SENSITIVITY OF SHEAR LOCALIZATION ON PRE-LOCALIZATION DEFORMATION MODE

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Abstract

As shear localization is observed in different deformation modes, an attempt is made to understand the conditions for shear localization in general deformation modes. Most emphasis in put upon the effects of pre-localization deformation mode on the onset of shear localization and all the other well-recognized effects of subtle constitutive features and imperfection sensitivity studied elsewhere are not investigated here. Rather, an approximate perturbation stability analysis is performed for simplified isotropic rigid-plastic solids subjected to general mode of homogeneous deformation. Shear localization is possible in any deformation mode if the material has strain softening. The incipient rate of shear localization and shear plane orientations are strongly dependent upon the pre-localization deformation mode. Significant strain softening is necessary for shear localization in homogeneous axisymmetric deformation modes while infinitesimal strain softening is necessary for shear localization in plane strain deformation mode. In any deformation mode, there are more than one shear plane orientation. Except for homogeneous axisymmetric deformation modes, there are two possible shear plane orientations with respect to the principal directions of stretching. Some well-known examples are discussed in the light of the current analysis.

1. Introduction

Shear localization is an important precursor to ductile fracture and is observed in a wide class of materials in different modes of deformation. It occurs in quasistatic deformation as well as in dynamic deformation, in metals, polymers, concretes and geological materials. Often times, shear localization is preceded by necking as in the case of plane strain tensile test specimens. Unlike the case of necking, shear localization does not involve geometrical softening (reduction of load carrying area) and hence the phenomenon has been attributed solely to materials' constitutive features such as strain hardening characteristics (kinematic hardening and yield surface corners), flow non-normality due to dilatancy and associated pressure sensitivity, rate sensitivity [1-8] and temperature dependence (in the case of adiabatic heating) [9-11]. Along this line, significant efforts have been made and now it is commonly understood that shear localization is extremely sensitive to these subtle constitutive features. Moreover, shear localization is also known to be very sensitive to the material's imperfection and inhomogeneity [8,12].

Most of the previous studies have been confined to one dimensional simple shear field or two dimensional plane strain deformation field. It is certain that these deformation modes are most favorable to shear localization when the conditions are met, but shear localization is observed also in the upsetting of circular cylindrical specimens [13,14] and in the necked region of biaxially stretched sheets [15]. Relatively little attentionhas been paid to the details of shear localization in general deformation field even though some of the studies have general three dimensional analytical frameworks (Rudnicki and Rice) . In this paper, the major premise is that pre-localization deformation mode has definite effect on the onset of shear localization. Along this line, an attempt will be made to the systematic understanding of this issue. For the simplicity of the analysis, the effects of various subtle constitutive features mentioned above will not be incorporated in the analysis. Rather, a simple approximate linear perturbation stability analysis will be performed for rate-independent rigid-plastic isotropic materials. Since the introduction of linear perturbation stability analysis for the prediction of thermo-mechanical shear instability in simple shear deformation of viscoplastic solids by Clifton [1980] and Bai [1982], linear perturbation stability analsis has received considerable attention as ananalytical tool for the prediction of the critical conditions for the onset of shear localization [16-18]. This analysis technique has been extended to cover shear localization in general three dimensional deformation field for isotropic viscoplastic solids recently by Anand, Kim and Shawki [1987]. In the following, their analysis methodis applied to rate-independent isotropic rigid-plastic solids subjected to quasi-staticdeformation.

2. Field Equations

It will be assumed that the effects of elasticity are negligible. For rate-independent isotropic solids, von Mises type J_2 flow rule will be used. For this class of material, stretching $\mathbf{D}$ is given as a function of applied stress such as

$$\mathbf{D} = \gamma\left[\mathbf{S} / 2\bar{\tau}\right] \tag{1}$$

Here $\mathbf{D}$ is stretching defined as

$$\mathbf{D} = \mathrm{sym}(\mathrm{grad}\ \mathbf{v}) \tag{2}$$

for the velocity field $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ and,

$$\mathbf{S} = \mathbf{T} + p\,\mathbf{1} \tag{3}$$

is the stress deviator of the Cauchy stress $\mathbf{T}$ with mean normal pressure

$$p = -\frac{1}{3}\mathrm{tr}\ \mathbf{T} \tag{4}$$

and,

$$\bar{\tau} = \sqrt{\frac{1}{2}\mathbf{S}\cdot\mathbf{S}} \tag{5}$$

is the equivalent shear stress and,

$$\dot{\gamma} = f\left(\frac{d\bar{\tau}}{dt}\right) = \frac{1}{h}\frac{d\bar{\tau}}{dt} \geq 0 \tag{6}$$

is the equivalent plastic strain rate with a hardening parameter h. From (5) equivalent plastic shear strain $\bar{\gamma}$ is defined as

$$\bar{\gamma}(t) = \int_0^t \dot{\gamma}\ dt \tag{7}$$

We assume that $\bar{\gamma}$ is a function of only $\bar{\tau}$ such as

$$\bar{\gamma} = \bar{\gamma}(\bar{\tau}) \tag{8a}$$

which can be obtained from the inversion of the stress-strain relation

$$\bar{\tau} = \bar{\tau}(\bar{\gamma}) \tag{8b}$$

From (6)-(8), we note that hardening h is given as

$$h = \frac{d\bar{\tau}}{d\bar{\gamma}} \tag{9}$$

The equation of motion for quasi-static deformation in the absence of body force is

