그래프에 의한 Stable Law 분포의 모수 추정

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본고에서는 그래프를 이용하여 **Stable law** 분포의 **index, skewness, scale, location** 모수들에 대한 추정방법을 제시한다. 먼저, **order statistics**의 함수인 **t ail length statistic** $\tilde{K}_t$, **skewness statistic** $\tilde{K}_s$ 를 이용하여 **index** α, **scale** β 를 추정한다. 다음에는, 추정된 α, β 를 **index** 로 하여 개발된 그래프들로 부터 **scale** σ, **location** μ 추정에 필요한 **order statistics**의 함수를 결정한다. 몇가지 예를 마지막에 예시한다.

I. INTRODUCTION

The stable laws are most conveniently defined in terms of their characteristic function ([3], [4])

$$\phi(u) = \exp\{i\mu u - |\sigma u|^{\alpha}(1+i\beta\frac{u}{|u|}\,\chi(u,\alpha))\} \tag{1.1}$$

where

$$\chi(u,\alpha) = \begin{cases} \tan\dfrac{\pi\alpha}{2} & \alpha \neq 1 \\ \dfrac{2}{\pi}\log|u| & \alpha = 1 \end{cases}$$

u is a real number, $i^2 = -1$, $0 < \alpha \leq 2$, $|\beta| \leq 1$, $\sigma > 0$, $|\mu| < \infty$.

The estimation procedure by [2] is based on a few order statistics but is, however, concerned only with the estimation of the parameters of the symmetric stable laws. (For the computationally intensive procedures, refer to [6].)

II. THE ESTIMATION PROCEDURE FOR $(\alpha, \beta, \sigma, \mu)$

The tables of [5] provide the x_p of the distribution function $S(x: \alpha, \beta, 1, 0)$ of (1.1) for $\alpha = .1(.1)1.9$, $\beta = -1(.1)0$, and values p of 0.0001 (0.0001).001(.001).01(.01).99(.001).999 (.0001).9999.
Define the tail length and skewness functions

$$K_t = K_t(\alpha, \beta) = \log \frac{x_e - x_a}{x_d - x_b} \tag{2.1}$$

$$K_s = K_s(\alpha, \beta) = \log \frac{x_e - x_c}{x_c - x_a} \tag{2.2}$$

For the estimates of α, β, σ and μ use either of Figures 1, 3, 4, 5, 6, 7 or 2, 3, 4, 5, 6, 7.

III. ESTIMATION FOR POSITIVE β

For positive β, use the relationship[7]

$$S(x: \alpha, \beta, \sigma, \mu) = 1 - S(-x: \alpha, -\beta, \sigma, -\mu) \tag{3.1}$$

If the sample value of $\tilde{K}_s < 0$, transform the original sample x_1, x_2, ..., x_n by $y_i = -x_i$, and use the nomograms as described in section 2. The estimators for the x-sample are then $\tilde{\alpha}_x = \tilde{\alpha}_y$, $\beta_x = -\beta_y$, $\tilde{\sigma}_x = \tilde{\sigma}_y$ and $\tilde{\mu}_x = -\tilde{\mu}_y$.

IV. APPLICATION AND EXAMPLES

For simulating the random deviates from $S(x: \alpha, \beta, \sigma, \mu)$ of (1.1), we use the Chambers et. al.'s[1] algoritm.

