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SSA기법에 의한 트러스 최적화

Truss Optimization based on Stochastic Simulated Annealing

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Abstract

A stochastic simulated annealing (SSA) is a recent approach to the solution of problems characterized by large number of interacting degrees of freedom. SSA simulates the degrees of freedom in a problem in a such a way that they are a collection of atoms slowly being coolded into a ground state which would correspond to the stationary point of the problem. In this paper, for a randomly disturbed current design, SSA optimization technique is used, which establishes a probabilistic criterion for acceptance or rejection of current design and iteratively improves it to arrive at a stationary point at which critical temperature is reached. Simple truss optimization problems which consider as their constraints only the tensile and compressive yielding strength of the members are tested using SSA. Satisfactory results are obtained and some discussions are given for the behavior of SSA on the tested truss structures.

1. INTRODUCTION

With the impressive progress of development in the computer technology together with the developement of efficient structural analysis of design methods in the last decade it becomes possible with varying degrees of success to efficiently handle the structural optimization which contains large degree of freedom and variables. In the area of structural engineering, the method of optimization has been steadily applied to various structural problems. Distinguishable linear and nonlinear optimization techniques have been successfully developed for finding optimal set of the material, topology, geometry or cross-sectional dimensions of different types of structures subject to particular loading system (Kavlie and Moe, 1971; Pedersen, 1973; Kirshc, 1981; Ding and Esping, 1985; and Scholz and Faller, 1986).

Along the main stream of linear programming (LP) and nonlinear programming (NLP) techniques, refined algorithms have been branched out in order to take into account for the discrete nature of structure, fabricated standardized structural conponents for example. Although great success has been achieved during the past decades in structural optimization, those techniques generally have difficulties in avoiding local minima and results are sometimes dependent upon the choice of the initial values in the design space.

Recently, with the aid of high speed conputers, a combinatorial optimization has been introduced in the field of computer science and has been successfully applied to the problems having a large number (possibly millions) of interacting decisions or degrees of freemdom. Example for this group includes the Travelling Salesman Problem(TSP),

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probably the best known problem in combinatorial optimization. A combinatorial optimization problem is formulized as a pair (R,C), where R is the finite-or possibly countable infinite-set of configurations (or configuration space) and C a cost function, C:R→ ⟨R⟩ , which assigns a real number to each configuration. The algorithm is based on randomization techniques and a number of aspects of iterative improvement algorithms is incorporated (Laarhoven and Arts, 1987).

Nearly optimum solutions to many combinatorial problems can be found using stochastic simulated annealing (SSA) (Kirkpatric,Gelatt and Vecchi, 1983), whose name is originated on the ground of thermodynamics. Stochastic simulated annealing finds a global minimum of an objective function (say, Hamiltonian H(S), S=(S1, S2, ..., Sn) by combining gradient descent with a random process. The method allows, under certain conditions, choices of S which actually increase H, thus providing SSA with a mechanism for escaping local minima. Decreasing slowly a parameter T (symbolically often referred to as the temperature) the severity and frequency of these uphill moves is gradually reduced so that the system settles into a global minimum.

2. Stochastic Simulated Annealing (SSA)

A brief theoretical review on SSA is given in this section. In thermodynamics, an ensemble is defined as a mental collection of a very large number, η, of systems, each contains N molecules and constructed to be a replica on a thermodynamis(macroscopic) level of the actual thermodynamic system. In particular, if a system is closed, and isothermal, this system is called "canonical ensemble."

It is well stated in statistical mechanis that probabiliy(P_j) observing a given quantum state Ej in an arbitrary system of a canonical ensemble is equal to:

$$P_j = \frac{1}{\eta} \cdot \frac{\Sigma\, \bar{\Omega_t . n_j}}{\Sigma\, \Omega_t} \qquad \text{——(1)}$$

where η = total number of systems in the supersystem;
n_j= number of systems with energy state of E_j;
Ω_t= possible no. of states in supersystem consitent with distribution (n1, ··· nk).

If $\eta \rightarrow \infty$, the most probable distribution(to which the largest Ω_t belongs) completely dominates in average computation and thus eq.(1) becomes:

$$P_j = \frac{e^{-E_j/kT}}{\sum_i e^{-E_i/kT}}, \quad j=1,2,\ldots \qquad \text{——(2)}$$

where E_j = energy state of Ej (for a given number of molecules(N) and a volume(V));
K = Boltsman constant; and
T = tenperature.

