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Robust Control of Flexible Joint Manipulators

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Abstract

In this paper robotic manipulators in which the joints exhibit a certain amount of elasticity are considered. Based on a feedback linearized model, sliding mode control system is designed. In the control system design, weak joint stiffness assumption does not needed. Simulation results are presented to verify the validity of the control scheme. A robustness analysis for a feedback linearized model is also given with respect to uncertainties on the robot parameters.

1 Introduction

Experimental results have shown that the joint elasticity should be taken into account in both the modeling and control of robotic manipulators if high performance is to be achieved. The elasticity in the joints may be caused by the harmonic drives, that are special type of gear mechanisms having high transmission ratio, low weight, and small size. As a counterpart, these gear-boxes introduce no negligible elasticity, due to their mechanical structure.

The introduction of joint flexibility in the robot modeling complicates considerably the equations of motion. In particular, the order of the related dynamics becomes twice that of rigid robots. Moreover, the property owned by rigid robots of being linearizable by static-state feedback is lost, in general. In fact experimental test, confirmed by simulation, show that in many cases control algorithms developed for rigid robots do not work satisfactorily in the presence of even small elasticity in the joints and, in particular, oscillations occur [1].

Consequently, the control laws proposed for elastic joint robots are more complex than those valid for rigid robots. Many control algorithms have been proposed to solve the problem of controlling robots with elastic joints. Previous approaches are that based on singular perturbation theory [2], [3] and on the concept of integral manifold [4], [5]. But this approach has an assumption that weak joint stiffness. Other used approach is that based on feedback linearization by dynamic state feedback [6]. A simple PD controller has been also proposed [7].

In this paper, sliding mode controller for flexible joint manipulator is proposed. The proposed control system is based on the feedback linearized model and does not require for the exact knowledge for joint stiffness but only the bound.

This paper is organized as follows: The mathematical model of a single-link flexible joint manipulator and its feedback linearized one are described in Section 2. On the basis of the feedback linearized model, sliding mode controller is designed in Section 3. In Section 4, simulation results on regulation and trajectory tracking control are presented. The conclusions including further study are given in Section 5.

2 Modeling of Flexible Joint Manipulators

Experimental investigations of industrial robots with harmonic drive transmission and other forms of gearing indicate that joint flexibility contributes significantly of the overall dynamics of the system. The dynamic equations of the flexible joint robots are given as [8], [9]:

$$D_l(q_l)\ddot{q}_l + C(q_l,\dot{q}_l)\dot{q}_l + g(q_l) + K(q_l - q_m) = 0, \quad (1)$$

$$D_m(\ddot{q_m}) + B_m\dot{q_m} - K(q_l - q_m) = u, \quad (2)$$

where an n-link manipulator becomes a $2n$-degrees of freedom system:

- D_m : Diagonal motor inertia matrix $\in \Re^{n\times m}$.
- B_m : Diagonal motor damping matrix $\in \Re^{n\times m}$.
- K : Diagonal drive shaft stiffness matrix $\in \Re^{n\times m}$.
- q_m : Vector of sensed motor angles $\in \Re^{n\times m}$.
- q_l : Vector of link joint angles $\in \Re^{n\times m}$.
- D_l : Link inertia matrix $\in \Re^{n\times m}$.
- $C(q_l,\dot{q}_l)$: Centrifugal and Coriolis terms matrix $\in \Re^{n\times m}$.
- $g(q_l)$: Gravitational vector term $\in \Re^{n\times m}$.

Matrices D_m, B_m, K are positive definite matrices. Further, D_l is symmetric, positive definite and both D_l and D_l^{-1} are both bounded as a function of q_l. When K tends toward infinity, the robot is considered to have rigid joints.

