Lot Size Scheduling Problem with Two Level Setup Cost/Time Structure

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본 연구에서는 가동준비 비용과 시간의 특수 구조를 갖는 다품종 제품군이 있을때의 생산일정 계획을 고려하며, 제품군 가동준비에 있어서의 작업순서간 상호의존성에 따라 일정계획 문제를 분류하고, 그 문제들을 정형화시킨다. 특히, 하나의 제품군만이 생산되어지는 작업환경에 대하여 발견적 기법을 개발하여 이를 적용한 생산계획을 마련한다. 또한, 그 결과를 정수선형 계획에 의한 최적해와 비교·분석한다.

We address scheduling problem on a single processor when there are multiple product families with a special cost/time structure. The setup structure consists of major setups between families and minor setups between products within a family. A major setup is required to change production from one family to another, but only a minor setup is required for products changes within a family. We call this problem "Family Lot Scheduling Problem"(FLSP) and consider the scheduling policy especially when product demand varies over time in a deterministic manner. The problem is to plan and schedule production over a finite time horizon that is subdivided into periods (days, weeks, or months), given that a single processor can process at most one product at any time.

FLSP in the presence of time-varying demand may be considered as a variant of the well-known Capacitated Lot Size Problem (CLSP). The CLSP has been studied for both single and multiple products with a single resource constraint in each period. Only the single family problem has been studied. The two-level setup cost and setup time characteristics of FLSP have not been treated. Different models result from assumptions about the sequence dependent nature of family setups. For example: (1) a major setup may or may not be required when a family that was in production at the end of one period is produced at the beginning of the next; (2) a major setup may or may not be required when idle time exists after a family's run and another run of the same family is begun at the end of the idle time; and (3) changeovers between periods may or may not reduce processor capacity. We give models for some of these variations.

1. Background

CLSP is the basic single machine production planning and scheduling problem in the time-varying demand case. This problem is to determine lot sizes and production timing under limited machine capacity. Even though there exist many extensions for various types of cost functions and capacity limits, most methods are computationally limited to solving small size problems. A basic version of the CLSP can be expressed as follows :

$$
\begin{aligned}
&\text{Min} \quad \Sigma_i\Sigma_t\ [a_{it}y_{it} + \Sigma_i\Sigma_t\ h_{it}I_{it}] \\
&\text{subject to} \\
&\quad I_{it-1} + X_{it} - I_{it} = d_{it} && \text{for all } i,t \\
&\quad \Sigma_i\ (s_i y_{it} + p_i X_{it}\ ) \le C_t && \text{for all } i,t \\
&\quad X_{it} - M\ y_{it} \le 0 && \text{for all } i,t \\
&\quad y_{it} = 0 \text{ or } 1 && \text{for all } i,t \\
&\quad X_{it},\ I_{it} \ge 0 && \text{for all } i,t \\
&\quad \text{where } M \text{ is a big number.}
\end{aligned}
$$

Let X_{it} and I_{it} represent production level and ending inventory of product i in period t. Binary variable y_{it} indicates whether or not a setup must be made in period t. The parameters a_{it} and h_{it} are the setup and holding costs for product i in period t. In period t, the demand for product i is d_{it} units and C_t is the machine capacity in time units. The required setup time and unit production time are given by s_i and p_i, respectively.

The CLSP is well-solved only in the single product, constant capacity with zero setup time case (Florian and Klein, 1971). Even the two product CLSP with zero setup time is proven NP-hard problem (Dixon, 1979). In addition, the CLSP for multiple products without setup times is also known NP-hard (Florian et al., 1980). CLSP becomes more complex if setup times are considered. With negligible setup times, CLSP has a feasible solution if and only if cumulative capacity is greater than or equal to cumulative demands at each period. However, finding a feasible solution to CLSP with setup times has recently been proven NP-complete (Salomon et al., 1989). Recognizing that CLSP is extremely difficult to solve in a direct way, both mathematical programming relaxations and heuristic methods have been proposed.

2. Classification and Formulation

Now we consider the Family Lot Size Scheduling Problem with a single capacity constraint. To analyze and formulate the problem, the following assumptions are generally made.

1. In any period, a major setup will be required in the event of a family changeover. Once the major setup is made in a period, any products within that family can be processed with only an additional minor setup as long as that family remains in continuous production.
2. A minor setup is always required before production of each product and it is assumed sequence independent.
3. No shortages are permitted in the production schedule.

