroded out, BESST assigns the shear strength of the new top layer by averaging those of two interfaces, one between water and the new top layer the other between the top and the second layer. This treatment may not cause the discontinuity.

Two methods for evaluating the transport of sediment mixtures have been proposed. One is to devide the modelling area into cohesive and non-cohesive regions, and two different formulae are applied. Representatives of this approach are Yum *et al.*(1985) and van Rijn and Meijer(1991). The other is to treat bottom materials separately as mud and sand according to each weight proportion. Yum *et al.*(1991a) and Tsuruya *et al.*(1991) adopted this method. Both of these methods, however, have inherent problems. In case of the former, real fields usually don't represent such distinct division except for special regions. And the latter disregards the interaction between cohesive and non-cohesive grains.

BESST adopts the latter, but it's a little different from the method in some points. Assuming selfweight sortings, the model devides sediments in a layer vertically according to the weight proportions, the upper part is for muds and the lower for sands. And then, lower sands won't move unless upper muds are thoroughly eroded out.

4. Application

BESST has been applied to Busan Harbor where an artificial island is under planning. The area is influenced by tidal currents and waves. The major source of the sediments is the Nakdong River lying about 6 km west of the harbor. The range of mean diameter is quite wide from 8 to 500 μm(Yum *et al.*,1990; 1991b). Depth field and mud percents in the modelling area are depicted in Fig.1 and Fig.2.

Pre-initial shear strengths of the eight layers are represented in Table 1. In this profiling, the Nakdong series of Lee's(1991) cores, the flume tests of Pachure and Mehta(1985), and the eq.6 of Hayter(1983) were referred. Assuming that serious erosions have not been happening, initial shear stress profiles of all the cells were re-set up by adjusting the pre-initial profile to the maximum tidal current of each cell.

BESST uses 2nd upwind scheme, $\Delta x = \Delta y = 70m$, and $\Delta t = 25.875s$. The floc settling velocity took its value of $0.31mm/s$ measured by Lee *et al.*(1991) near the Nakdong. The value of $0.15N/m^2$ was adopted as critical deposition shear stress, τ_d, through the model calibration. For constant M in the eq.3, the value of $10^{-5}kg/sm^2$ was used by assuming low erosion rate. For coefficients E_o and α, values of $0.42 \times 10^{-5} g/cm^2$min and $8.3m/N^{0.5}$ were selected which had been found in a bay mud, a lake mud and estuarial muds(Pachure and Mehta,1985). For δ in the eq.5, the same value of Thorn and Parsons' was used while the half of theirs for ζ. β in the eq.7 took its value according to the mean diameter of sand fraction(Heathershaw,1981). General logarithmic velocity profile and the depth-averaged current velocity computed by the model named KORFLO of Kang *et al.*(1991) were used for the calculation of u_{100}. For u_{100cr} in also the eq.7, the modified Shields' curve of Yalin(1972) was helpful. As input concentrations of suspended sediments at three open boundaries, the observed results of Yum *et al.*(1990,1991b), Lee *et al.*(1991), and Park(1985) were used.

There are two necessary sufficient conditions to confirm the validity of a sediment model. One is to verify in the point of sediments concentration, and the other is of the rate of sedimentation. Unfortunately around Busan, the sediment concentrations are very low, being about 10*mg/l*, in dry seasons, and time variations are so vague that the differences between the maximum and the minimum values fall within computing error limit. In case of rainy season, only one observation at western open boundary was carried out(Yum *et al.*,1991b). But fortunately, there have been some studies on the rate of the sedimentation(KECC,1989; Suk *et al.*,1991; Park and Chu,1991). Mud layer of the area has been deposited since 5,000~7,000 years before, the major source of this Holocene sequence is the Nakdong, and the annual deposition rate is about 2 *mm/yr* according to their results.

