

# 시뮬레이션 기초이론

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## 시뮬레이션의 정의

- Simulation is the *process* of designing a *model* of a *real system* and conducting *experiments* with this model for the purpose of understanding the *behavior of the system* or of *evaluating various strategies* for the operation of the system.
- *Experimentation* with *dynamic models*.
  
- 문제 해결을 위한 도구(Tool),  
모델을 바탕으로 한 실험,  
시스템의 이해, 대안의 예측 또는  
최적화를 목적으로 함.

## 컴퓨터 시뮬레이션의 수행이유

### 실제 실험 수행의 문제 :

- 위험성 ( 예 : war game )
- 시간 ( 예 : 생태환경 )
- 비용 ( 예 : space shuttle )
- 부작용 ( 예 : hauthon effect )

### 컴퓨터 시뮬레이션만의 기능 :

- Back Track and Replay
- Freeze
- Malfunction
- Override

## 시뮬레이션의 대상

### 시스템의 정의 :

- 특정한 목적을 위해 서로 연관관계를 갖는 element들의 집합으로 주위 환경으로부터 구분되어지는 것
- A collection of interacting elements that function together for some purpose
- A collection of inputs whose pass through certain processing phases to produce outputs

## 시스템의 구성

요소 : Entity  
Attributes  
Activity

예 :

| System    | Entity                             | Attribute                    | Activities                               |
|-----------|------------------------------------|------------------------------|--|
| Aircraft  | Autopilot<br>Aerilons<br>Airframe  | Error<br>Angle<br>Heading    | Signaling<br>Forcing<br>Turning          |
| Factory   | Workpieces<br>Machines<br>Markets  | Number<br>Capacity<br>Size   | Ordering<br>Machining<br>Scheduling      |
| Business  | Products<br>Customers<br>Markets   | Price<br>Demand<br>Size      | Manufacturing<br>Selling<br>Advertising  |
| Political | Parties<br>Issues<br>Social groups | Size<br>Acceptance<br>Income | Fund raising<br>Campaigning<br>Migrating |

## 시뮬레이션의 적용분야

- Computer System ( network,circuit design )
- Flow process ( oil,gas,water etc. )
- Game, Educational Modeling
- Health care system
- Manufacturing system
- Marketing and Sales modeling
- Military
- Traffic control
- Aircraft and Airport operation
- Financial modeling
- Governmental and urban planning
- Etc
- .
- .
- .

## 시뮬레이션의 종류

- Monte Carlo System Simulation
- Discrete System Simulation
- Continuous System Simulation
- Combined Discrete-Continuous Simulation

## 시뮬레이션 과정

- 시스템 분석과 문제의 정의
- 모델 구축
- 타당성 검증
- 실험 계획
- 실험 수행
- 결과 분석
- 문서화



## 문제의 정의와 시스템 분석

- Formulate of problem

$$P_t = | D_t - A_t |$$

- System의 Boundary와 Environment의 구분

- Real System

||

|                       |       |                   |
|-----------------------|-------|-------------------|
| <i>Abstraction</i>    | ..... | <i>Entity</i>     |
| <i>Simplification</i> | ..... | <i>Activity</i>   |
| <i>Approximation</i>  | ..... | <i>Attributes</i> |



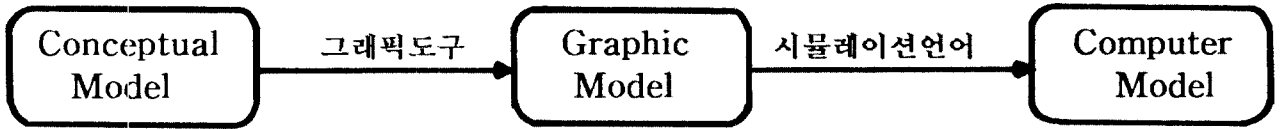
Logical flow-diagram

or

Static Model

# 구축 단계

실시스템에  
대한 이해



PERT  
CP/M  
Petri-Net  
.  
.  
.

