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주 관 정

(Effect of Random Geometry Pertubation on Acoustic Scattering)

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1. Introduction

In recent years, the finite element method has become one of the most popular numerical technique for obtaining solutions of engineering science problems. However, there exist various uncertainties in modeling the problems, such as the dimensions(geometry shape), the material properties, boundary conditions, etc.

The consideration for the uncertainties inherent in the problems can be made by understanding the influences of uncertain parameters [1].

Determining the influences of uncertainties as statistical quantities using the standard finite element method requires enormous computing time, while the probabilistic finite element method is realized as an efficient scheme [2, 3] yielding statistical solution with just a few direct computations.

In this paper, a formulation of the probabilistic fluid-structure interaction problem accounting for the first order perturbation of geometric shape is derived, and especially probabilistical acoustic pressure scattering from the structure with surrounding fluid is focused on. In Section 2, governing equations for the fluid-structure problems are given. In Section 3, a finite element formulation, based on the functional, is presented. First order pertubation of geometric shape with randomness is incorporated into the finite element formulation in conjunction with discretzation of the random fields in Section 4 and 5.

Finally, the proposed formulation is applied to a acoustic pressure scattering problem from an infinitely long cylindrical shell structure with randomness of radial perturbation.

2. Governing Equations

Assuming the fluid is inviscid, incompressible, and has no body forces acting on it, the irrotational flow which obeys a barotropic state and undergoes small motions

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can be governed by the following momentum equation and the continuity equation:

$$p + \rho_F \ddot{\phi} = 0 \quad \text{in } \Omega_F \qquad (2.1)$$

and

$$\frac{1}{B} p + \nabla^2 \phi = 0 \quad \text{in } \Omega_F \qquad (2.2)$$

respectively where p is the pressure; B is the bulk modulus; ρ_F is the fluid density; ϕ is a flow potential such that

$$\{v\} = \nabla \dot{\phi} \quad \text{in } \Omega_F \qquad (2.3)$$

where {v} is the velocity of the fluid particle; Ω_F is the fluid region; ∇^2 denotes the Laplacian operator; the superposed dot denotes time differentiation; and ∇^2 denotes the Laplacian operator; the superpoed dot denotes time differentiation; and ∇ denotes the gradient operator.

In the structure region, the motion of the solid can be governed by

$$\text{div}\{\sigma\} + \{b\} = \rho\{\ddot{u}\} \quad \text{in } \Omega_s \qquad (2.4)$$

$$\{\sigma\} = [D]\{\epsilon\} \quad \text{in } \Omega_s \qquad (2.5)$$

$$\{\epsilon\} = \frac{1}{2}\left[\nabla\{u\} + (\nabla\{u\})^T\right] \quad \text{in } \Omega_s \qquad (2.6)$$

with prescribed boundary conditions

$$[\sigma]\{n\} = \{\bar{t}\} \quad \text{in } \Omega_t \qquad (2.7)$$

$$\{u\} = \{\bar{u}\} \quad \text{in } \Omega_u \qquad (2.6)$$

where $[\sigma]$ is the symmetric Cauchy stress; {b} is the body force; ρ is the density; {u} is the displacement; [D] is the tangent moduli;

$\{n\}$ is the outnormal unit vector; and the structure region Ω_s is decomposed into Ω_t and Ω_u.

At the fluid-structure interface, the following boundary condition can be obtained from the momentum and the continuity consideration:

$$\{n\}\nabla p + \rho_F\{n\}\{\ddot{u}\} = 0 \quad \text{on } \partial\Omega_I \quad (2.9)$$

where $\partial\Omega_I$ is the interface between the structure and the fluid regions.