2.1 Single-Link Flexible Joint Robot

Consider the single link manipulator with flexible joint shown in Figure 1. Ignoring damping for simplicity the equations of motion are as follows [10] :

$$J_l\ddot{q}_l + MgL\sin(q_l) + K(q_l - q_m) = 0$$

$$J_m\ddot{q}_m - K(q_l - q_m) = u. \quad (3)$$

Since the nonlinearity enters into the first equation the control input u cannot simply be chosen to cancel it as in case of the rigid manipulator equations. In other words, there is no obvious analogue of the inverse dynamics control for the system in this form.

In the state space, let

$$\begin{aligned} x_1 &= q_l \\ x_2 &= \dot{q}_l \\ x_3 &= q_m \\ x_4 &= \dot{q_m} \end{aligned}$$

and write the system (3) as

$$\dot{x}_1 = x_2 \quad (4)$$

$$\dot{x}_2 = -\frac{MgL}{J_l}\sin(x_1) - \frac{K}{J_l}(x_1 - x_3) \quad (5)$$

$$\dot{x}_3 = x_4 \quad (6)$$

$$\dot{x}_4 = \frac{K}{J_m}(x_1 - x_3) + \frac{1}{J_m}u. \quad (7)$$

The system is thus of the form

$$\dot{x} = f(x) + g(x)u$$

with

$$f(x) = \begin{bmatrix} x_2 \\ -\frac{MgL}{J_l}\sin(x_1) - \frac{K}{J_l}(x_1 - x_3) \\ x_4 \\ \frac{K}{J_m}(x_1 - x_3) \end{bmatrix} \quad g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

2.2 Feedback Linearized Model

From above system configuration, $n = 4$ and the necessary and sufficient conditions for feedback linearization of the system are that

$$\text{rank}\{g, ad_f(g), ad_f^2(g), ad_f^3(g)\} = 4$$

and that the set

$$\{g, ad_f(g), ad_f^2(g)\}$$

be involutive. Performing the indicated calculations it is easy to check that

$$[g, ad_f(g), ad_f^2(g), ad_f^3(g)] = \begin{bmatrix} 0 & 0 & 0 & \frac{K}{J_lJ_m} \\ 0 & 0 & \frac{K}{J_lJ_m} & 0 \\ 0 & \frac{1}{J_m} & 0 & -\frac{K}{J_m^2} \\ \frac{1}{J_m} & 0 & -\frac{K}{J_m^2} & 0 \end{bmatrix}$$

which has rank 4 for $K > 0$, $J_l, J_m < \infty$. Also, since the vector fields $\{g, ad_f(g), ad_f^2(g)\}$ are constant, they form an involutive set. It follows that the system (3) is feedback linearizable. The new coordinates

$$y_i = T_i \qquad i = 1, \cdots, 4$$

are found from

$$\begin{aligned} < dT_1, g > &= 0 \\ < dT_1, ad_f(g) > &= 0 \\ < dT_1, ad_f^2(g) > &= 0 \\ < dT_1, ad_f^3(g) > &\neq 0. \end{aligned}$$

Carrying out the above calculations leads to the system of equations

$$\frac{\partial T_1}{\partial x_2} = 0, \quad \frac{\partial T_1}{\partial x_3} = 0, \quad \frac{\partial T_1}{\partial x_4} = 0, \quad \frac{\partial T_1}{\partial x_1} \neq 0,$$

Therefore, following simplest solution can be easily obtained.

$$\begin{aligned} y_1 &= T_1 = x_1 \\ y_2 &= T_2 =< dT_1, f >= x_2 \\ y_3 &= T_3 =< dT_2, f >= -\frac{MgL}{J_l}\sin(x_1) - \frac{K}{J_m}(x_1 - x_3) \\ y_4 &= T_4 =< dT_3, f >= -\frac{MgL}{J_l}\cos(x_1)x_2 - \frac{K}{J_m}(x_2 - x_4) \end{aligned}$$