$$\mathrm{div}\ \mathbf{T} = \mathbf{0} \tag{10}$$

3. Perturbation Stability Analysis

Let B_t and B_τ denote configurations of a body at times t and $\tau > t$, respectively. The relative motion of the body is characterized by a function

$$\mathbf{p}_t(\mathbf{x}, \tau) \tag{11}$$

which gives the place occupied at time τ by a material particle which at time t occupied the place $\mathbf{x}$. The vector valued functions

$$\left.\begin{aligned} \mathbf{u}_t(\mathbf{x},\tau) &= \mathbf{p}_t(\mathbf{x},\tau) - \mathbf{x} \\ \mathbf{u}_t(\mathbf{x},\tau) &= \frac{\partial \mathbf{u}_t(\mathbf{x},\tau)}{\partial \tau} \end{aligned}\right\} \tag{12}$$

describe the relative displacement and the relative velocity respectively. Equation (10) for the equilibrium at time τ may be expressed as

$$\operatorname{div} \overline{\mathbf{S}}_t(\mathbf{x},\tau) = 0 \tag{13}$$

where $\mathbf{S}_t$ is the relative first Piola Kirchhoff stress tensor which describe the actual forces in the configuration B_τ per unit area of the configuration B_t . It is denoted by the relation

$$\overline{\mathbf{S}}_t(\mathbf{x},\tau) = \left(\det \mathbf{F}_t(\mathbf{x},\tau)\right)\ \mathbf{T}\left(\mathbf{p}_t(\mathbf{x},\tau),\tau\right)\ \mathbf{F}_t^{-T}(\mathbf{x},\tau) \tag{14}$$

where $\mathbf{F}_t = \dfrac{\partial \mathbf{p}_t}{\partial \mathbf{x}} \det \mathbf{F}_t(\mathbf{x},\tau)\mathbf{F}_t^{-T}$ is the relative deformation gradient, $\left(\det \mathbf{F}_t\right)$ its determinant, $\mathbf{F}_t^{-T}$ the transpose of its inverse. For the perturbation stability analysis we consider that the body is homogeneous and homogeneously deformed in its current configuration B_t . If the body is subjected to boundary conditions which could give rise to continued homogeneous deformation, then the field equations together with the appropriate boundary conditions determine the homogeneous solution $\left[\mathbf{u}_t^\circ, \mathbf{S}_t^\circ\right]$. Next, we wish to determine that if this homogeneous solution is perturbed so that the configuration B_τ of the body, with $\Delta t = (\tau - t) \to 0$, differs only by infinitesimal displacements of a shear band mode relative to B_t, then can this perturbation grow while the field variables still satisfythe field equations? Let the normal to the perturbation shear band have an orientation $\mathbf{n}$ in B_t. The homogeneous solution $\left[\mathbf{u}_t^o, \mathbf{S}_t^o\right]$ is assumed to be perturbed by a small fluctuation which varies with $(\mathbf{x}-\mathbf{0})\cdot\mathbf{n}$, that is, with position across the band. Accordingly, we assume that the relative velocity field can be written as

$$\dot{\mathbf{u}}_t(\mathbf{x},\tau) = \dot{\mathbf{u}}_t^o(\mathbf{x},\tau) + \varepsilon\,\tilde{\mathbf{v}} \quad , \quad \varepsilon << 1 \tag{15a}$$

corresponding to which

$$\dot{\mathbf{F}}_t(\mathbf{x},\tau) = \dot{\mathbf{F}}_t^o(\mathbf{x},\tau) + \varepsilon \text{ grad } \tilde{\mathbf{v}} \tag{15b}$$

For the perturbation velocity field (15a) to be of a form which may lead to shear band formation, we require that

$$\text{grad } \mathbf{v} = \mathbf{a}\cdot\mathbf{n} \tag{16a}$$

where

$$\mathbf{a} = \mathbf{a}\left((\mathbf{x}-\mathbf{0})\cdot\mathbf{n}\,,\Delta t\right) \tag{16b}$$

is an amplitude vector which is parallel to the shear plane such that

$$\mathbf{a}\cdot\mathbf{n} = 0 \tag{16c}$$

As a result of velocity perturbation, the stress field will also be perturbed as

with

$$\left.\begin{aligned} \overline{\mathbf{S}}_t(\mathbf{x},\tau) &= \overline{\mathbf{S}}_t^o(\mathbf{x},\tau) + \varepsilon\,\tilde{\mathbf{T}} \\ \tilde{\mathbf{T}} &= \tilde{\mathbf{T}}\left((\mathbf{x}-\mathbf{0})\cdot\mathbf{n}\,,\Delta t\right) \end{aligned}\right\} \tag{17}$$

