초기고장률과 와이불분포의 적합성 검토

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기술의 발전으로 전자기기(또는 부품)의 수명은 상당히 길어졌다. 따라서 장기간에 걸친 기기의 신뢰성을 예측하기 위해 초기고장률로 부터 일정시간 지난후 부터를 상수고장률로 추정하여 사용하고 있다. 그러므로 초기고장률은 기기의 수명예측에 중요한 역할을 하고 있다.
본고에서는 초기고장률 모형으로서 와이불분포의 적합성을 검토하는 새로운 통계적 방법을 소개한다.

I. Infant mortality

Historically, the bathtub curve has been used as a failure rate model with great popularity owing to its physical implication. The curve is composed of three phases with different characteristics, i.e., infant mortality, steady-state, and wearout phase as shown in Figure 1-a. Infant mortality phase is characterized by an initially high but rapidly decreasing failure rate. This is because of the existence of weak parts in the system, which will cause failure within a short period of operation. In the steady-state phase, initial weak failed parts have been replaced already, and hence further failures occur at a much lower rate. The failure rate remains constant or at the most changes very slowly. After a long period of operation, wearout phase starts and failure rate increases monotonically.

However for most of the electrical systems, the life time is so long that the wearout phase is hardly observed in the field. The constant failure rate makes the model for the steady-state phase simple, though its consistency is being severely criticized recently[3]. To understand the behavior during the steady-state, we heavily depend on various accelerating life test methods as an important source of information. Often this has been the only useful tool as the system becomes more and more reliable. However, it is still a difficult job to have a fairly good estimate of

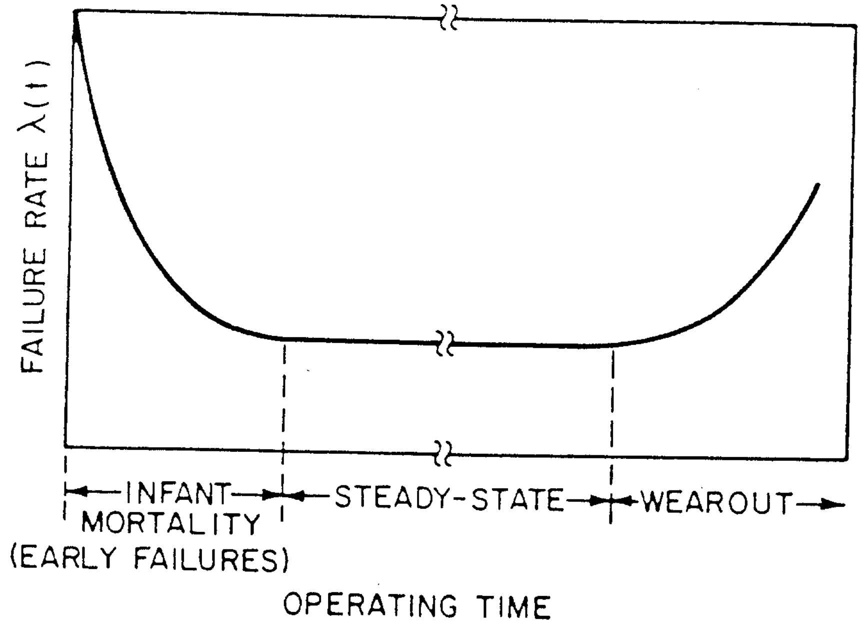

Fig. 1-a. Reliability bathtub curve

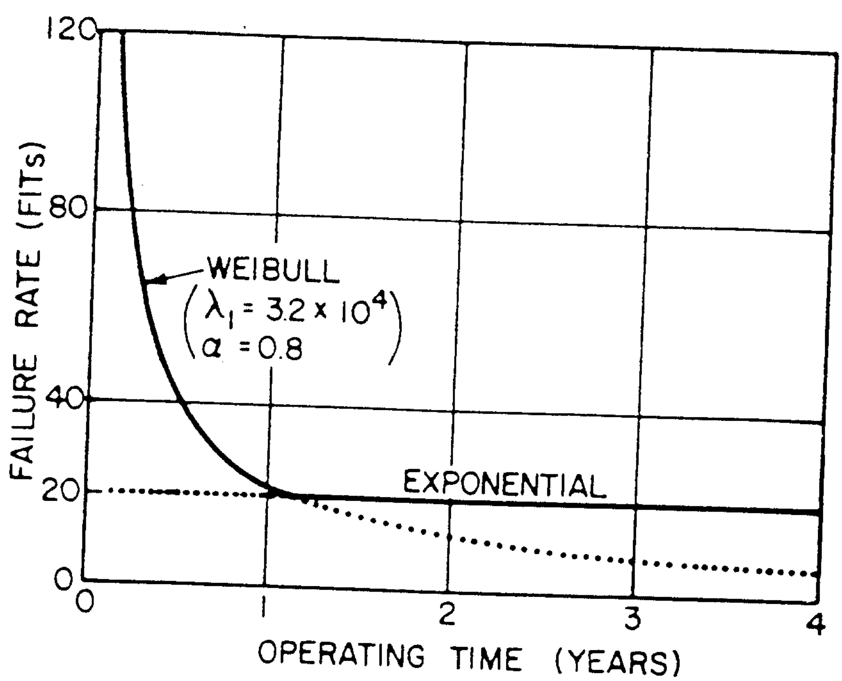

Fig. 1-b. Revised bathtub curve

the constant failure rate considering the long steady-state period of operation. In the auther's opinion, it is rather more pratical and intuitively correct if we estimate the constant failure rate from the infant mortality model.