The feedback linearizing control input u is found from the condition

$$u = \frac{1}{< dT_4, g >}(\nu - < dT_4, f >)$$

as

$$u = \frac{J_l J_m}{K}(\nu - a(x)) = \beta(x)\nu + \alpha(x) \tag{8}$$

where

$$\begin{aligned} a(x) \;=\; & \frac{MgL}{J_l}\sin(x_1)\left(x_2^2 + \frac{MgL}{J_l}\cos(x_1) + \frac{K}{J_l}\right) \\ & + \frac{K}{J_l}(x_1 - x_3)\left(\frac{K}{J_l} + \frac{K}{J_m} + \frac{MgL}{J_l}\cos(x_1)\right). \end{aligned}$$

Therefore, in the coordinates $y_1, \cdots, y_4$ with the control law (8) the system becomes

$$\dot{y} = Ay + b\nu \tag{9}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

The above feedback linearization is actually global and the transformed variables $y_1, \cdots, y_4$ are themselves physically meaningful because

$$\begin{aligned} y_1 &= \text{link position} \\ y_2 &= \text{link velocity} \\ y_3 &= \text{link acceleration} \\ y_4 &= \text{link jerk.} \end{aligned}$$

3 Design of a Robust Controller

When the system parameters are exactly known, pole assignment technique is easily applied and desired performance can be obtained. However, uncertainties in the system parameters, nonlinearities such as a nonlinear spring characteristic or backlash, will reduce the achievable performance from the above design. Therefore, control system for flexible joint manipulators should be robust against the parameter uncertainties.

Let's assign the sliding surface as follows :

$$s = ce = \sum_{i=1}^{4} c_i e_i \tag{10}$$

where,

$$c^t = [\, c_1\, c_2\, c_3\, c_4 \,],$$

$$c_i > 0 \quad \text{for } 1 \leq i \leq 3, \quad c_4 = 1$$

$$e = \begin{bmatrix} y_1 - y_{1d} \\ y_2 - y_{2d} \\ \vdots \\ y_4 - y_{4d} \end{bmatrix}$$

y_{id} represent the desired trajectory for y_i.

^{t} represents the transpose.

where c_i is chosen so that $s = ce$ is stable.

Now the control law

$$u = \alpha(x) + \beta(x)\nu \tag{11}$$

that is, (8), which ideally linearizes the system is unachievable in practice due to parameter uncertainty, computational roundoff, unknown disturbances, etc. It is more reasonable to assume a control law of the form

$$u = \hat{\alpha}(x) + \hat{\beta}(x)\nu \tag{12}$$

where $\hat{\alpha}(x)$ and $\hat{\beta}(x)$ are estimated or computed values of

$\alpha(x)$ and $\beta(x)$, respectively. Let's assume the following conditions are satisfied for a known function $h(y) > 0$.

- $\beta_{min} \leq |\beta(x)| \leq \beta_{MAX}$.
- $|\hat{\alpha} - \alpha| \leq h(y) < \infty$.

Now substitute the control law (12) into (9) which results in

$$\begin{aligned} \dot{y}_4 &= \frac{\hat{\beta}}{\beta}\nu + \frac{\hat{\alpha} - \alpha}{\beta} \\ &= \nu + \left(\frac{\hat{\beta}}{\beta} - 1\right)\nu + \frac{\hat{\alpha} - \alpha}{\beta}. \end{aligned} \tag{13}$$

Noting that

$$\left|\frac{\beta}{\hat{\beta}} - 1\right| < 1 \tag{14}$$

can always be obtained by letting $\hat{\beta}$ as

$$\hat{\beta} = \frac{\beta_{MAX} + \beta_{min}}{2}.$$

Set the control input ν as following :

$$\nu = \dot{y}_{4d} - \sum_{i=1}^{3} c_i e_{i+1} - K sgn(s) \tag{15}$$

Then for the existense of sliding mode, following lemma can be obtained.

Lemma 1 *For the flexible joint system (3) and the control law (15) sliding mode exists provided that*

$$K \geq \left|\dot{y}_{4d} - \sum_{i=1}^{3} c_i e_{i+1}\right| + \frac{1}{\beta}h(y) + \eta \tag{16}$$

where $\eta > 0$.