Eq.(2) states that the probability of obeserving a given quantum state in a canonical ensemble decreases exponentially with the energy of the quantum state.
The average energy in the system of canonical ensemble is therefore given as:

$$\langle E \rangle = \int E(x)\cdot P(x)dx$$

$$= \int \frac{E(x)\cdot e^{-E(x)/kT}}{\int e^{-E(x)/kT} dx} dx \qquad \text{——(4)}$$

where E,P = energy and probability in enery of E, respectively ; and
X = configurational variables = (X1, X2,.., Xn).

Eq.(4) shows how the energy ⟨E⟩ changes as a temperature T changes and it represents simulation of an annealing process of a heated solid. The difficulties in evaluating P(X) in eq.(4) and the fact that P(x) is proportional to the exp(-E/kT) and thus varies very rapidly with E(x) lead to employ importance sampling technique forming a Markov chain (Berne, 1977). Construction of a Markov chain is accomplished by the use of a transition matrix P_{ij} by Metropolis et. al. The Markov chain constructed below has unique limit distribution π ($=c\cdot e^{-E/kT}$) and does not depend on the configurational integration:

$$P_{ij} = P_{ij}^* \qquad \pi_j \geq \pi_i \quad j \neq i \qquad \text{——(5)}$$
$$P_{ij}^*.\pi_j/\pi_i \qquad \pi_j < \pi_i \quad j \neq i$$
$$P_{ii} = 1-\sum_{j\neq i} P_{ij}$$

where $P_{ij} = Pr[j, t+1/ i, t]$; ——(6)
$\pi_j(t) = \Sigma \pi_i(t-1)P_{ij}$; and
P_{ij}^* = transition matrix of the underlying Markov chain.

The SSA algorithm which realizes above equations is schematically presented below and based on which optimal solutions of trusses under the FSD criterion are sought. The algorithm was programmed with C.

SSA Algorithm:

```
T = starting temperature
while (T > Tmin)
    do until (equilibrium is reached)
        purturb A[i]
        obtain Enew (A[i])
        if Enew < Eold
            accept A[i]
        else
            r = random number
            if exp (-(Enew-Eold)/T) > r
                accept A[i]
            else
                reject
Lower T
```

3. Truss Optimization: Examples

In general optimum structural design can be stated as minimizing the cost function by properly choosing design and corresponding variables subjected to constraints which are related to the design and the behavior of structure. Design variables of interest in this study are chosen to be member areas and stress constraints in terms of tensile and compressive yielding strengths are only imposed, limiting examples to the FSD (fully stressed design) criteria. The problem formulation then can be defined as:

$$\min Z = \sum_{i=1}^{m} A_i \cdot L_i$$

$$\text{subject to } g_i(A[j]) \leq 0, \; i=1,2,\ldots,nc \qquad (7)$$
$$j=1,2,\ldots,m$$
$$A_j \geq A_{lower\ limit}.$$

Using a penalty function (Reklaitis et.al., 1983), the above constrained optimum problem is transformed to a unconstrained problem:

$$\min \; z = \sum A_i \cdot L_i + R \cdot \sum_i \langle g_i \rangle^2 \qquad (8)$$

where A_i = cross sectional area of member i;
L_i = length of member i;
R = penalty parameter; and
$\langle g_i \rangle = 0$ if $g_i \leq 0$
g_i if $g_i > 0$

(1). Three bar truss: SSA is applied to a rather simple three-bar truss which is subjected to the load P=20K. (see Fig. 1). The conditions are :

$$\min Z = \sum_{i=1}^{3} A_i L_i \qquad (9)$$

$$\text{subject to } -15 \leq \sigma 1 \leq 20$$
$$-15 \leq \sigma 2 \leq 20$$
$$A1, A2 \geq 0$$

The exact solution is given by Schmit (1960) and it is reproduced in Fig 2. The minimum volume $263.9 in^3$ is obtained at A1=0.788 in^2 and A2=0.410 in^2. For the same truss the results from SSA is presented in Fig. 3. Minimum volume attained from SSA is $264.8 in^3$ at A1=0.76in^2 and A2=0.5in^2. Near the optimum, it can be seen from Fig.2 that small move of objective function in the design space bring a relatively sensitive changes in the value of A2.

(2). Ten bar Truss: Truss geometry and loading conditions are given in Fig. 4. Fig. 5 presents results from optimization by SSA under the stress constraints $|\sigma i| \leq$ 25ksi, i=1,2,..,10 with initial areas given by A_i^o = 5.0. Column (2) in Table 1 represents corresponding cross sectional areas at the optimum volume of 15962 in^3 for this truss. To see if there can be an improvement in final design, the values of colum (2) are used as inputs and the second run is made for the same ranges of the temperature given for the case in column(2). The results are given in column (3) and in Fig. 6. There seems to be essentially no difference between two designs. This supports independency of SSA on the initial values and that the method leads the design variables to a global minimum as time T approaches to the freezing temperature. Member forces in column (4) in Table 1 is obtained using the areas given in column (2) in Table 1 and its corresponding stresses are shown in column (5) in Table 1. Except members 3,4,8 and 9, all other members are in fully stressed states.