Substituting from (17) into (13), we obtain the following equilibrium equation for the perturbed stress field

$$\text{div } \tilde{\mathbf{T}} = \mathbf{0} \tag{18}$$

In order to analyze the stability of the homogeneous solution, we consider solutions of (18) of the form

$$\tilde{\mathbf{v}} = \phi\,\mathbf{v}_* \tag{19a}$$

$$\phi = \begin{cases} \exp\left\{i\,\xi(\mathbf{x}-\mathbf{0})\cdot\mathbf{n} + \eta\,\Delta t\right\} & \text{inside the band} \\ 0 & \text{outside the band} \end{cases} \tag{19b}$$

$$\mathbf{v}_* = \text{const.} \quad , \quad \mathbf{v}_* \cdot \mathbf{n} = 0 \tag{19c}$$

For this assumed form of the perturbation in the velocity

$$\text{grad}\ \tilde{\mathbf{v}} = \mathbf{a} \otimes \mathbf{n} \tag{19d}$$

with

$$\mathbf{a} = i\ \xi\ \tilde{\mathbf{v}} \ , \text{ and } \quad \mathbf{a} \cdot \mathbf{n} = 0 \tag{19e}$$

and this satisfies the requirement (16). Here, ξ is the wave number of the periodic perturbation in the direction normal to the shear band. If a solution in the form of (19) exists with η real and positive, then the perturbation may grow with time and a shear band type instability is possible. However, if η is real and negative, the perturbationis likely to decay with time and the homogeneous solution is considered stable. For this type of velocity perturbation, the perturbation in the stress takes the form

$$\tilde{\mathbf{T}} = \phi\ \mathbf{T}_* \quad , \quad \mathbf{T}_* = \text{const.} \tag{20}$$

where the constant coefficient tensor $\mathbf{T}_*$ can be obtained as follows. Since $\dot{\gamma} = \sqrt{2\mathbf{D} \cdot \mathbf{D}}$,the perturbation in the equivalent plastic shear strain rate $\tilde{\dot{\gamma}}$ can be written as

$$\left.\begin{aligned} &\tilde{\dot{\gamma}} = \phi\ \dot{\gamma}_* \\ \text{where} \quad &\dot{\gamma}_* = (i\ \xi)\ \mathbf{g} \cdot \mathbf{v}_* \\ \text{with} \quad &\mathbf{g} = \frac{2}{\dot{\gamma}^o} \mathbf{D}^o \cdot \mathbf{n} \end{aligned}\right\} \tag{21}$$

Also, since $\frac{1}{\eta}$ has dimensions of time, the perturbation in the equivalent plastic shear strain $\tilde{\gamma}$ can be estimated by

$$\tilde{\gamma} \cong \frac{\tilde{\dot{\gamma}}}{\eta} = \phi\left(\frac{\dot{\gamma}_*}{\eta}\right) \tag{22}$$

Further, from the constitutive equations, the perturbation in the equivalent shear stress $\tilde{\tau}$ is where

$$\left.\begin{aligned}\tilde{\tilde{\tau}} &= \phi\, \overline{\tau}_* \\ \overline{\tau}_* &= \left(\frac{h^\circ}{\eta}\right)\dot{\overline{\gamma}}_*\end{aligned}\right\} \tag{23}$$

where h° is strain hardening evaluated at the homogeneous solution at time t. It is important to note that these are time varying quantities. Next, from the constitutive equations (1) - (9) and equations (19) - (23) it follows that

with

$$\left.\begin{aligned}\mathbf{T}_* &= \left(\frac{h^\circ}{\eta} - \frac{\dot{\tau}^\circ}{\dot{\gamma}^\circ}\right)\overline{\gamma}_*\, \overline{\mathbf{D}}^\circ + (i\,\xi)\left(\frac{\tau^\circ}{\dot{\gamma}^\circ}\right)(\mathbf{v}_* \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{v}_*) \\ \overline{\mathbf{D}}^\circ &= \left(\frac{2\mathbf{D}^\circ}{\dot{\gamma}^\circ}\right)\end{aligned}\right\} \tag{24}$$

Upon substituting (20) in (18) and further substitution from (21) and (24) for $\dot{\overline{\gamma}}_*$ and $\mathbf{T}_*$ we obtain an equilibrium condition inside the shear band

with

$$\left.\begin{aligned}(\xi^2)\,\mathbf{A}\,\mathbf{v}_* &= \mathbf{0} \\ \mathbf{A} &= \left[\left(\frac{h^\circ}{\eta} - \frac{\dot{\tau}^\circ}{\dot{\gamma}^\circ}\right)\right]\mathbf{g} \otimes \mathbf{g} + \left(\frac{\tau^\circ}{\dot{\gamma}^\circ}\right)\mathbf{1}\end{aligned}\right\} \tag{25}$$

For non-trivial velocity perturbation $\mathbf{v}_*$, equation (25) implies that

$$\det \mathbf{A} = 0 \tag{26}$$

Further, the perturbation in the normal traction $\tilde{\mathbf{N}}$ on the shear plane can be obtained from (19), (20) and (24) as

$$\tilde{\mathbf{N}} = \mathbf{n}\cdot(\mathbf{T}\,\mathbf{n}) = (\mathrm{i}\,\xi)\,\phi\left(\frac{h^\circ}{\eta} - \frac{\dot{\tau}^\circ}{\dot{\gamma}^\circ}\right)(\mathbf{g}\cdot\mathbf{v}_*)(\mathbf{g}\cdot\mathbf{n}) \tag{27}$$