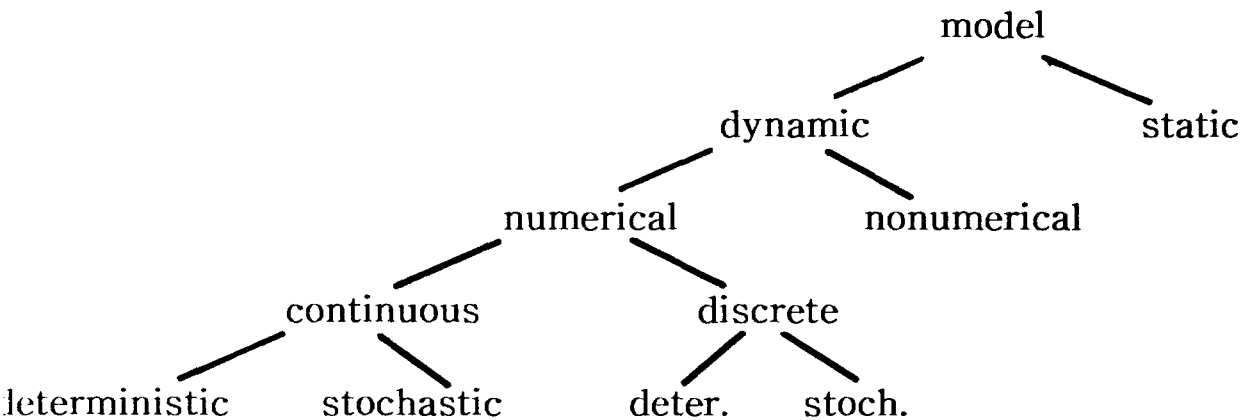
GPSS  
SIMSCRIPT  
SLAM  
SIMAN  
.  
.

# 모 델

정의 :

- A model is a representation of an object, system or idea in some form other than that of the entity itself
- A simplified representation of a system

구분 :



## 타당성 검증

Verification

Validation

- Concept
- Methodology
- Results
- Inference
- Data

Problem Analysis

## 실험계획

원칙 - 최소 수행 횟수

목적 - 분석, 관찰

방법 - parameter, 변수의 level들의  
다양한 조합 수행  
- 초기, 종료 조건  
- sample size  
- 결과의 차이 축소

## 실험 수행

Execution

Sensitivity Analysis

## 결과 분석과 문서화

# 시뮬레이션을 위한 기초 통계

개념 :

Continous Distribution

Discrete Distribution

PDF( Probability Density Function )

CDF( Cumulative Distribution Function )

Random Number

Random Variate

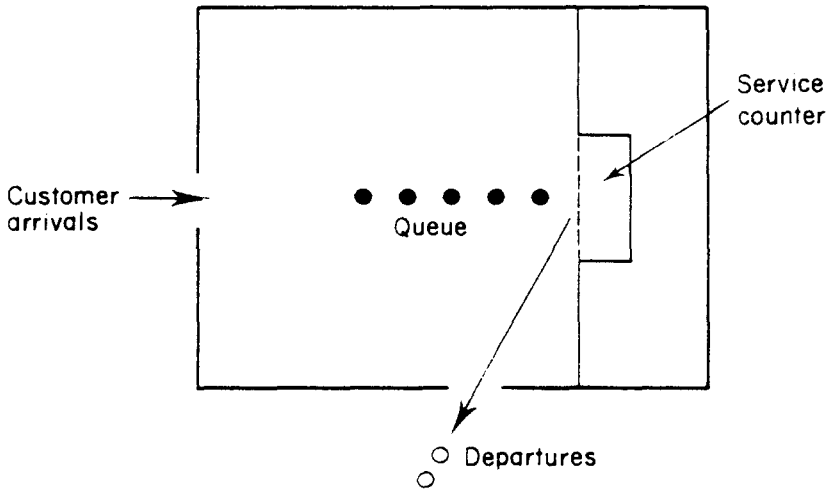
Correlation

Covariate

Hypothesis Test

Confidential Interval

# DISCRETE SYSTEM SIMULATION의 예



Post office system.

- single-server-single-queue system
- FIFO
- 목적(관찰 대상)
  - customer arrival rate
  - service rate
  - server utilization
  - average queue length
  - average number in the system
  - average waiting time
  - average time spent in the system