(3). Ten bar truss with different yielding strength: Another example is given for the same truss under the same loading condition except that $|\sigma_i| \leq 25$, $(i=1,2,\cdots,9)$ and $|\sigma_{10}| \leq 50$. The areas at true optimum (column (2), Table 2), the areas obtained from SSA with starting initial areas of A^o_i=5 and their corresponding stresses are given in columns (3),(4) and (5), in Table 2, respectively. It can be seen from this table that except members 8 and 10, all others are of fully or almost fully stressed state. Fig. 7 shows the trends of convergence of SSA in this particular problem. The column (6) in Table 2 shows the results obtained by assigning relatively small initial areas ($A^o_i=1$) to each member for ten bar truss. The result reveals that regardless of the choice of initial areas, SSA allows the design variables to converge to a global minimum. It is worth mentioning that since the SSA decreases acceptance probability steadily to zero as the temperature gets lowered this makes SSA itself enable to break out of local minimum and converge upon the global minimum.

In general, values of cross sectional areas of truss are not obtained exactly from SSA in this study although minimum volume is satisfactorily attained. This is mainly due to the nature of SSA in selecting random number in finite sizes during the iterative cooling process and also in terminating the cooling process at a relatively high temperature. With the expense of longer computing time, taking a smaller perturbation on the cross sectional areas as well as further lowering of freezing temperature would lead to a better solution. Since at a relatively low temperature (close to a freezing) the rate of changes both in volume and cross sectional areas of truss are small, one may use one of the nonlinear programming techniques to accelerate a convergence to an exact minimum in the vicinity of a global minimum.

4. CONCLUSIONS

Stochastic simulated annealing (SSA) is an algorithm based on a combinatorial optimization and bases its derivation on the cooling process of heated solid. SSA is experimentally applied to a rather simple truss optimization problems known as fully stressed design (FSD) without considering any buckling problems.

Applications of SSA to optimization problems in this class were successfully performed in obtaining optimum sectional areas of trusses. SSA has a capability of escaping local minima by accepting uphill moves during the cooling process. In relation to this it was observed in this study that in a limited number of different initial areas used for starting point at high temperature, volumes at global minimum are obtained regardless of choice of these initial values.

More realistic conditions on structural constraints are being considered as an extention to this study by the authors and it seems that some elaborated work related to reducing the computational time is needed in order to make SSA to converge within reasonable time frames.

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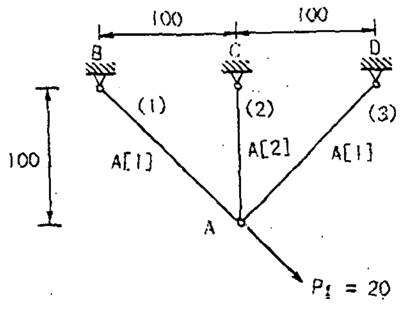


Figure 1. three bar truss

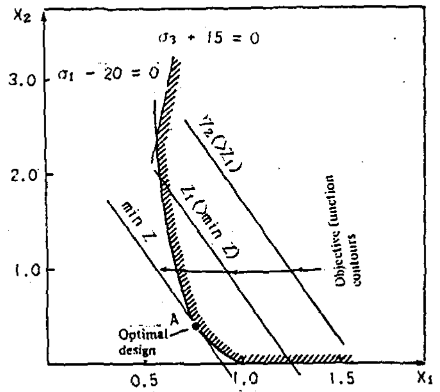


Figure 2. design space and optimum solution for the three bar truss

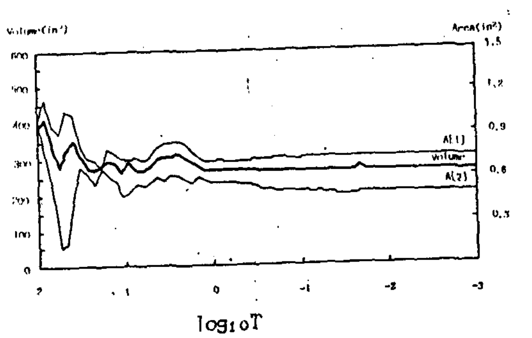


Figure 3. convergence to a minimum volume during the annealing process for the three bar truss. (σ_y^t=20, σ_y^c=15)