We note here that there can be no jump in the value of normal traction across the shear band, and thus

$$\left(\frac{h^\circ}{\eta} - \frac{\dot{\tau}^\circ}{\dot{\gamma}^\circ}\right)(\mathbf{g}\cdot\mathbf{v}_*)(\mathbf{g}\cdot\mathbf{n}) = 0 \tag{28}$$

From (25a) and (25b), we have

$$\mathbf{A}\,\mathbf{v}_* = \mathbf{0} = \left(\frac{h^\circ}{\eta} - \frac{\dot{\tau}^\circ}{\dot{\gamma}^\circ}\right)(\mathbf{g}\cdot\mathbf{v}_*)\,\mathbf{g} + \left(\frac{\dot{\tau}^\circ}{\dot{\gamma}^\circ}\right)\mathbf{v}_*$$

and it is readily seen that for non-trivial solution for $\mathbf{v}_*$, the term $\left(\frac{h^\circ}{\eta} - \frac{\dot{\tau}^\circ}{\dot{\gamma}^\circ}\right)(\mathbf{g}\cdot\mathbf{v}_*)$ cannot have zero value. Hence the normal traction continuity condition across the shear band requires that

$$\mathbf{g}\cdot\mathbf{n} = \frac{2}{\dot{\gamma}^\circ}\left(\mathbf{D}^\circ\,\mathbf{n}\right)\cdot\mathbf{n} = 0 \tag{29}$$

Let

$$\mathbf{D}^\circ = \sum_{i=1}^{3}\alpha_i\mathbf{e}_i\otimes\mathbf{e}_i \tag{30}$$

denote a spectral representation of the homogeneous stretching. Here, $\{\mathbf{e}_i\}$ with i=1, 2, 3 are the eigenvectors and $\{\alpha_i\}$ are the eigenvalues of $\mathbf{D}^\circ$ with $\alpha_1 \geq \alpha_2 \geq \alpha_3$. Then equation (26) can be rearranged as

$$C_1\,y^3 + C_1\,y^3 + 1 = 0 \tag{31a}$$

with

$$y = \frac{h^\circ\,\dot{\gamma}^\circ}{\dot{\tau}^\circ\,\eta} - 1 \tag{31b}$$

and

$$C_1 = 4\left(d_1 \; d_2 \; d_3\right)^2 \left(n_1 \; n_2 \; n_3\right)^2 \tag{31c}$$

$$C_0 = \left(d_1 \; n_1\right)^2 + \left(d_2 \; n_2\right)^2 + \left(d_3 \; n_3\right)^2 \tag{31d}$$

$$d_i = \frac{2\alpha_i}{\dot{\gamma}^o} \quad , \quad i = 1,2,3 \tag{31e}$$

$$n_i = \mathbf{n} \cdot \mathbf{e}_i \quad , \quad i = 1,2,3 \tag{31f}$$

As $\mathbf{n}$ is a unit vector,

$$\mathbf{n} \cdot \mathbf{n} = (n_1)^2 + (n_2)^2 + (n_3)^2 = 1 \tag{32}$$

Also equation (29) can be rearranged as

$$d_1 \, (n_1)^2 + d_2 \, (n_2)^2 + d_3 \, (n_3)^2 = 0 \tag{33}$$

Here, we define a Lode's variable ν

$$\nu = \frac{\alpha_1 - \alpha_2}{\alpha_1 - \alpha_3} = \frac{d_1 - d_2}{d_1 - d_3} \tag{34}$$

For imcompressible materials

$$\mathrm{tr}\, \mathbf{D}^o = \alpha_1 + \alpha_2 + \alpha_3 = \frac{2}{\dot{\gamma}^o}\left(d_1 + d_2 + d_3\right) = 0 \tag{35a}$$

and from (31e) and $\dot{\gamma}^o = \sqrt{2\left(\mathbf{D}^o \cdot \mathbf{D}^o\right)}$,

$$(d_1)^2 + (d_2)^2 + (d_3)^2 = 2 \tag{35b}$$

From (34) and (35), d_1, d_2 and d_3 can be expressed in terms of the Lode's variable such as

$$\left.\begin{aligned} d_1 &= \frac{1+\nu}{\sqrt[4]{3\left(1-\nu+\nu^2\right)}} \\ d_2 &= \frac{1-2\nu}{\sqrt[4]{3\left(1-\nu+\nu^2\right)}} \\ d_3 &= \frac{-(2-\nu)}{\sqrt[4]{3\left(1-\nu+\nu^2\right)}} \end{aligned}\right\} \tag{36}$$

From (32), (33) and (36), $(n_1)^2$ and $(n_3)^2$ can be expressed in terms of $(n_2)^2$ as

$$\left.\begin{aligned} (n_1)^2 &= \frac{2-\nu}{3} - (1-\nu)\,(n_2)^2 \\ (n_3)^2 &= \frac{1+\nu}{3} - \nu\,(n_2)^2 \end{aligned}\right\} \tag{37}$$

Equations (36) and (37) show that the coefficients C_1 and C_0 of cubic equation (31a) for y are functions of only ν and $(n_2)^2$. As C_1 and C_0 are non-negative, the cubic equation has only one real root y_{real}. For given values of ν and $(n_2)^2$ the cubic equation can be solved for $y_{real} = y_{real}\left(\nu\,,(n_2)^2\right)$. From this solution and (31b), we can have

$$\frac{\eta}{\dot{\gamma}^{\circ}} = \frac{h^{\circ} / \overline{\tau}^{\circ}}{1 + y_{real}} \tag{38}$$