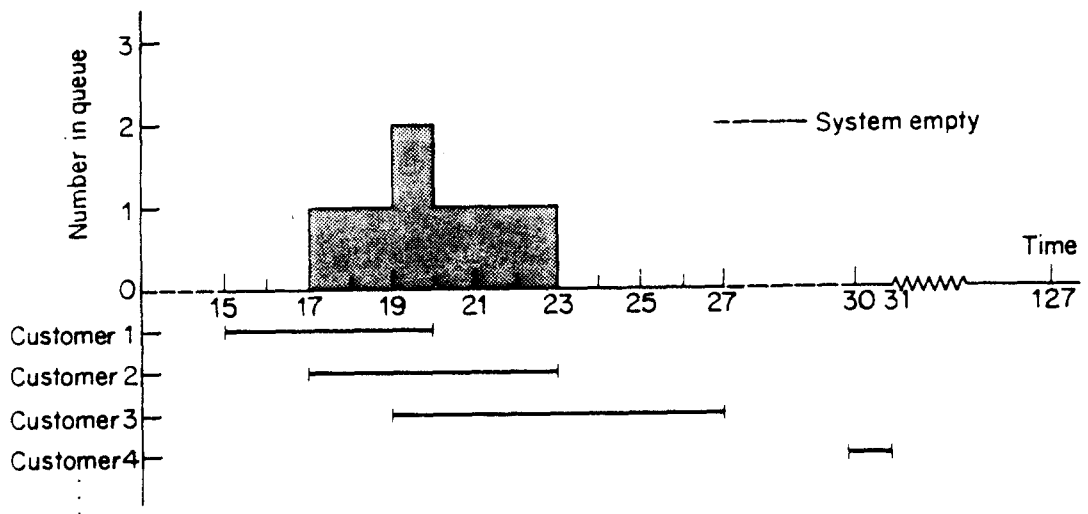
## Observed Data for 2 hours

| Customer<br>$i$ | Arrival<br>time | Inter-<br>arrival<br>time | Time<br>service<br>begins | Time<br>service<br>ends | Service<br>time,<br>$S_i$ | Waiting<br>time in<br>queue, $W_i$ | Time spent<br>in system,<br>$(W_i + S_i)$ |
|-----------------|-----------------|---------------------------|---------------------------|-------------------------|---------------------------|------------------------------------|---|
| 1               | 15              | 15                        | 15                        | 20                      | 5                         | 0                                  | 5   |
| 2               | 17              | 2                         | 20                        | 23                      | 3                         | 3                                  | 6   |
| 3               | 19              | 2                         | 23                        | 27                      | 4                         | 4                                  | 8   |
| 4               | 30              | 11                        | 30                        | 31                      | 1                         | 0                                  | 1   |
| ⋮               | ⋮               | ⋮                         | ⋮                         | ⋮                       | ⋮                         | ⋮                                  | ⋮   |
| ⋮               | ⋮               | ⋮                         | ⋮                         | ⋮                       | ⋮                         | ⋮                                  | ⋮   |
| ⋮               | ⋮               | ⋮                         | ⋮                         | ⋮                       | ⋮                         | ⋮                                  | ⋮   |
| ⋮               | ⋮               | ⋮                         | ⋮                         | ⋮                       | ⋮                         | ⋮                                  | ⋮   |
| 25              | 120             | 3                         | 122                       | 127                     | 5                         | 2                                  | 7   |
|                 |                 |                           |                           |                         | $\Sigma S_i =$<br>71      | $\Sigma W_i =$<br>100              | $\Sigma (W_i + S_i)$<br>= 171             |

Observed data (time in minutes) for post office system.

- Total number of arrivals :  $N = 25$
- Arrival time of the last customer :  $T' = 120$
- Departure time of the last customer:  $T = 127$





Observed number of customers in the queue.

| $i$ | No. of customers in queue, $m_i$ | No. of customers in system, $n_i$ | Cumulative time, $t_i$ |
|-----|----------------------------------|-----------------------------------|------------------------|
| 1   | 0                                | 0                                 | 56                     |
| 2   | 0                                | 1                                 | 21                     |
| 3   | 1                                | 2                                 | 15                     |
| 4   | 2                                | 3                                 | 20                     |
| 5   | 3                                | 4                                 | 15                     |
|     |                                  |                                   | $\Sigma t_i = T = 127$ |

Observed cumulative times for customers in queue and in system.

|   |                    | Approximate Method          | Theoretical Method           |
|---|--------------------|-----------------------------|------------------------------|
| Average arrival rate                    | $\lambda$          | $\frac{N}{T}$               | $\lambda$                    |
| Average service rate                    | $\mu$              | $\frac{N}{\sum S_i}$        | $\mu$                        |
| Utilization of server                   | $\lambda$<br>$\mu$ | $\frac{\lambda}{\mu}$       | $\frac{\lambda}{\mu} = \rho$ |
| Average waiting time<br>in queue        | $W_q$              | $\frac{\sum W_i}{N}$        | $\frac{\rho}{(1-\rho)\mu}$   |
| Average time spent<br>in system         | $W$                | $\frac{\sum(S_i + W_i)}{N}$ | $\frac{1}{\mu - \lambda}$    |
| Average number of<br>customers in queue | $L_q$              | $\frac{\sum(m_i t_i)}{T}$   | $\frac{\rho^2}{1-\rho}$      |
| Average number of<br>customer in system | $L$                | $\frac{\sum(n_i t_i)}{T}$   | $\frac{\rho}{1-\rho}$        |

|   | Approximate<br>method | Theoretical<br>method | Simulation<br>method |
|---|-----------------------|-----------------------|----------------------|
| Average no. of customers<br>in system, $L$  | 1.35                  | 1.43                  | 1.353                |
| Average no. of customers<br>in queue, $L_q$ | 0.79                  | 0.84                  | 0.793                |
| Average time spent in system, $W$           | 6.84                  | 6.94                  | 6.72                 |
| Average time spent in queue, $W_q$          | 4.0                   | 4.08                  | 3.94                 |
| Utilization of postmaster $\rho$            | 0.59                  | 0.59                  | 0.56                 |

Comparison of results (post office) by different methods.