Example 1. Here we simulate variates from a stable distribution with parameter vector $(\alpha, \beta, \sigma, \mu) = (1.5, 0.5, 1.0, 0)$. From the ordered sample of x-values we obtain the values $\tilde{K}_t = 2.08$ and $\tilde{K}_s = -0.626$ respectively. Since $\tilde{K}_s$ has a negative value, by taking the negative of the variates as in section 3, we obtain $\tilde{K}_s = 0.626$. Then from Figure 2 we obtain the estimates $\tilde{\alpha} = 1.61$ and $\beta = -0.52$. From Figure 3 we obtain the percentile value p = 0.57 which results in an estimate $\tilde{\mu} = 0.672$. From Figure 4 and Figure 5 we obtain percentile values p = 0.29 and q =

0.79 which results in $x_{(np)} = -0.416$ and $x_{(nq)} = 1.50$. We find $\tilde{\sigma} = 0.96$. By the results of section 3 we have finally $\tilde{\alpha}_x = 1.61$, $\beta_x = +0.52$, $\tilde{\sigma}_x = 0.96$, $\tilde{\mu}_x = -0.672$.

Example 2. Here we simulate variates from a stable distribution with parameter vector $(\alpha, \beta, \sigma, \mu) = (0.5, -1, 1.0, 0)$. We find $\tilde{K}_t = 7.65$ and $\tilde{K}_s = 8.79$, from Figure 2 we find the estimates $\tilde{\alpha} = 0.47$ and $\beta = -1$. From Figure 3 we get $p = 0$. Thus the estimate of μ for this sample is the minimal order statistic, i.e. $\tilde{\mu} = 0.086$. From Figure 5 we $q = 0.337$ and thus compute $x_{(nq)} = 1.087$. We obtain $\tilde{\sigma} = 1.087 - 0.086 = 1.001$.

References

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[2] Fama E. F. and R. Roll, Parameter estimation for symmetric stable distributions. *Journal of the American Statistical Association*, **66**, 331-338, 1971.

[3] Gnedenko, B. V. and A. N. Kolmogorov, *Limiting distributions of independent random variables*. Addison-Wesley, Reading, MA, 1954.

[4] Lukacs E., *Characteristic functions*. London, Charles W. Griffin & Co., 1970.

[5] Paulson A. S., T. A. Delehanty and Brothers, Rensselaer Polytechnic Institute, School of Management, *Technical report*, 1988.

[6] Zolotarev V. M., On the representation of stable laws by integrals. *Selected Transactions in Mathematical Statistics and Probability*, **6**, 84-88, 1964.

[7] Won H. G., *Methods and methodologies for the goodness of fit test of some parametric models*, unpublished thesis, RPI, 1988.

Figure 1. (continued)

(c) α = 1.0 – 2.0

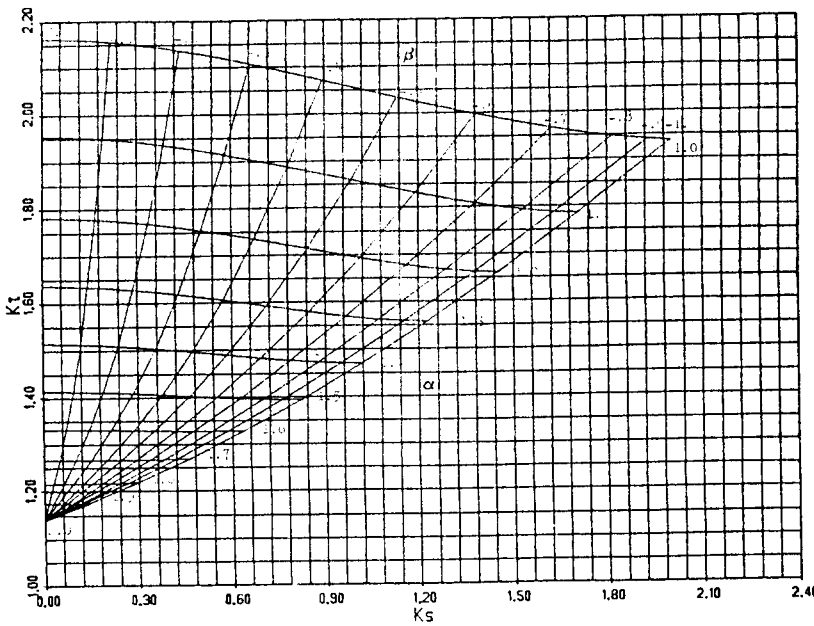


Figure 2. $\tilde{\alpha}$, β by K_s and K_t (Case B)