The term $\frac{\eta}{\dot{\gamma}^{\circ}}$ represents the incipient rate of perturbation growth η normalized with respect to the homogeneous deformation rate $\dot{\gamma}^{\circ}$. Depending upon the signs of h° and $\left(1+y_{real}\right)$, $\frac{\eta}{\dot{\gamma}^{\circ}}$ can have either positive or negative values. Numerical solution of (31a) for y reveals that the sign of term $(1+y_{real})$ is negative for the entire ranges of values of ν and $(n_2)^2$, i.e.,

$$y_{real} < -1 \quad \text{for} \quad 0 \le \nu \le 1 \quad \text{and} \quad 0 \le (n_2)^2 \le 1 \tag{39}$$

and thus

$$\frac{\eta}{\dot{\bar{\gamma}}^{o}} \begin{cases} < 0 & \text{for} \quad h^{o} > 0 \\ > 0 & \text{for} \quad h^{o} < 0 \end{cases} \tag{40}$$

From (40), it is obvious that *strain softening* (with $h^{o} < 0$) is a necessary condition for the growth of shear band perturbation. Thus for the case of strain softening materials, we can rearrange (38) to have

$$-\frac{\eta / \dot{\bar{\gamma}}^{o}}{h^{o} / \bar{\tau}^{o}} = -\frac{1}{1+y_{real}} > 0 \quad \text{for} \quad h^{o} < 0 \tag{41}$$

Figure 1 shows $-\frac{\eta / \dot{\bar{\gamma}}^{o}}{h^{o} / \bar{\tau}^{o}}$ as a function of $(n_2)^2$ and ν obtained from the numerical solution of (31) and (41). The incipient rate of shear band perturbation growth η is strongly dependent upon the values of ν and $(n_2)^2$ as indicated in Figure 1. For a given value of ν it has its maximum value at a certain value of $(n_2)^2$. For each value of ν, the maximum value of η has been normalized as in (41) and plotted against the valueof ν as shown in Figure 2. It is obvious that as the deformation mode approaches plane strain condition (ν=0.5) $\left[-\frac{\eta / \dot{\bar{\gamma}}^{o}}{h^{o} / \bar{\tau}^{o}}\right]_{max}$, the maximum value of $-\frac{\eta / \dot{\bar{\gamma}}^{o}}{h^{o} / \bar{\tau}^{o}}$, becomes infinite. In this case, the rate of formation of shear bands can be infinite once the material has *infinitesimal strain softening* such that $\frac{h^{o}}{\bar{\tau}^{o}}$ has infinitesimal positive value. On the other hand, for deformation modes with $\nu = 0$ or 1 the value of $\left[-\frac{\eta / \dot{\bar{\gamma}}^{o}}{h^{o} / \bar{\tau}^{o}}\right]_{max}$

approaches a finite value in the vicinity of 2 and thus the formation of shear band requires *finite strain softening* such that $\frac{h^{o}}{\bar{\tau}^{o}}$ has finite positive value. All the other deformation modes (with $0 < \nu < 0.5$ or $0.5 < \nu < 1$) fall between these two extremes. Deformation modes with $\nu = 0$ or $\nu = 1$ are most resistant to shear band formation even though material's strain hardening characteristics plays primary role in all deformation modes. Here, it is to be noted that for the materials which does not show significant strain softening, shear localization is not likely to occur except in the plane strain deformation mode.

Regarding the incipient shear plane orientations, we assume that shear bands are most likely to form in the direction where the value of $-\frac{\eta / \dot{\bar{\gamma}}^{o}}{h^{o} / \bar{\tau}^{o}}$ becomes maximum. In this context, the values of $(n_1)^2$, $(n_2)^2$, and $(n_3)^2$ shown in Figure 3 represent the shear band orientations for the entire range of ν.

It can be seen from Figure 3 that except for the values of ν very close to 0 (balanced biaxial stretching or, uniaxial compression) or 1 (uniaxial tensile deformation) the value of η has its maximum at $n_2 = 0$. Thus for the deformation modes in the range $0.079 \le \nu \le 0.921$, there are *two* possible shear plane orientations which are parallel to the intermediate principal direction $\mathbf{e}_2$ of stretching tensor.

$$(n_2)^2 = 0\,,\ (n_1)^2 = \frac{2-\nu}{3}\,,\ (n_3)^2 = \frac{1+\nu}{3} \quad \text{for} \quad 0.079 \le \nu \le 0.921 \qquad (42a)$$

If we denote the angle to the shear plane measured from the major principal direction $\mathbf{e}_1$ in the $\mathbf{e}_1 - \mathbf{e}_3$ plane by θ, we have from (42a) (See Figure 4)

$$\tan\theta = \pm\left|\frac{n_1}{n_3}\right| = \pm\left|\frac{2-\nu}{1+\nu}\right|^{1/2} \quad \text{for} \quad 0.079 \le \nu \le 0.921 \qquad (42b)$$

Outside this range of ν, the value of $(n_2)^2$ at which the value of $-\frac{\eta / \dot{\bar{\gamma}}^{o}}{h^{o} / \bar{\tau}^{o}}$ becomes maximum are not non-zero and the corresponding values of $(n_1)^2$ and $(n_3)^2$ are obtained from the value of $(n_2)^2$ thru (37) (See Figure 3).