(A) No restriction on setup operations

Problem A

$$\text{Min } \Sigma_t \Sigma_i \; [a_{io} Y_{it} + \Sigma_{j \in i}(a_{ij} y_{ijt} + h_{ij} I_{ijt})]$$

subject to

$$I_{ijt-1} + X_{ijt} - I_{ijt} = d_{ijt} \qquad \text{for all } i,j,t$$

$$\Sigma_{j \in i} X_{ijt} \leq M \; Y_{it} \qquad \text{for all } i,t$$

$$X_{ijt} \leq M \; y_{ijt} \qquad \text{for all } i,j,t$$

$$\Sigma_i \; (s_{io} Y_{it} + \Sigma_{j \in i}(s_{ij} y_{ijt} + p_{ij} X_{ijt})] \leq C_t \qquad \text{for all } t$$

$$X_{ijt}, \; I_{ijt} \geq 0 \quad \text{and} \quad Y_{it}, \; y_{ijt} = \{0,1\}$$

In Problem A, any number of products and families can be produced in a period. This would be appropriate if the time periods were reasonably long. However, a major setup is required each period in which one of its products are produced, regardless of previous period. Problem A becomes the CLSP formulation if it has a single family and one-level setup structure.

(B) Restriction on setup operations

Assume that due to manufacturing conditions, a major setup is not required again if the family scheduled at the start of a period is also scheduled at the end of the preceding period. Also an extra major setup is not required if production resumes without changing the family after an idle time period.

Problem B-1

Assume that at most one product can be produced in each period. This is most suitable for a planning horizon segmented into daily or half-daily periods.

Problem B-2

Assume that at most one family can be produced in a period. In this case, time period of several days or a week would be reasonable.

Problem B-3

Here, we further relax scheduling conditions such that at most K families can be produced in a time period. For family dependent setup, a major setup is always required unless a family is produced in consecutive production runs, possibly separated by idle time. We allow K divisions of a period for such a family schedule property.

3. Scheduling Procedure for Problem B-2

In this section, we develop a planning and scheduling heuristic for the B-2 case in the FLSP classification above. This is the situation where at most one family can be scheduled in a period, and where an additional major setup is not required if the same family is produced in two consecutive production periods. The principal reason for choosing this particular problem is that it represents a common scheduling practice. Another reason is that a scheduling procedure for this case may be modified for other FLSP assumptions. Our main purpose here is to develop a heuristic method applicable to a particular scheduling situation and to compare a performance of this heuristic to the optimal solutions generated by a mixed-integer model in a series of test problems.

3.1 Basic concept

The general approach of the algorithm is to sequentially construct a family production schedule by assigning families to periods within the planning horizon. Since a major setup operation is dependent upon the preceding family production sequence, it is reasonable to follow a forward pass procedure. At the current period t, we extend the (t-1)-period production schedule by assigning a family to be produced in period t, or else we leave period t idle. To decide what to do, we first identify products that would have unsatisfied demand in period t unless they are produced. If shortages are expected to occur for any product, its family is a candidate for scheduling in the current period. We assume that initial inventory is always available to satisfy demand requirements for products not produced in the first period.

The production lot sizes for products within a selected family are then calculated by the least average cost per period method. This method determines a production lot by choosing the lot cycle time to minimize average setup and

ventory costs per period over the cycle. The lot size is also limited by currently available machine capacity.
If more than one family candidate exist, we need to identify additional production resources to satisfy the current demand requirements.

There sometimes are cases where machine capacity is not sufficient to produce the shifted lots in any earlier production periods at all. This identifies a scheduling infeasibility. There are several ways to handle this situation, that is, to escape from infeasibilty, and the method selected will depend upon a choice of the scheduler and/or the environment of the manufacturing process. For example, we can treat such shortages as lost sales, or subcontract to purchase from outside sources. Increasing machine capacity using overtime or increasing the initial inventories are other available ways to escape from an infeasible schedule. Allowing shortages (backorders or lost sales) in some time periods can also be utilized. Here, we decide to take lost sales in case of unavoidable shortages.