Fig.3 depicts the spatial distributions of the sediment concentrations at low and high water of mean spring tide in rainy season. Although a quantitative verfication, of course, was not carried out, the qualitative pattern at high water seems to be relatively good in comparison with LANDSAT image(Park and Chu,1991). Fig.4 represents contours of the anuual deposition rates by BESST, and Fig.5 depicts the sampling sites of cores and the thicknesses of the mud layers around proposed area of the artificial island and along its detached breakwater(KECC,1989). The comparions of computed annual deposition rates with measured mud thicknesses are depicted in Fig.6 along selected sections. The applicability of BESST seems to be confirmed to some degrees by Fig.6. The sedimentation patterns during transition periods before the settling down of the new equilibrium bed conditions after the construction, is depicted in Fig.7. Dotted lines in the figure mean erosions. These areas are potential erosion zones , and the quantitative prediction should'nt be trusted because the profiles of sediment properties have not been known thoroughly until now.

5. Conclusions and Recommendations

The phenomenon of sediment transport has inherent difficult problems to be considered in numerical models. Furthermore, some of them have not been grasped yet theoritically, of which representatives are related cohesive sediments. Therefore, it seems too early to simulate the transport of cohesive and non-cohesive mixtures. BEEST may be a step to improve this situation. It is, howerver, strongly required to exame the behavior of the mixtures and liquefaction, and to raise the validity of BESST. In addition to such problems, the effect of waves should be included in BESST.

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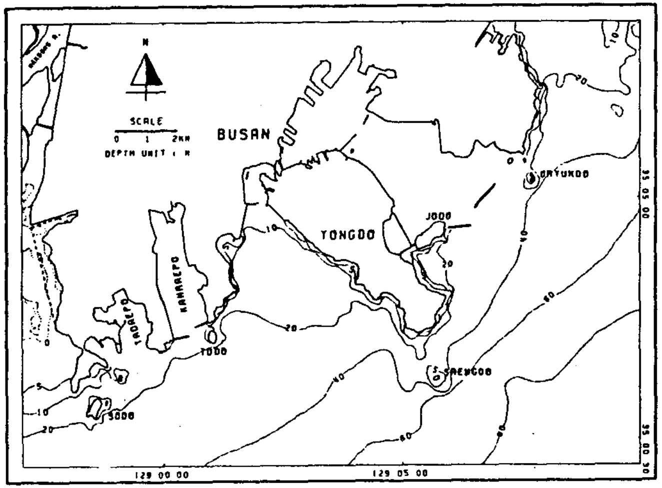

Fig.1 Bathymetric chart around Busan Harbour.

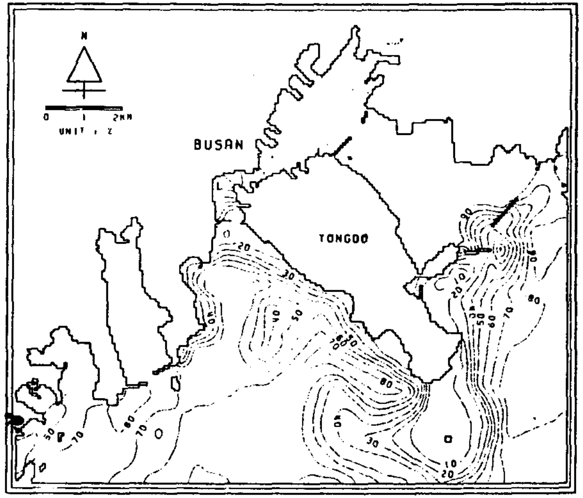

Fig.2 Mud percents in the modelling area.