For the homogeneous axisymmetric deformation modes with $\nu = 0$ or $\nu = 1$, shear bands can be in any orientation as far as they maintain a certain angle from $\mathbf{e}_3$ axis or $\mathbf{e}_1$ axis respectively. in these cases, we have

$$(n_2)^2 = (n_1)^2 = (n_3)^2 = \frac{1}{3} \quad \text{for} \quad \nu = 0 \text{ or } 1 \tag{43a}$$

In the case of deformation with $\nu = 0$, the deformation field is symmetric about the $\mathbf{e}_3$ axis and the principal directions $\mathbf{e}_1$ and $\mathbf{e}_2$ are not unique. The angle θ_3 measured from the $\mathbf{e}_3$ axis to the shear planes has the values

$$\tan\theta_3 = \pm\left|\frac{n_3}{\sqrt{(n_1)^2 + (n_2)^2}}\right| = \pm\frac{1}{\sqrt{2}}, \quad \text{or} \quad \theta_3 = \pm 35.3^\circ \quad \text{for} \quad \nu = 0 \tag{43b}$$

Likewise, in the case of $\nu = 1$ the angle θ_1 measured from the $\mathbf{e}_1$ axis to the shear planes has the values

$$\tan\theta_1 = \pm\left|\frac{n_1}{\sqrt{(n_2)^2 + (n_3)^2}}\right| = \pm\frac{1}{\sqrt{2}}, \quad \text{or} \quad \theta_1 = \pm 35.3^\circ \quad \text{for} \quad \nu = 1 \tag{43c}$$

In the close vicinity of uniaxial deformation modes with $0 < \nu < 0.079$ or, $0.921 < \nu < 1$ the growth rate becomes maximum in four different directions depending upon the signs of n_1, n_2 and n_3 as indicated in Figure 3. However, as shown in Figure 1, the growth rate has minimal variation for the entire range of $(n_2)^2$ values when ν is very close to 0 or 1, and thus has minimal dependence on the shear band orientation. Hence, in these ranges of ν,shear bands can develop in different directions other than these four directions.

4. Discussion

In the light of current analysis, we can revisit some of the well-known examples of shear localization and present systematic understanding of these phenomena.

During the plane strain tension test of a metallic material, neck develops first in the gauge section of the specimen after a certain amount of deformation. As necking is a result of competition between material's strain hardening capability and progressive reduction of load carrying area, it does not require complete loss of strain hardening for its occurence. Hence, necking usually occurs first and is a precursor to the subsequent strain softening. As the deformation field is already in plane strain mode, shear bands can develop at infinite speed in the necked region once the material exhibits strain softening. The shear fracture can be understood as the result of shear localization. The shear fracture surface is parallel to the $\mathbf{e}_2$ axis and makes 45° with respect to the $\mathbf{e}_1$ axis (See Figure 5).

Another common example is necking and subsequent cup-cone type fracture in the axisymmetric tensile test specimens of metallic materials (See Figure 6). As in the case of plane strain tensile test specimens, necking develops first in the gauge section. For materials with cup-cone type fracture, it can be understood that strain softening large enough for shear localization *in uniaxial mode* ($\nu = 1$) is never achieved even at the core of the neck where the value of mean stress is highest. On the other hand, at the pheriphery of the neck, the mode of deformation deviates gradually from the uniaxial modeas the neck develops and the value of ν decreases. Hence in this region, incipient rate of shear band formation increases (See Figure 2) with the progress of necking and, lessand less amount of strain softening is required for shear localization. Moreover, mean stress is lower at the periphery of the neck compared to its core and thus normal type fracture is less likely to precede shear localization. Hence in this region, shear fracture due to shear localization takes place instead of normal fracture.

Another example is shear fracture during upsetting of some high strength materials [13,14]. During uniaxial compression of a circular cylinder with appropriate aspect ratio, material can deform beyond its strain hardening limit without geometric instability. If the material exhibits strain softening before the major development of barreling, the deformation field remains relatively homogeneous at the onset of shear localization and thus one or two shear bands are likely to form across the entire specimen (See Figure 7(a)). On the other hand, if the material can sustain relatively largedeformation before it exhibits strain softening, the deformation field becomes inhomogeneous due to the friction at the dies-workpiece interface and thus the condition for shear localization cannot be achieved simultaneously throughout the specimen. In this case, the deformation mode at the periphery of the specimen will deviate from thehomogeneous axisymmetric mode ($\nu = 0$) gradually with the progress of barreling (SeeFigure 7(b)) and the incipient growth rate of shear bands will be maximum in this region. With the aid of positive hoop stress developed with barreling, shear localization leads to shear fracture at the periphery of the specimen. Two conjugate shear plane orientationsare possible at each point on the periphery.