3. Finite Element Formulation with Scattering Pressure

Euler equation and boundary conditions given as in the previous section can be derived from the corresponding functional; Equations (2.1) and (2.2) can be obtained by taking the first variation of the functional given by

$$J_F[p,\phi,\lambda] = \int_{t_0}^{t_1}\left[\int_{\Omega_F}\left(\frac{1}{2}\rho_F\nabla\dot{\phi}^T\nabla\dot{\phi} - \frac{1}{2B}p^2\right)d\Omega\right.$$

$$\left. + \int_{\Omega_F}\lambda_1(p+\rho_F\ddot{\phi})\,d\Omega\right]dt \quad (3.1)$$

with $t_0 \le t \le t_1$ and $\delta\phi(t_0) = \delta\phi(t_1) = 0$ where δ implies the first variation; the first and second terms in the first integral represent the kinetic and potential energies of the fluid, respectively ; and λ_1 is the Lagrange multipliers to be determined such that the equlibrium equation (2.1) is satisfied in the fluid.

The fulid-structure interaction can be included by adding following terms to the functional equation (3.1)

$$J_I[p,\phi,\{u\},\lambda_2] = \int_{t_0}^{t_1}\left[-\int_{\partial\Omega_I}\{u\}^T\{n\}p\,d\Gamma\right.$$

$$\left. + \int_{\partial\Omega_I}\lambda_2(p+\rho_F\ddot{\phi})\,d\Gamma\right]dt \quad (3.2)$$

where the first integral is the work done by the normal component of the structural displacement and the pressure at the interface. The equilibrium condition between p and ϕ is enforced with the Lagrange multiplier λ_2 .

For the Structural part, the functional is expressed as

$$J_s[\{u\}] = \int_{t_0}^{t_1}\left[\int_{\Omega_s}\left(-\frac{1}{2}\rho_F\{\dot{u}\}^T\{\dot{u}\}+\{\epsilon\}^T\{\sigma\}\right.\right.$$

$$\left.\left. -\{u\}^T\{b\}\right)d\Omega - \int_{\partial\Omega_t}\{u\}^T\{\bar{t}\}\,d\Gamma\right]dt \quad (3.3)$$

where the first integral contains the kinetic and potential energies and the potential of the body force; and the second term represents the potential of the external traction.

A three field mixed functional $J[p,\phi,\{u\},\lambda_1,\lambda_2]$ can be derived by combining equations (3.1), (3.2) and (3.3). The Lagrange multipliers λ_1 and λ_2 can be determined to satisfy equations (2.1) - (2.9). The resulting three field mixed functional is given by

$$J = \int_{t_0}^{t_1}\left[\int_{\Omega_F}\left(\frac{1}{2}\rho_F\nabla\dot{\phi}^T\nabla\dot{\phi} + \frac{1}{2B}p^2 + \frac{\rho_F}{B}p\ddot{\phi}\right)d\Omega\right.$$

$$- \int_{\partial\Omega_I}\rho_F\{u\}^T\{u\}\ddot{\phi}\,d\Gamma$$

$$+ \int_{\Omega_s}\left(-\frac{1}{2}\rho_F\{\dot{u}\}^T\{\dot{u}\} + \{\epsilon\}^T\{\sigma\} - \{u\}^T\{b\}\right)d\Omega$$

$$\left. - \int_{\partial\Omega_t}\{u\}^T\{\bar{t}\}\,d\Gamma\right]dt \quad (3.4)$$

For convenience, the pressure p is spilitted into the incident part and the scattered part

$$p = p^i + p^s \quad (3.5)$$

and the infinite fluid boundary consideration is incorporated by using the silent boundary condition [4].

With the Galerkin type of finite element approximation,

$$\{u\} = \Sigma\,[N]_I\{u\}_I \quad (3.6)$$

$$\{p\}^s = \Sigma\,[L]_I\{p\}_I^s \quad (3.7)$$

and with a new fluid unknown such that

$$\{p\}^s = -\{\dot{q}\} \quad (3.8)$$

the necessary and sufficient condition for the stationarity of the functional, J, and the integration by parts in time yield two set of equation:

$$\begin{bmatrix}[M] & [O]\\ [O] & [-Q]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}\}\\ \{\ddot{q}\}\end{Bmatrix}+\begin{bmatrix}[O] & [M]_{sF}\\ [M]_{sF}^{T} & [-C]_{F}\end{bmatrix}\begin{Bmatrix}\{\dot{u}\}\\ \{\dot{q}\}\end{Bmatrix}$$