As we note, the failure rate is decreasing monotonically early in the operating period. Hence, if we observe any time t such that the decreasing failure rate at t is smaller than a small predetermined value, we prefer estimating the constant failure rate for the stady-state phase with the failure rate at t. It may be also possible to decide the time t from the field experience. Bellcore[1] suggests 10,000 hours or 1 year of operation. Following this strategy, two points are worth of mentioning. First, we can have a conservative failure rate during the steady-state phase. Secondly, the constant failure rate will be more effectively estimated because it is being based on actual failure data. It is pretty much possible to keep a tract of the real failures during the relatively short period of early operation. It is our belief that if the above mentioned strategy is supplemented by the information obtained from the accelerating life test, we will have a better prediction for the long steady-state phase.

The idea of using the infant mortality for predicting the failure rate during the steady-state phase implies that the identification of the correct model for the infant mortality is very critical and important. As an infant mortality model, Weibull, log-normal, gamma, or any other model with decreasing failure rate can be a typical approach. But the Weibull model is the most widly used one. The Weibull failure rate can be expressed as

$$r(t) = \beta_1 t^{\alpha - 1} \quad (1)$$

As noted in Figure 2, r(t) is decreasing, constant, and increasing according as $0<\alpha<1$, $\alpha=1$, and $1<\alpha$ respectively. In the next section, we suggest a simple statistic to identify the weibullness vis-a-vis a complete set of lifetime data.

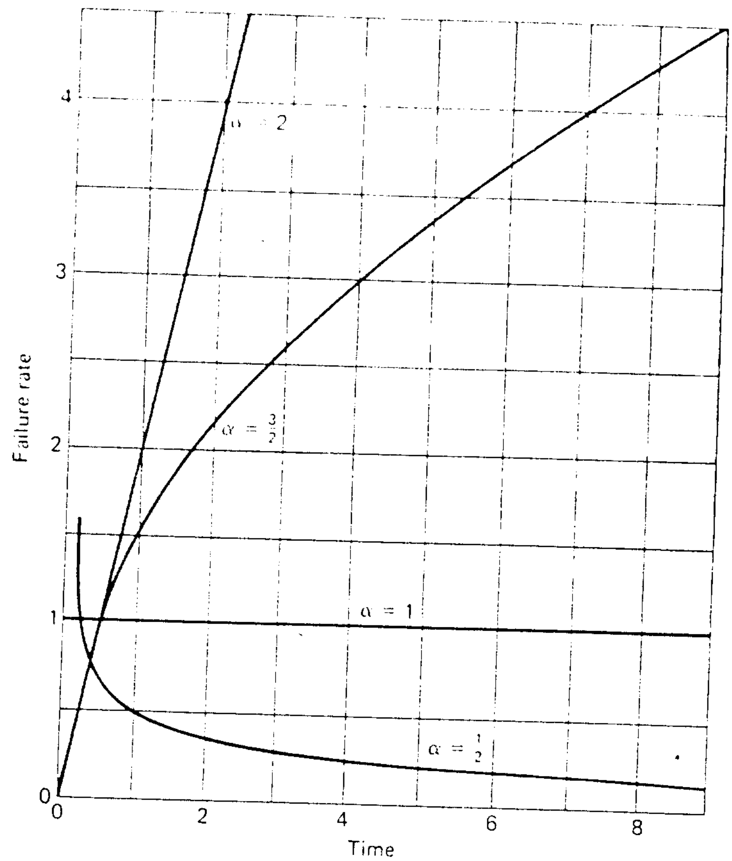

Fig. 2. Failure rate curves of the Weibull distribution

II. Test of fit for the Weibull model

Gnedenko, et al.[2] states two reasons for the popularity of the Weibull family as a life time model. First, it generalizes the popular exponential distribution family. Considering the positive power of life time data, we have more room for better fit between data and model than we have with the exponential family, which is characterized with a constant failure rate. Secondly, the Weibull family has theoretical legitimacy as a limiting distribution of the lifetime of a system. Even though the convergence rate is found to be slow, if the system is composed of a large number of independently working components and if it fails when one of the components fails, the Weibull model is the only one we can conceive. Even the life of a system with a relatively small number of visible components is conceived to be limited by a large number of molecular-size elements.