(a) α = 0.1 – 0.5

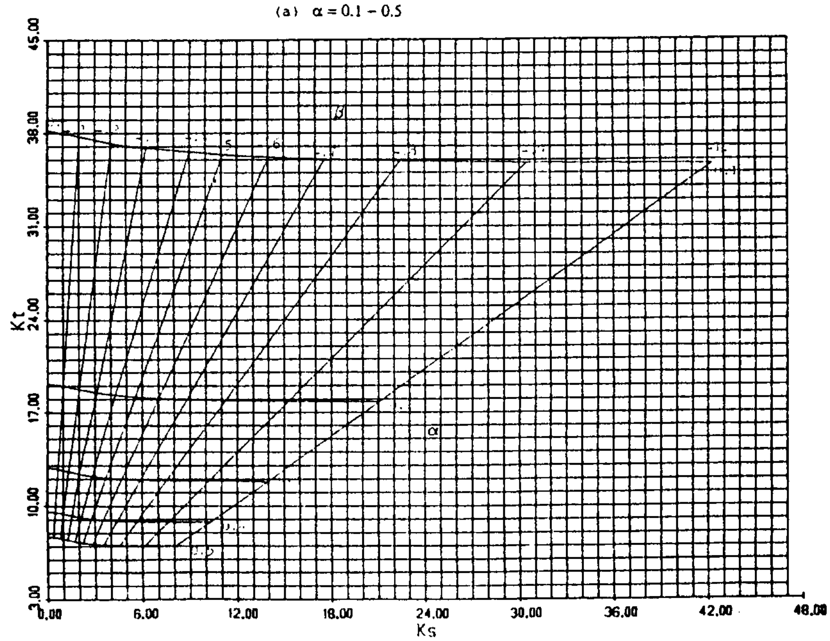


Figure 2. (continued)

(b) $\alpha = 0.5 - 1.0$

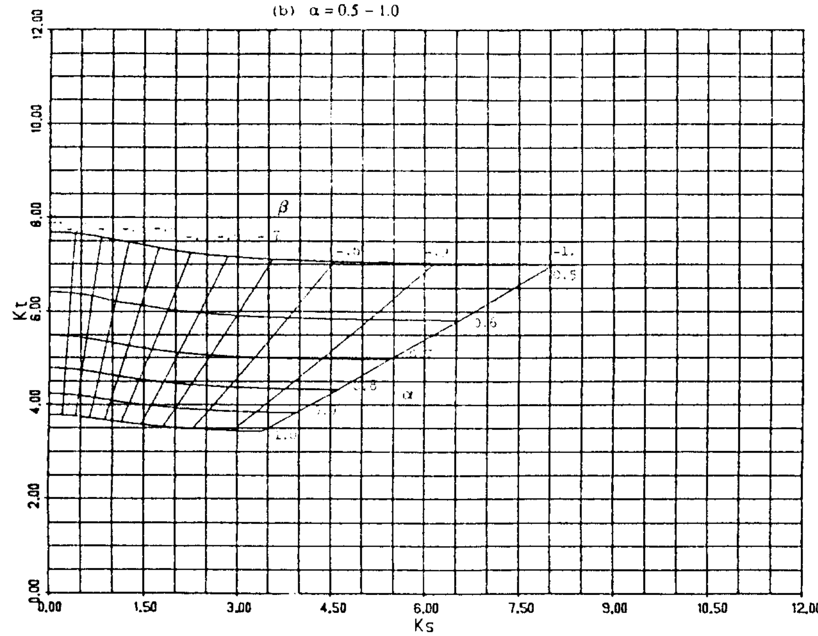
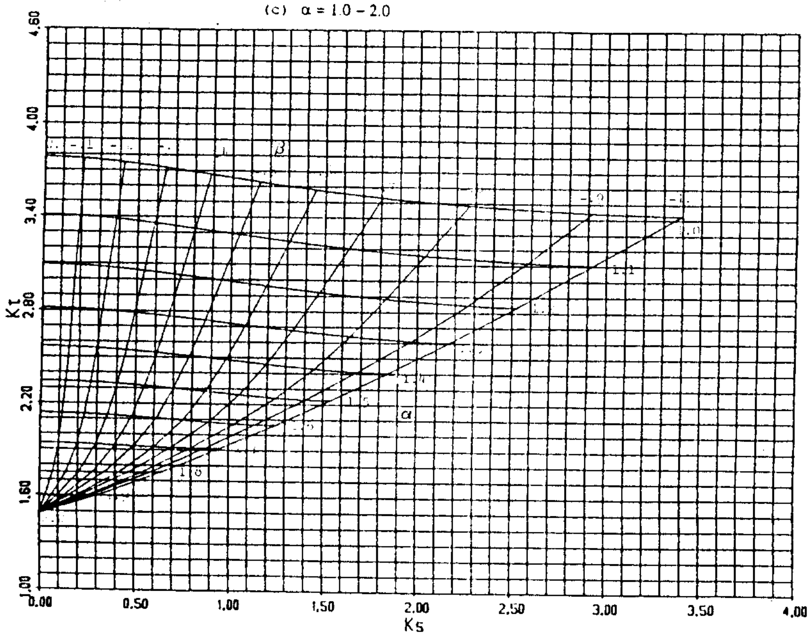