$$+\begin{bmatrix}[K] & [O]\\ [O] & [-G]\end{bmatrix}\begin{Bmatrix}\{u\}\\ \{q\}\end{Bmatrix}=\begin{Bmatrix}\{f\}\\ \{f\}_{F}\end{Bmatrix} \quad (3.9)$$

or simply,

$$[M]^{*}\{\ddot{d}\}+[C]^{*}\{\dot{d}\}+[K]^{*}\{d\}=[F] \quad (3.10)$$

where

$$[M]=\int_{\Omega_s}\rho[N]^{T}[N]\,d\Omega \quad (3.11)$$

$$[Q]=\frac{1}{B}\int_{\Omega_F}[L]^{T}[L]\,d\Omega \quad (3.12)$$

$$[M]_{sF}=\int_{\partial\Omega_I}[N]^{T}[L]\{n\}\,d\Gamma \quad (3.13)$$

$$[C]=\frac{1}{\rho_F c}\int_{\partial\Omega_{FB}}[L]^{T}[L]\,d\Gamma \quad (3.14)$$

$$[K]=\int_{\Omega_s}[B]^{T}[D][B]\,d\Omega \quad (3.15)$$

$$[G]=\frac{1}{\rho_F}\int_{\partial\Omega_F}[A]^{T}[A]\,d\Omega \quad (3.16)$$

$$\{f\}=-\int_{\partial\Omega_I}[N]^{T}\{p\}^{i}\{n\}\,d\Gamma+\int_{\Omega_s}[N]^{T}\{b\}\,d\Omega+\int_{\partial\Omega_t}[N]^{T}\{\bar{t}\}\,d\Gamma \quad (3.17)$$

$$\{f\}_F=\int_{\partial\Omega_F}[L]^{T}\{\dot{u}\}^{i}\{n\}\,d\Gamma \quad (3.18)$$

In the above, matrices $[B]$ and $[A]$ are the gradient of the matrices $[N]$ and $[L]$, respectively.

4. First Order perturbation of Geometry

The perturbation methods [5] presume that the pertubation is a continuous function of a parameter γ which measures the strength of the perturbation.

The application of first order pertubation techniques involves the expansion of all random functions about the mean of random field $\bar{b}(x)$, where $b(x)$ is the random field and retaining up to first order terms. For example, the random functions $\{u\}$ and $\{p\}^s$ are expanded about $\bar{b}(x)$ as follows:

$$\{u\} = \{u\}^0 + \gamma\{u\}' \quad (4.1)$$

$$p^s = p^{s^0} + \gamma\, p^{s'} \quad (4.2)$$

where γ is a small parameter representing the magnitude of randomness in $b(x)$; the zeroth and first primes represent the zeroth and first order variations, respectly, due to variations in $b(x)$.

The geometry-dependant quantities such as the surface and volume Jacobian and the gradient of the functional variables are expanded in a similar manner to equations (4.1) and (4.2) [3].

5. Discretizing in Random Fields

The frequency transfer function approach is often used for the vibration analysis of linear systems. the efficiency of the approach depends on the frequency band over which the vibration analysis is required.

In this section a random finite element formulation is carried out for the transfer function matrix.

The system's equation (3.9) can be expressed, in frequency domain, as

$$[H(i\omega)][d(i\omega)] = [F(i\omega)] \quad (i=\sqrt{-1}) \quad (5.1)$$

where $[d(i\omega)]$ and $[F(i\omega)]$ are the Fourier transforms of $[d(t)]$ and $[F(t)]$, respectively; and $[H(i\omega)]$ is the transfer function matrix for the frequency of harmonic excitation, ω.

If random function $b(x)$ where x is the spatial coordinates is introduced during the process of finite element discretization, the transformed response vector, the forcing matrix can be denoted by $[d(b)]$, $[F(b)]$ and $[H(x, b)]$, respectively.