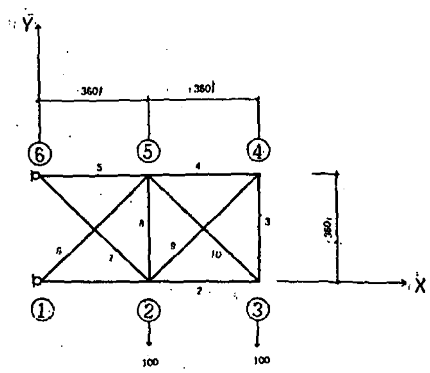


Figure 4. ten bar truss

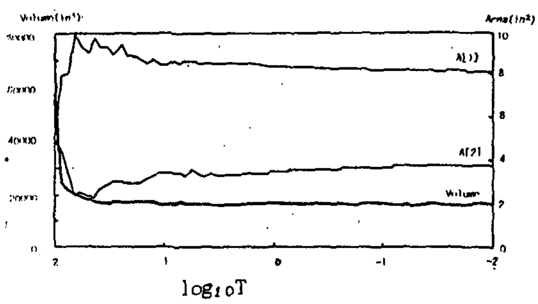


Figure 5. convergence to a minimum volume during the annealing process for the ten bar truss. (σ_y^t=25, σ_y^c=25)

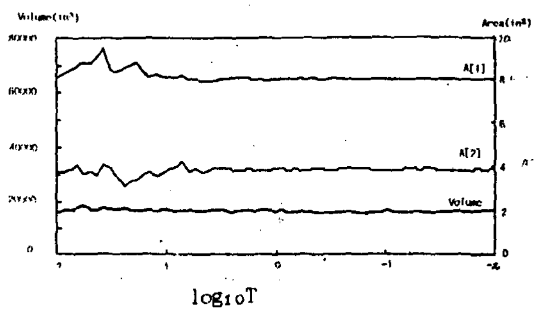
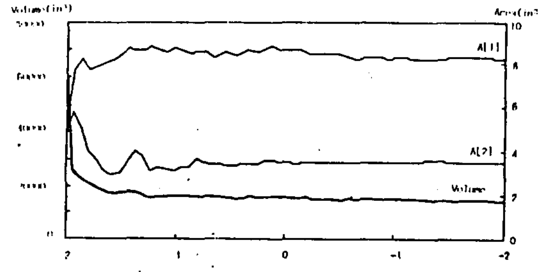


Figure 6. convergence to a minimum volume during the annealing process for the ten bar truss with initial areas given from previous solution set.(σ_y^t=25, σ_y^c=25)



$\log_{10}T$

Figure 7. convergence to a minimum volume during the annealing process for the ten bar truss. $\sigma_y{}^i=25$ for $i=1,2,\cdots 9$ and $\sigma_y{}^{10}=50$ for tension and compression.

Table 1. results from SSA for ten bar truss. ($\sigma_y{}^t=25$, $\sigma_y{}^c=25$)

(1) member no.	(2) sectional area	(3) sectional area	(4) member force	(5) stress
1	8.156	8.08	-201.86	-25
2	3.844	3.92	-98.13	-25
3	0.215	0.103	1.87	18
4	0.1976	0.108	1.87	18
5	7.848	7.92	198	25
6	5.437	5.56	-138.8	-25
7	5.889	5.76	144.1	25
8	0.124	0.1037	0.004	0.04
9	0.295	0.134	-2.644	-19.7
10	5.4368	5.55	138.78	25
Total volume	15962	15945		

Table 2. results from SSA for ten bar truss. $\sigma_y{}^i=25$ for $i=1,2,\cdots 9$ and $\sigma_y{}^{10}=50$ for tension and compression

(1) member no.	(2) true optimum	(3) sectional area ($A_i{}^o=5$)	(4) member force ($A_i{}^o=5$)	(5) stress ($A_i{}^o=5$)	(6) sectional area ($A_i{}^o=1$)
1	8.06	8.12	-203.1	-25	8.11
2	3.94	3.89	-96.8	-25	3.98
3	0.10	0.13	3.21	24.7	0.106
4	0.10	0.13	3.21	24.7	0.105
5	7.94	7.9	196.9	25	7.94
6	5.57	5.53	-136.9	-24.8	5.51
7	5.74	5.82	145.9	23.8	5.85
8	0.10	0.11	0.0744	0.676	0.105
9	0.1	0.18	-4.54	25	0.15
10	5.57	3.62	136.9	37.8	3.67
total volume	14976	15020			15052