5. Conclusion

A linear peturbation stability analysis of shear localization in isotropic rate-independent rigid-plastic solids has been performed for general three dimensional homogeneous deformation modes. Shear localization is possible in any mode, and a necessary condition for shear localization is *strain softening*. Incipient growth rate for shear band is strongly dependent upon pre-localization deformation mode. For plnae strain deformation mode, shear localization requires *infinitesimal* strain softening and the incipient growth rate for shear localization can be infinite. In other deformation modes, *finite* strain softening is required and incipient growth rate becomes finite. Uniaxial deformation modes with $\nu = 0$ (uniaxial compression or balanced biaxial stretching) or $\nu = 1$ (uniaxial tensile deformation) are most resistant to shear localization with the lowest incipient growth rate. Shear band orientation is strongly dependent upon the pre-localization deformation mode and there are more than one shear band orientation in any deformation mode. For materials which do not exhibit significant strain softening, shear localization is not likely to occur except in the plane strain deformation mode. In the light of current analysis, qualitative but systematic understanding of some well-known examples of shear localization has been presented.

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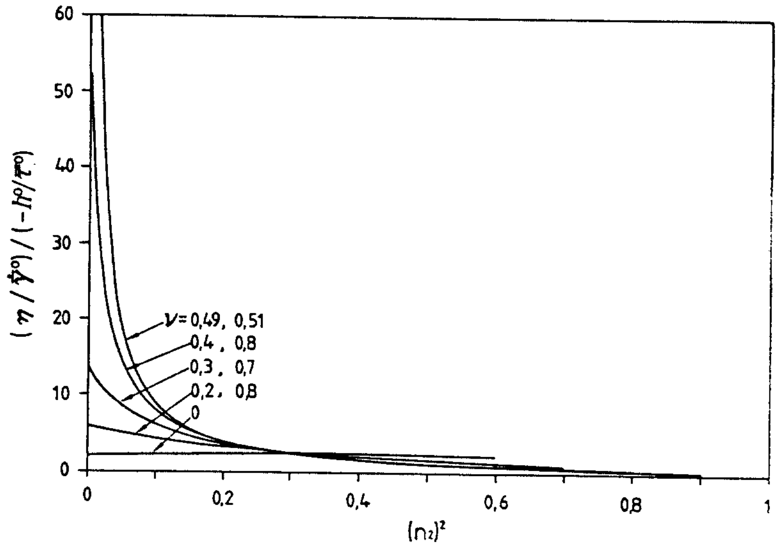

Figure 1. Normalized incipient rate of shear band formation $(\eta/\dot{\gamma}^o)/(-h^o/\tau^o)$ as a function of Lode's variable ν and the intermediate direction cosine n_z of unit normal vector $\underline{n}$ to the shear plane. Data is obtained from the numerical solution of equation (31) and (41).

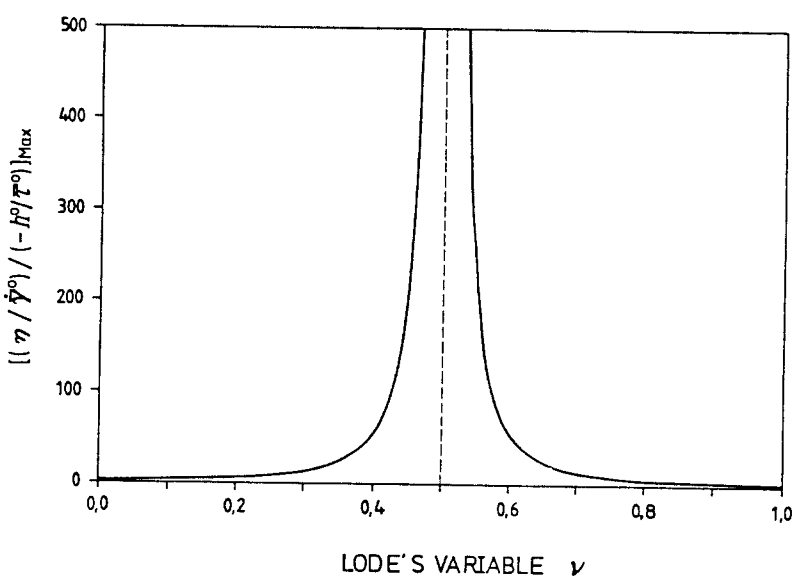

Figure 2. Maximum incipient rate of shear band formation $[(\eta/\dot{\gamma}^o)/(-h^o/\tau^o)]_{max}$ as a function of Lode's variable ν. For homogeneous axisymmetric deformation modes with ν = 0 or 1, the maximum rate converges to 2 while it converges to infinity for plane strain deformation mode with ν = 0.5.