As in the finite element method, the random function $b(x)$, the expectation of $b(x)$, and the covariance of $b(x)$ are approximated by

$$b(x) = \sum_{i=1}^{q} N_i(x)\, b_i \quad (5.2)$$

$$E[b(x)] = \sum_{i=1}^{q} N_i(x) E[b_i] \quad (5.3)$$

and

$$\text{cov}(b(x_k), b(x_l)) = \sum_{i=1}^{q} N_i(x_k) N_j(x_l) \text{cov}(b_i, b_j) \quad (5.4)$$

where $N_i(x)$ represents the prescribed shape functions and b_i is the values of b at $\{x\}_i$, $i=1,2,\cdots,q$ where q is the number of random functions.

Appling the Taylor series expansion to equation (5.1) the expansion of $[H]$, $\{d\}$ and $[F]$ about the mean value of the random function, $\{b\}$, with a small perturbation ε yields the following equation:

$$[H(b)] = [\overline{H}] + \varepsilon \sum_{i=1}^{q} [\overline{H}]_{bi} \triangle b_i + O(\varepsilon^2) \quad (5.5)$$

$$\{d(b)\} = \{\overline{d}\} + \varepsilon \sum_{i=1}^{q} \{\overline{d}\}_{bi} \triangle b_i + O(\varepsilon^2) \quad (5.6)$$

$$[F(b)] = [\overline{F}] + \varepsilon \sum_{i=1}^{q} [\overline{F}]_{bi} \triangle b_i + O(\varepsilon^2) \quad (5.7)$$

where $[\overline{H}]$, $\{\overline{d}\}$ and $[\overline{F}]$ denote the mean value of $[H]$, $\{d\}$ and $[F]$ respectively; $[\overline{H}]_{bi}$, $\{\overline{d}\}_{bi}$ and $[\overline{F}]_{bi}$ are the first derivatives with respect to b_i evaluated at $\{\overline{b}\}$, respectively.

The $[\overline{H}]_{bi}$ can be calculated by

$$[\overline{H}]_{bi} = -\omega^2 \frac{\partial [M]^*}{\partial \{m\}^{*e}} \frac{\partial m^{*e}}{\partial b_i} + i\omega \frac{\partial C^*}{\partial c^{*e}} \frac{\partial c^{*e}}{\partial b_i} + \frac{\partial K^*}{\partial k^{*e}} \frac{\partial k^{*e}}{\partial b_i} \quad (5.8)$$

Substituting equations (5.5) - (5.7) into equation (5.1) and collecting terms of zeroth and first orders of ε yield the following equations:

$$[\overline{H}]\{\overline{d}\} = [\overline{F}] \quad (5.9)$$

$$[\overline{H}]\{\overline{d}\}_{bi} = [\overline{F}]_{Ii} \quad (i = 1,\cdots,q) \quad (5.10)$$

where

$$[\overline{F}]_{Ii} = [\overline{F}]_{bi} - [\overline{H}]_{bi}\{\overline{d}\} \quad (5.11)$$

Remarks :

Equation (5.9) - (5.11) can also be obtained by substiting the first order expansion of a random function-dependant quantities such as equations (4.1) and (4.2) into the first variation of functional eq. (3.4) and collecting terms of zeroth and second terms of γ.

6. Numerical Application

The probabilistic finite element formulation derived in Section 2 - 5 was applied to the 2-D problem of scattering from an infinitely long cylinderical shell structure with surrounding fluid, as depicted in Fig.1. Radius and thickness of the structure are 0.5m and 0.01m, respectively. The material properties are $E = 2.0 \times 10^{12}$ N/m^2, $\nu = 0.3$, and $\rho = 7500$ kg/m^3. For the fluid, wave speed is 1500 m/sec and density is 1030 kg/m^3. The loading is assumed to be a plane wave, $p = e^{i(kx+\bar{w}t)}$ with kd = 2.0.