Figure 2. (continued)

(c) $\alpha = 1.0 - 2.0$



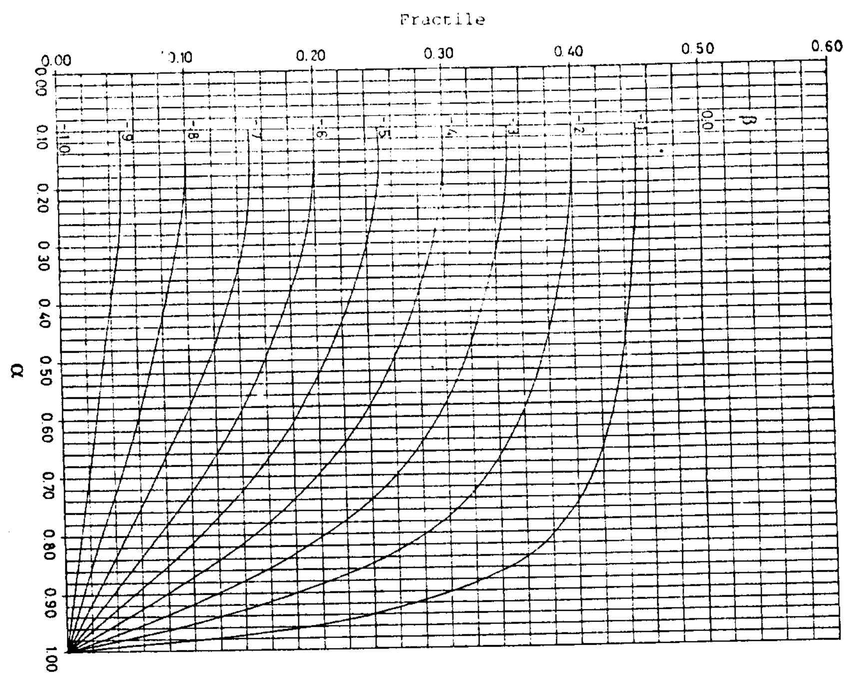


Figure 3. Fractile p of μ, given α and β

(a) α = 0.1 - 1.0

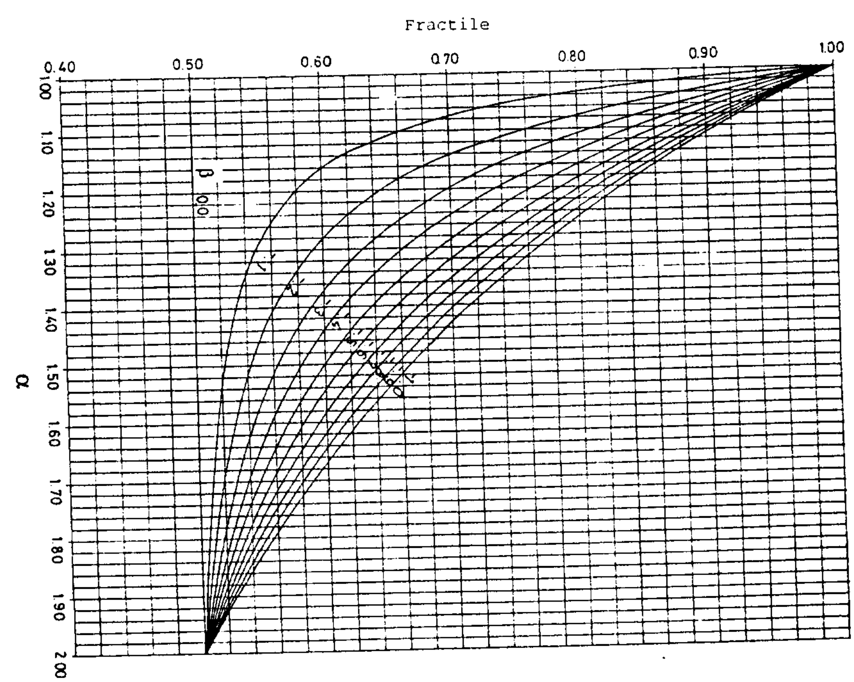