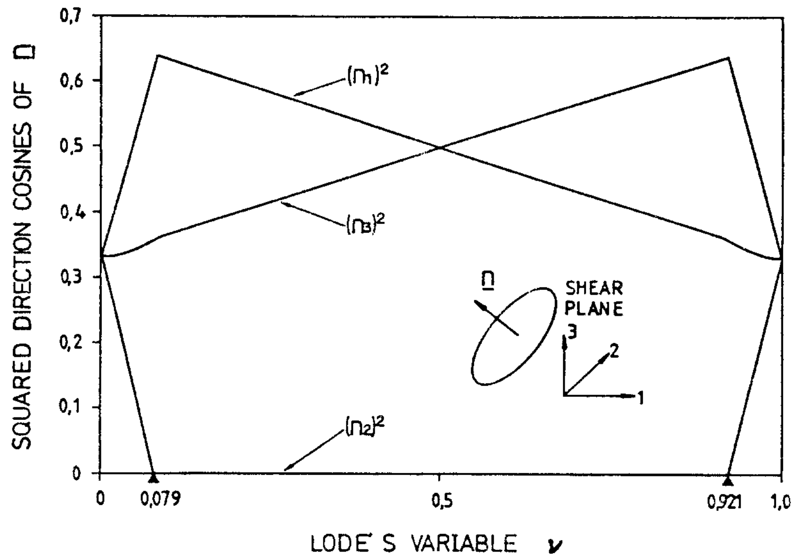

Figure 3. Squared direction cosines $(n_1)^2$, $(n_2)^2$ and $(n_3)^2$ with respect to the principal directions of stretching tensor at which the value of normalized incipient rate of shear band formation $(\eta/\dot{\gamma}^{o})/(-h^{o}/\dot{\tau}^{o})$ achieves its maximum $[(\eta/\dot{\gamma}^{o})/(-h^{o}/\dot{\tau}^{o})]_{max}$ as functions of Lode's variable ν. Here, n is a unit vector normal to the shear plane.

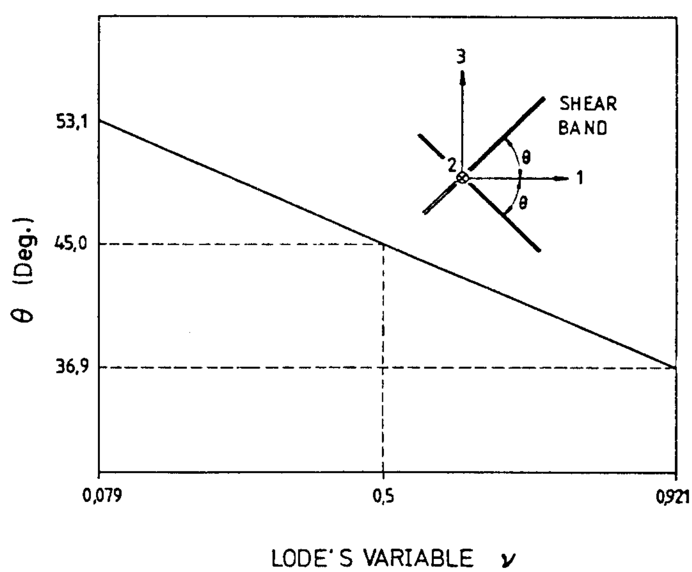

Figure 4. Possible shear band orientations with respect to the principal directions of stretching tensor D (eq. (30)) in the Lode's variable range $0.079 \leq \nu \leq 0.921$. Shear planes are parallel to the intermediate principal direction 2 and symmetrically oriented with respect to the major principal direction 1.

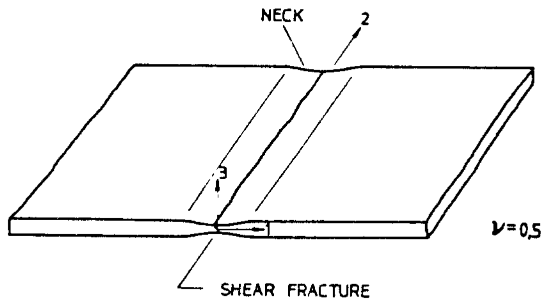

Figure 5. Necking and shear fracture in a plane strain tensile test specimen.

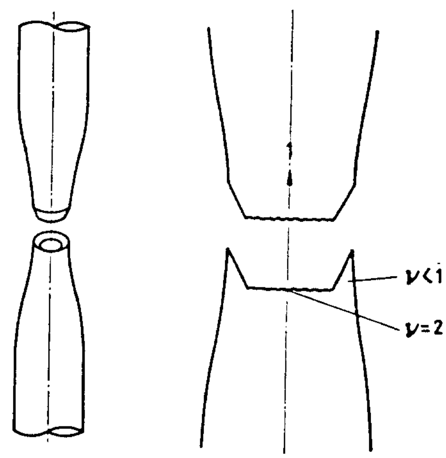

Figure 6. Necking and cup-cone type fracture in an axisymmetric tensile test specimen.

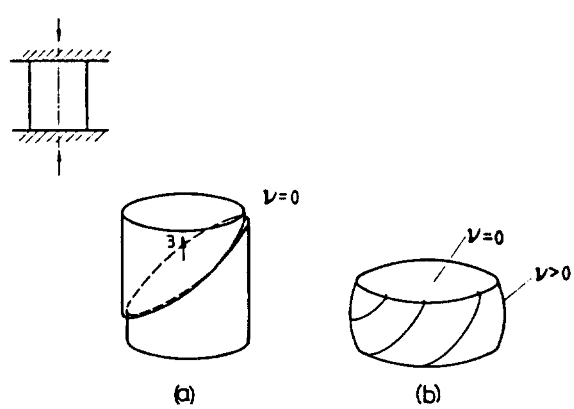

Figure 7. Shear fracture during the upsetting of circular cylinders of high strength materials. (a) Materials with lower strain hardening index. (b) Materials with higher strain hardening index.