The shell structure was modeled with the degenerated three node 2-D shell elements [6] and the fluid with 2-D quadrilateral nine node plane strain elements. The thickness in the z-direction for the model is chosen as 1m. By symmetry only half of the problem needs to be modeled with 30 shell elements and 210 fluid elements. Rayleigh damping is included in the shell structure.

The random radius of the shell structure is normally distributed with a coefficient of variance equal to 0.01. The random field is discretized such that the number of random variables (q) is 30.

The enfluence of radius, with a constant pertubation of $\pm$ 0.05 m, on the scattering acoustic pressure is illustrated in Fig. 2.

The $\pm 3\sigma$ (σ : standard deviation) bounds of the scattering pressure are plotted in Fig. 3 for the pertubation of $0.05\cos 3\theta$ in the circumferential direction, with 1% of standard deviation in the magnitude of the pertubation. The $\pm 3\sigma$ bounds for the pertubation of $0.025\cos 5\theta$ with 5% standard deviation are shown in Fig. 4. The enfluence of the correlation coefficient (γ) between the two pertubation patterns is illustrated in Fig. 5(a) and Fig. 5(b) for γ = 0.5 and γ = -0.5, respectively.

7. Concluding Remarks

The proposed probabilistic first order pertubation technique can be easily incorporated into a widely used finite element analysis software. The advantage of this method is that the effect of the uncertainty of shape can be observed such that the solutions can be expected within bounds, for example, with the confidence level of 3σ.

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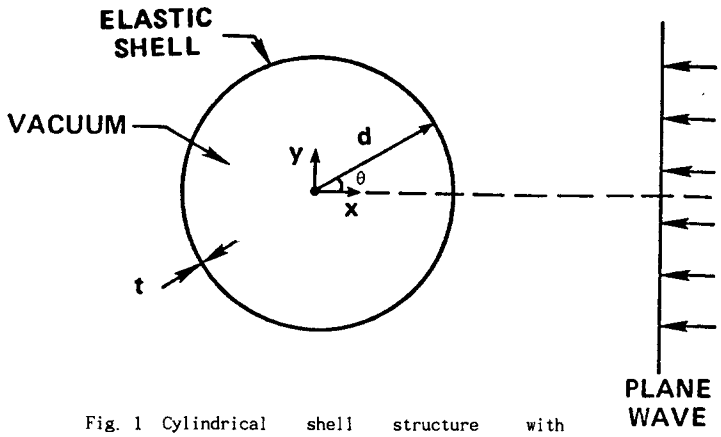


Fig. 1 Cylindrical shell structure with surrounding fuild, being subjected to a plane acoustic wave

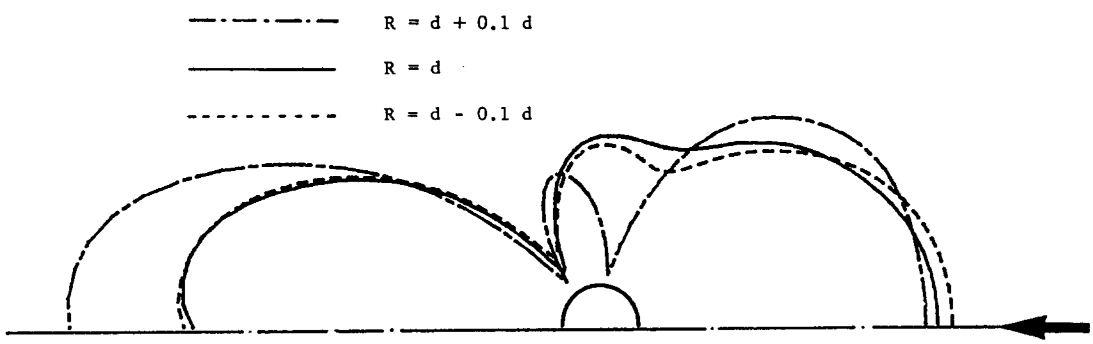
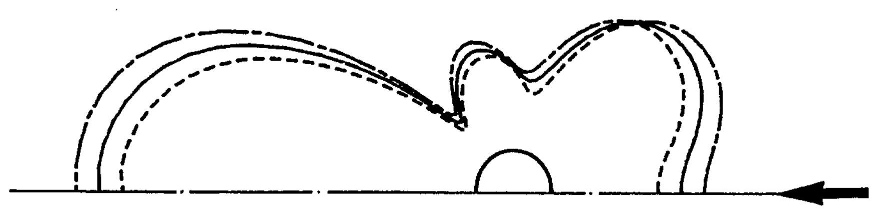