Figure 3. (continued)

(b) α = 1.0 - 2.0

Figure 4. Fractile p of σ for the case $x_p = -1$

(a) α = 0.1 - 1.0

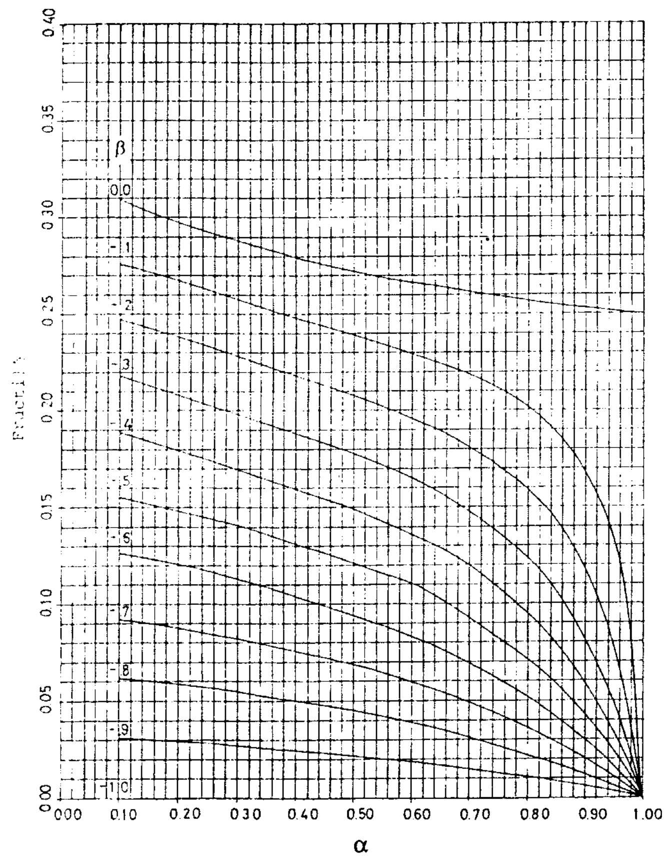
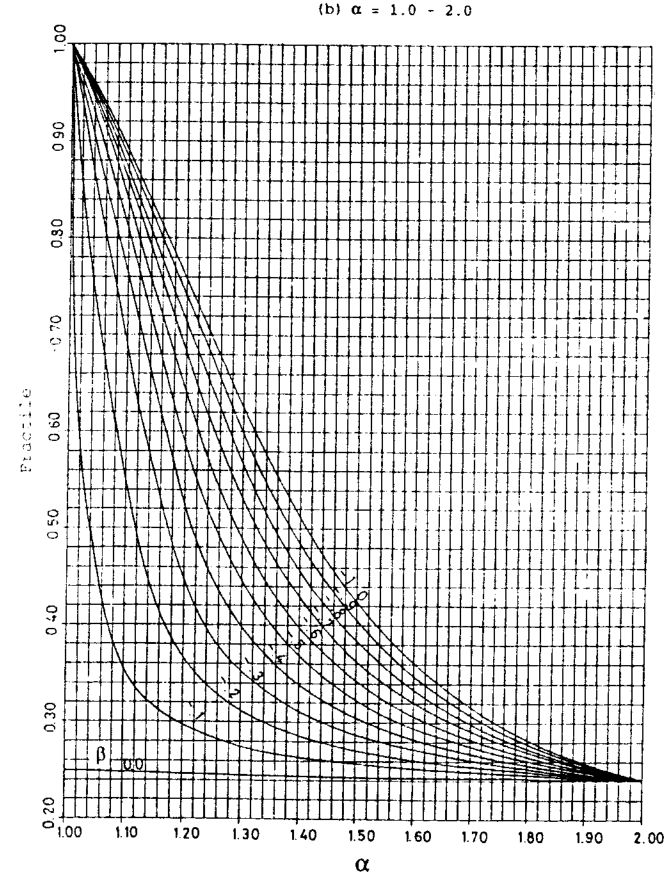