The probability density and the distribution function for the Weibull family we are considering are

$$f(x: \alpha, \beta) = \beta\alpha(\beta x)^{\alpha-1}\exp[-(\beta x)^{\alpha}] \tag{2}$$

$$F(x: \alpha, \beta) = 1 - \exp[-(\beta x)^{\alpha}], \tag{3}$$

where $x>0$, $\alpha>0$ and $\beta>0$ which are referred to as shape and scale parameters repectively. Note that $\alpha=1$ leads $f(x)$ to exponential. When we have lifetime data, often more convenience is achieved statistically and numerically if we consider logarithmic transformatin of the original data. Suppose X is a random variable arising from the lifetime data with the probability density function $f(x: \alpha, \beta)$ of (2). Then its logarithmic transformation $Y = \ln X$, leads to a new density and a distribution function

$$g(y:u,b) = \frac{1}{b}\exp[\frac{y-u}{b} - \exp(\frac{y-u}{b})] \tag{4}$$

$$G(y:u,b) = 1 - \exp[-\exp(\frac{y-u}{b})] \tag{5}$$

where $u = \ln \beta$ and $b = 1/\alpha$ are the location and scale parameters with $-\infty < u < \infty$, $b > 0$ respectively. This distribution is called extreme value distribution. The extreme value distribution has mean and variance

$$E(Y) = u - \gamma b \tag{6}$$

$$Var(Y) = \pi^2 b^2 / 6, \tag{7}$$

where γ is the Euler constant with 0.57721··· . Further it has a constant skewness

$$\kappa = \frac{\mu_3}{\sigma^3} = 1.3, \tag{8}$$

where μ_3 and σ^3 are the third central moment and the standard deviation repectively. From the skewness, κ, we obtain the sample skewness statistic, $\hat{\kappa}$, simply by substituting μ_3 and σ with the corresponding unbiased sample statistic

$$\hat{\mu}_3 = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} (x_i - \bar{x})^3 \tag{10}$$

$$\hat{\sigma} = [\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2]^{1/2} \tag{11}$$

Then, to test the weibullness of the data, the statistic we recommend is

$$TAU = (\frac{\hat{\mu}_3}{\hat{\sigma}^3} - 1.3)^2 \tag{13}$$

It is clear that TAU will be positive and closely located to 0 under the null hypothesis. Therefore, to test the weibullness of a set of random lifetimes, first we take logarithm for each lifetime and calculate the TAU statistic. Secondly, if TAU is far from 0, we reject the weibullness. In order to decide how far the TAU could be allowed from 0 for its acceptancy, we tabulate the critical values through the Monte Carlo simulation. Table 1 shows the critical values for the significance level of 0.01, 0.025, 0.05, and 0.1 for the sample sizes of 10, 12, 15, $\cdots$, 120.

Table 1. Percentage points for TAU

sample size	probability			
n	0.90	0.95	0.975	0.99
10	2.511	3.360	4.130	5.250
12	2.256	2.950	3.631	4.777
15	1.951	2.537	3.123	3.898
20	1.632	2.142	2.683	3.266
24	1.467	1.910	2.342	2.984
30	1.272	1.676	2.046	2.575
40	1.086	1.380	1.773	2.382
60	0.811	1.038	1.306	1.869
120	0.540	0.713	0.921	1.364

In order to compare the effectiveness of the statistic TAU with the existing ones, small power study was made against 6 alternative distributions. They are

1. N(0,1): exp(X), X has normal distribution with p.d.f

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp[\frac{-1}{2}(\frac{x-\mu}{\sigma})^2],\ \mu=0,\ \sigma=1$$

2. LOGISTIC: exp(X), X has logistic p.d.f

$$f(x) = e^{-x}(1+e^{-x})^{-2}$$

4. CAUCHY: exp(X), X has Cauchy p.d.f

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$$

5. $\chi^2(1)$: chi square with 1 degree of freedom

$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1}\exp(-x/2), \quad \nu = 1$$

6. $\chi^2(4)$: chi square with 4 degrees fo freedom

$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1}\exp(-x/2), \quad \nu = 4$$

5000 random sample of size 25 are generated from each alternative distribution and values of TAU are calculated. Comparing to the respective critical value with the significance level 0.05, the portion of rejections is counted. Since the critical value for sample size of 25 is not readily available from the Table 1, interpolation is made with the two neighboring sample sizes of 24 and 30. The simulation result is shown in Table 2. The first 5 statistics are from Tiku and Singh[4] and J statistic from Won[5]. Even TAU behaves with bias for the alternative chi square distribution with degree of freedom 1, the relatively high power for other alternatives shows its usefulness as an identifier for the weibullness of the lifetime data. However, the real power of the statistic TAU lies in its simplicity for calculation.

Table 2. Power of TAU for n=25

alternatives	statistics						
	A^2	T	S	Zw*	Zw	J	TAU
N(0,1)	.286	.119	.384	.392	.404	.412	.437
LOGISTIC	.376	.207	.427	.466	.470	.497	.464
DEXP	.568	.346	.485	.548	.524	.518	.497
CAUCHY	.919	1.000	.555	.761	.769	.874	.756
$\chi^2(1)$	.094	.091	.018	.100	.099	.070	.030
$\chi^2(4)$	.058	.030	.102	.079	.074	.103	.097

where A^2 : Modified Anderson-Darling statistic
T : Smith and Bain's statistic
S : Mann, Scheuer, and Fertig's statistic
Zw*, Zw : Tiku and Singh's statistic
J : Won's Statistic

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