With lost sales allowed, the mathematical programming model for Problem B-2 becomes the following, where decision variable L_{ijt} is the number of units of lost sales, and parameter π_{ij} is the unit cost of a lost sale.

$$\text{Min} \quad \Sigma_t \Sigma_i \quad [a_{io} Y_{it} + \Sigma_{j\in i}(a_{ij} y_{ijt} + h_{ij} I_{ijt} + \pi_{ij} L_{ijt})]$$

subject to

$$I_{ijt-1} + X_{ijt} + L_{ijt} - I_{ijt} = d_{ijt} \qquad \text{for all } i,j,t$$
$$\Sigma_i Z_{it} = 1 \qquad \text{for all } t$$
$$\Sigma_i Y_{it} \le 1 \qquad \text{for all } t$$
$$Z_{it} - Z_{it-1} \le Y_{it} \qquad \text{for all } i,t$$
$$\Sigma_{j\in i} X_{ijt} \le M Z_{it} \qquad \text{for all } i,t$$
$$X_{ijt} \le M y_{ijt} \qquad \text{for all } i,j,t$$
$$\Sigma_i (s_{io} Y_{it} + \Sigma_{j\in i}(s_{ij} y_{ijt} + p_{ij} X_{ijt})] \le C_t \qquad \text{for all } t$$
$$X_{ijt},\ I_{ijt},\ L_{ijt} \ge 0 \quad \text{and} \quad Y_{it},\ y_{ijt} = \{0,1\}$$

M is a big number

3.2 Scheduling algorithm

This section contains a formal statement of the scheduling procedure described above.

Step 0. Find net demand requirements after allocation of initial inventory.
Set $t \leftarrow 1$.

Step 1. Compute $I_{ijt} = I_{ijt-1} - d_{ijt}$ for each family i

Step 2. If $I_{ijt} \ge 0$ for all $j \in i$, then set $t \leftarrow t+1$ and go to next period.
Else, shortages are expected to occur.

(Case A) Only one family with shortages

Lot Sizing :

a) Lot size is first determined by current demand requirement.
Products are sorted and reindexed in the order of descending unit inventory holding cost and the order of ascending of processing time in case of ties.

Set $RC_t \leftarrow C_t$. (where RC_t = Remaining machine capacity at time t)

For $j=1,2, \ldots, g_i$ do begin

$X_{ijt} \leftarrow \text{Min} \{ d_{ijt}, [RC_t - (s_{io}Y_{it}+s_{ij}y_{ijt})]/p_{ij} \}$

Update $RC_t \leftarrow RC_t - \{ s_{io}Y_{it}+s_{ij}y_{ijt}+p_{ij}X_{ijt} \}$

where $Y_{it}=1$ if $X_{ijt}>0$ & $Y_{it-1}=0$, 0 otherwise.
$y_{ijt}=1$ if $X_{ijt}>0$, 0 otherwise.

If ($d_{ijt} > X_{ijt}$ or $RC_t \leq 0$), then do begin

i) Shift lot of ($d_{ijt}-X_{ijt}$) to its earlier production periods t'<t and treat unsatisfied requirement as lost sale
if no more shifting is available.
ii) Goto Step 3.

end.
end.

b) To see whether or not to combine future requirements into current production, apply the Silver-Meal method.

The average cost per period is given as

$$AC(k,t) = \frac{A + h \sum_{w=1}^{k-1} w \, d_{t+w}}{k}$$

where k = cycle length d_t= demand
A = setup cost, h = inventory cost

The optimum cycle length $k^*(t)$ is the value minimizing the cost, AC(k,t). The size of the production lot at the current period is the sum of demand requirements up to the next period of production.

For products in family i,

While { $(RC_t - s_{ij}) > 0$ }

for $t' = t+1, \ldots, k(t^*)$ do begin

$Q_{ijt'} \leftarrow \text{Min} \{ d_{ijt'}, (RC_t - s_{ij})/p_{ij} \}$
Update $RC_t \leftarrow RC_t - p_{ij}Q_{ijt'}$.
Set $X_{ijt} \leftarrow X_{ijt} + Q_{ijt'}$

end.

(Case B) more than one family with shortages

For such families,

1. Compute $\Delta_{it} = A_{it} - B_{it}$ at t.

where
A_{it} = cost of satisfying current requirements from earlier production schedules $t'<t$ by shifting the lots

= incurred inventory cost + cost of lost sale, if any

$= \Sigma_{t'} \Sigma_j \, h_{ij} Q_{ijt} (t-t') + \pi_{ij} L_{it}$

Q_{ijt}=quantity which can be shifted to earlier schedules at t'
π_{ij}= unit cost due to lost sales
L_{ijt}=quantity of lost sales

B_{it} =cost of scheduling the production at t
$= a_{io} Y_{it} + \Sigma_j \, a_{ij} y_{ijt}$

2. Choose a family i with $\text{Argmax}_i\{\Delta_{it}\}$, plan to produce lots for such family i according to the aforementioned lot sizing rule, and shift lots to earlier periods for other families having expected shortages.