Figure 4. (continued)

(b) α = 1.0 - 2.0



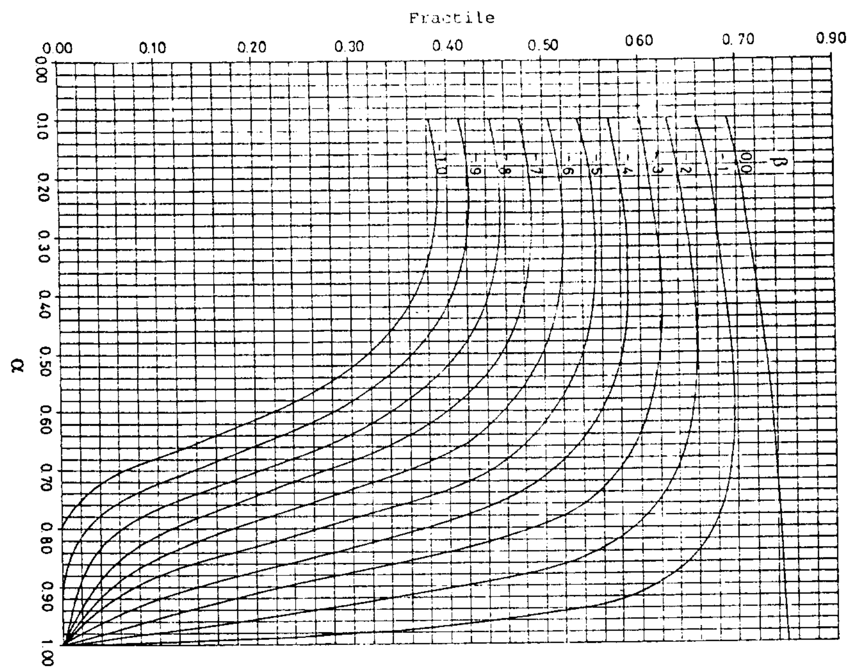


Figure 5. Fractile q of α for the case $x_q = +1$

(a) $\alpha = 0.1 - 1.0$

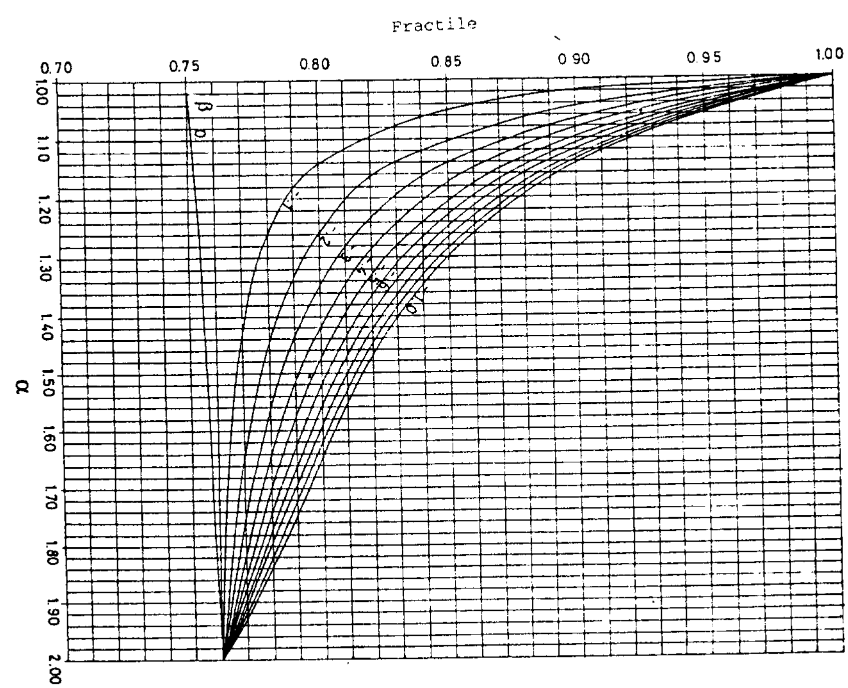


Figure 5. (continued)