Fig. 2 Scattered pressures on the surface of a shell structure with radius of d ± 0.05 m and k_d = 2.0



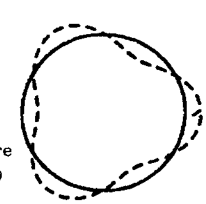
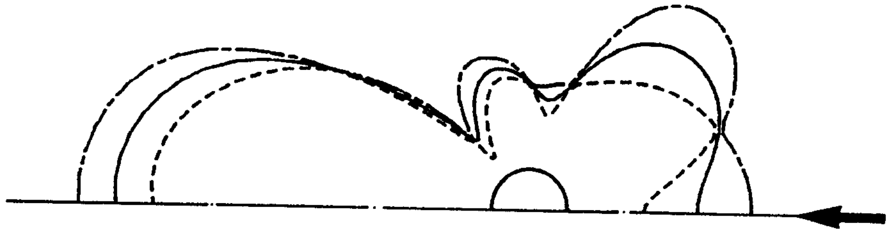


Fig. 3 ± 3σ bounds for scattered pressure with the radius pertubation of 0.05cosθ and σ = 0.01



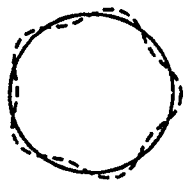
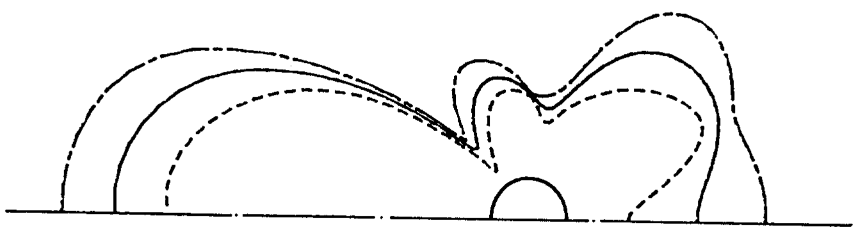
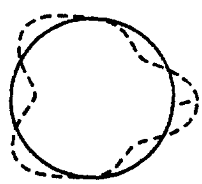
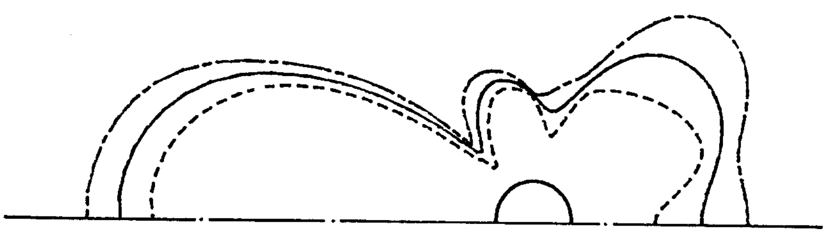


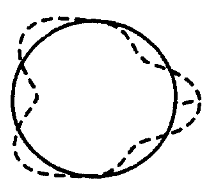
Fig. 4 $\pm 3\sigma$ bounds for scattered pressure with the radius pertubation of $0.025\cos\theta$ and $\sigma = 0.01$





(a) correlation coefficient = 0.5





(b) correlation coefficient = -0.5

Fig. 5 $\pm 3\sigma$ bounds for scattered pressure with the radius pertubation of $0.05\cos\theta + 0.025\cos 5\theta$ and $\sigma = 0.01$