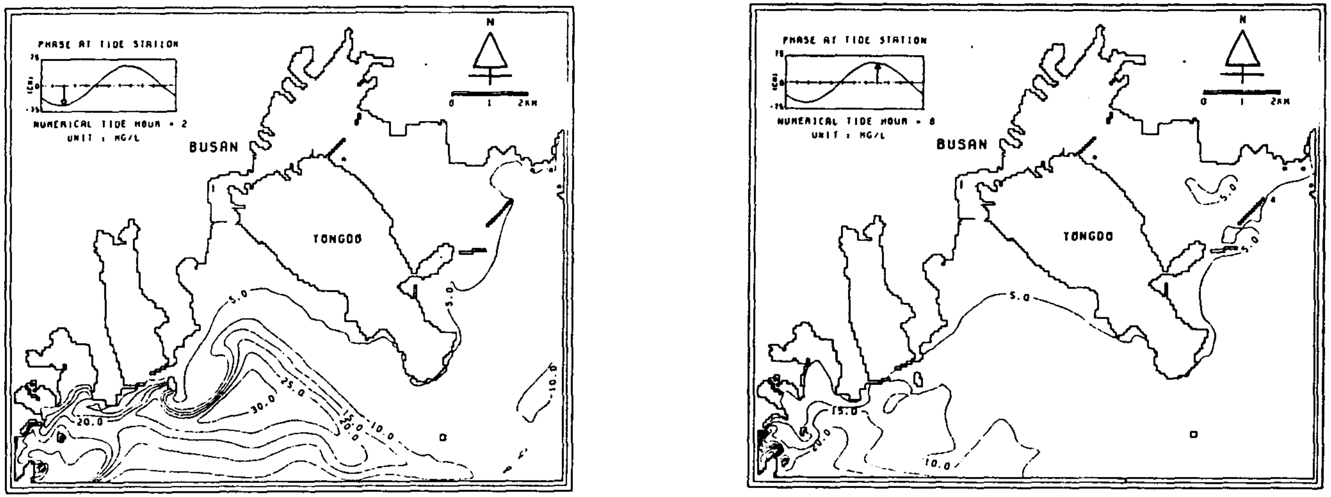

Fig.3 Spatial distributions of sediment concentration at low and high waters of mean spring tide in rainy season.

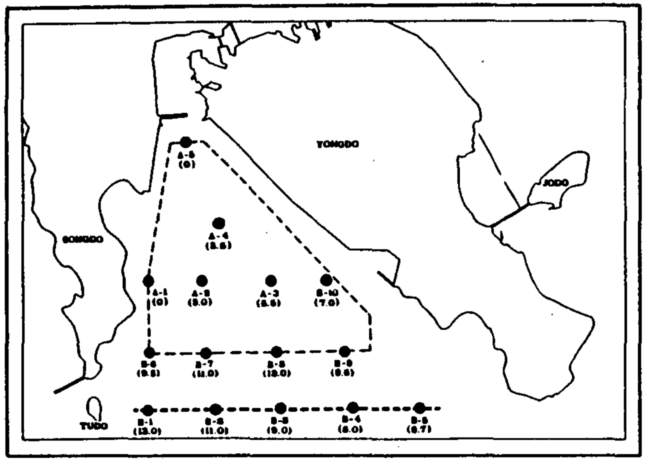

Fig.4 Sampling sites of cores and thicknesses of mud layers (KEOC, 1989).

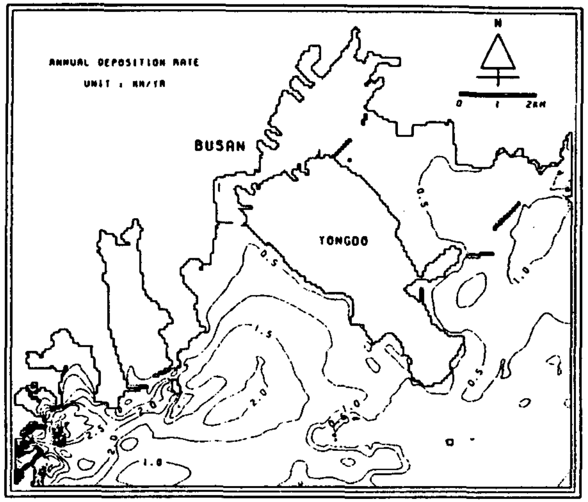

Fig.5 Computed annual sedimentation rates.