(b) $\alpha = 1.0 - 2.0$

Figure 6. Fractile q of σ for the case $x_q = +6$

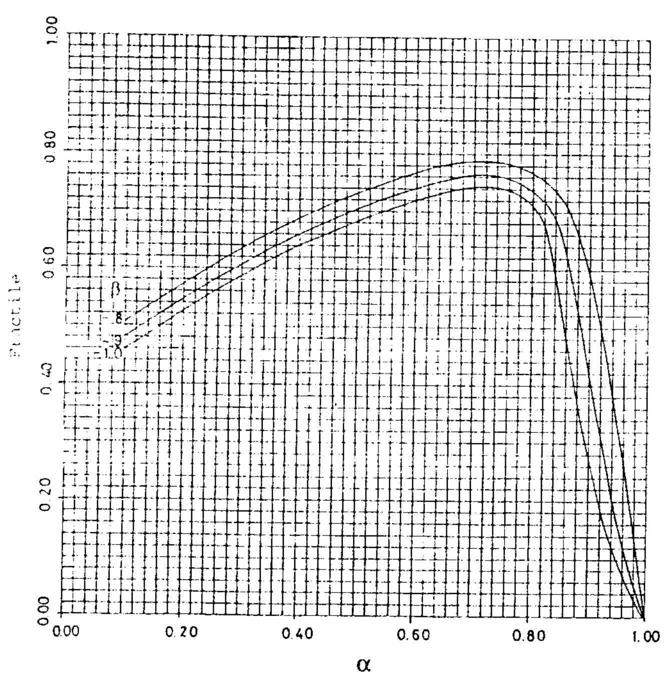


Figure 7. Fractiles p, q and r of σ for the cases $x_p = 0$, $x_q = -1$, $x_r = +1$ when $\alpha = 1$, $\beta < 0$