Step 3. Update demand requirements by eliminating requirements which are satisfied by planned production.

Step 4. Stop the procedure if it reaches at end of planning horizon.
Otherwise, set $t \leftarrow t+1$ and go back to Step 1.

■

4. Computational Experience

Performance of the proposed scheduling algorithm was examined for a series of test problems having different problem characteristics. It was then compared to the optimal solutions obtained using a mixed-integer model solved by a mathematical programming package, ZOOM version 4.0 (Zero/One Optimization Method) on a Sequent S81 computer under DYNIX V3.0, a UNIX-based operating system. Each problem is composed of three families, having two products each and a planning horizon of six periods. The total number of variables is 180, including 72 binary variables, and there are 132 constraints.

The literature shows very little test data for CLSP with setup times case. Since we perform a test of FLSP with setup times, we decide to generate necessary sets of data using a uniform distribution. In order to construct diversified test problems, we classify problem characteristics in the following ways.

A. Ratio of major to minor setup costs
   - High (> 5)
   - Low (≤ 5)

B. Ratio of minor setup to inventory costs
   - High (> 10)
   - Low (≤ 10)

C. Average level of machine utilization
   - High (≥ 80%)
   - Low (< 80%)

In accordance with three characteristics with two levels each, an experiment was designed as 2x2x2 combinations with 4 replications, for a total of 32 problems. Major and minor setup costs were generated uniformly from the integer values, 50, 60, 70, 80, 90, and 100, and 5, 10, 15, and 20, respectively. Holding costs were selected uniformly from the values 0.2, 0.5, 0.8 and 1.0, while run times were chosen from 0.1, 0.2, 0.3, 0.4, and 0.5. Demand requirements were generated period-by-period with integer values from 10 to 50. For demand variability and scheduling flexibility, some demands are replaced by zero in 5 out of 12 for the first two periods, and 6 out of 24 for the remaining four periods. Finally, machine capacity was set to 50 hours in each period and setup times equal to 3 and 0.5 hours for major and minor, respectively.

Based upon the experimental design, test problems were generated and solved using both the heuristic procedure and an integer programming code, ZOOM. The quality of the solutions in the heuristic scheduling method is represented by **gap**(%), which is the relative deviation in percent from the optimal solution obtained by ZOOM.

It was found that the heuristic method averaged 2.8% from optimality and obtained the optimal solution on 39% (11/28) of solved problems. Four out of 32 test problems resulted in "overflow" because they exceeded maximum nodes in the branching tree while running ZOOM. In the test problems, 11 lost sale cases were treated and they were mainly for the high level of machine capacity utilization. In case of unavoidable shortages in the production scheduling, we treat them as lost sales and penalize those quantities multiplying by unit cost of lost sale in the objective function. Since such multiplier, π_{ij}, unit cost of lost sale, is set to a very high value(100 times of the unit holding cost, h_{ij}), even small quantities of lost sales would be a big proportion of the total cost value, and this might affect the solution gap to measure performance of the proposed scheduling algorithm. In order to obtain pure effects in the solution quality, we excluded the lost sale cases and analyzed the solution performance. With exclusion of lost sales, the heuristic method averaged 2.0% from optimality and obtained the optimal solution on 41% (7/17) of the test problems.

5. Summary

We treated the FLSP when product demand varies over time in a deterministic manner. Depending upon assumptions about the sequence dependent nature of family setup, FLSP was classified and formulated as mixed-integer mathematical programming mdels. As seen in the literature review, even CLSP which is a simple variant of the FLSP was known to be difficult to solve optimally using current mathematical programming methods. The case that at most one family can be scheduled in a period was then chosen as representative of common scheduling practice in order to develop a planning and scheduling procedure. The general approach of the heuristic algorithm is to sequentially construct a family production schedule by assigning families to periods within the planning horizon using a forward pass procedure. The performance of the proposed algorithm was evaluated using a set of test problems. Performance was measured by the solution gap, defined as the percent difference between the algorithm's solution and the optimality.

Since the heuristic averaged 3.8% from the optimality in twenty-eight test problems and large amounts of computer time were required to obtain optimal solutions by a mathematical programming code, the proposed scheduling procedure could be justified in terms of quality and computation times. Our finding indicated that the solution gap was large when machine capacity was tightly constrained, and the effects of major vs. minor setup costs and minor setup vs. inventory holding costs were not significant.

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