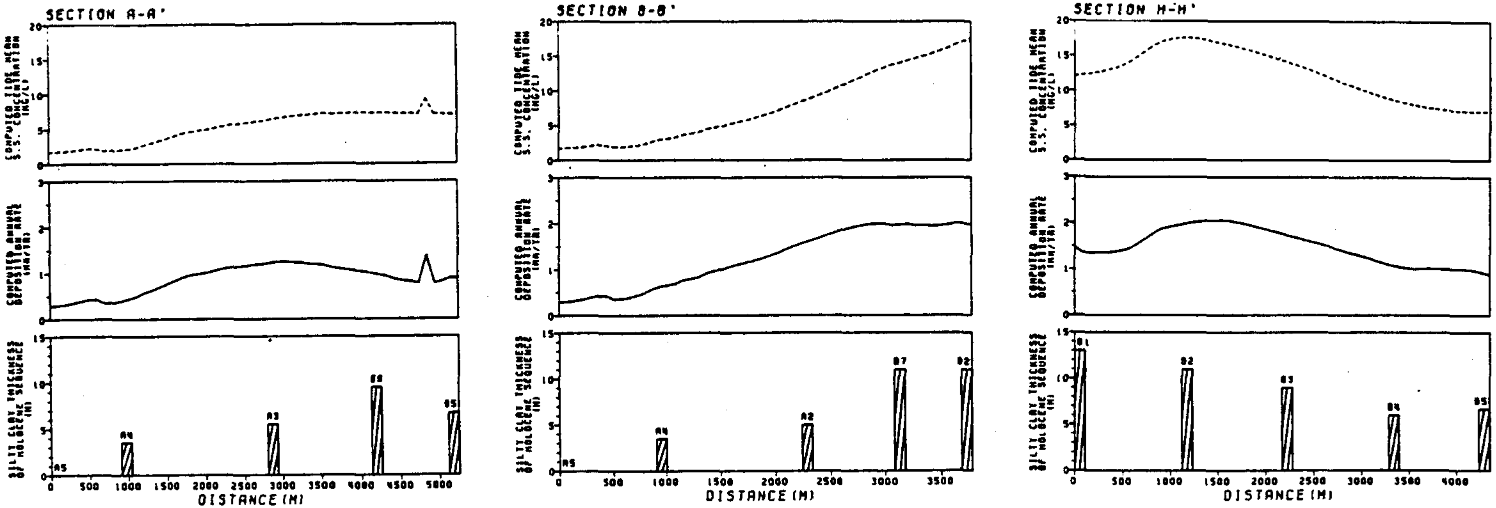

Fig.6 Comparisons of computed deposition rates with cores.

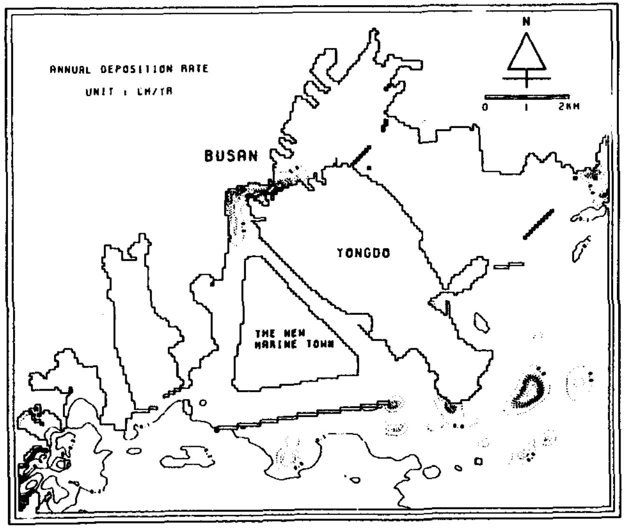

Fig.7 Computed sedimentation rates after the construction.

Table 1 Pre-initial bed condition in the modelling area.

Bed Name	Layer Number	Thickness (cm)	Mean ρ_d (kg/m^3)	Mean τ_s (N/m^2)
Stationary Suspension Bed	1	0.057	162.4	0.296
	2	0.057	232.6	0.672
	3	0.057	302.9	1.226
Consolidating Bed	4	1.000	347.4	1.678
	5	1.000	370.9	1.947
	6	1.000	404.8	2.376
	7	1.000	462.3	3.216
Settled Bed	8	1.000	615.1	5.209