Proof

It is sufficient to show that $s\dot{s} \leq 0$. From equation (10), (13), (15) and (14),

$$\begin{aligned} s\dot{s} &= s\left(\sum_{i=1}^{3} c_i e_{i+1} + \dot{y}_4 - \dot{y}_{4d}\right) \\ &= s\left(\sum_{i=1}^{3} c_i e_{i+1} + \nu + \left(\frac{\hat{\beta}}{\beta} - 1\right)\nu + \frac{1}{\beta}(\hat{\alpha} - \alpha)\right) - \dot{y}_{4d} \\ &= -K|s| + s\left(\frac{\hat{\beta}}{\beta} - 1\right)\left[\dot{y}_{4d} - \sum_{i=1}^{3} c_1 e_{i+1} - K sgn(s)\right] \\ &\quad + s\frac{1}{\beta}(\hat{\alpha} - \alpha) \\ &= s\left(\frac{\hat{\beta}}{\beta} - 1\right)\left[\dot{y}_{4d} - \sum_{i=1}^{3} c_1 e_{i+1}\right] + s\frac{1}{\beta}(\hat{\alpha} - \alpha) - \frac{\hat{\beta}}{\beta}K|s| \\ &= s\frac{\hat{\beta}}{\beta}\left[\left(1 - \frac{\beta}{\hat{\beta}}\right)\left[\dot{y}_{4d} - \sum_{i=1}^{3} c_i e_{i+1}\right] + \frac{1}{\hat{\beta}}(\hat{\alpha} - \alpha) - K sgn(s)\right] \\ &\leq -\eta|s|. \end{aligned}$$

□

Lemma 2 *For the flexible joint system (3) and the proposed control law (15), the overall system is globally asymptotically stable.*

Proof

From the **Lemma 1**, we can easily know that the sliding function s reaches to zero in finite time. Furthermore, we designed the sliding surface so that the system is asymptotically stable in the sliding mode. So, we can conclude that the link position error e_1 goes to zero as $t \to \infty$.

□

4 Simulation Results

In order to demonstrate the effectiveness of the controller developed in the previous section, numerical simulation has been performed on a single-link flexible joint manipulator. Numerical parameters for the manipulator are given in Table 1.

Table 1: Parameters for the flexible joint manipulator

Parameters	Symbol	Numerical Value
Stiffness	K	100 – 200
Minimum Stiffness	K_{min}	100
Maximum Stiffness	K_{max}	200
Inertia of Motor	J_m	2.0
Inertia of Link	J_l	1.0
Gravity	G	9.8
Link Length	L	1.0
Mass of the Link	M	1.0

In the Table 1, all the values has SI units.

It is assumed that the flexible joint manipulator is initially at rest. The controller in equation (15) is discontinuous and it is well known that synthesis of such a controller gives rise to chattering of trajectory about sliding surface $s = 0$. In order to avoid the chattering phenomenon, the function $sgn(s)$ in the controller (15) has been replaced by $sat(s)$. The function $sat(s)$ is defined as follows:

$$sat(s) = \begin{cases} 1 & \text{if } s > \delta \\ s/\delta & \text{if } |s| < \delta \\ -1 & \text{if } s < -\delta \end{cases}$$

where $\delta > 0$.

Figs. 1~4 show the performance of the proposed controller. Simulations on regulation are shown in Figure 1 and 2. These figures demonstrate that the link angle is successfully regulated for bounded unknown stiffness under gravity. By use of $sat(s)$ function instead of $sgn(s)$, the control input is significantly smoothed as shown in Figure 2.

For trajectory tracking control, simulation results are shown in Figure 3 and 4. It is clear that the proposed controller presents a good performance for the trajectory tracking.

5 Conclusions

In this paper, sliding mode control system for a single-link flexible joint manipulator is proposed. Based on the feedback linearized system, the controller is designed. In course of control system design, weak joint stiffness assumption does not needed. From the simulation results, it is clear that the proposed controller exhibits stable and good performance in both regulation and trajectory tracking under bounded unknown system stiffness K.

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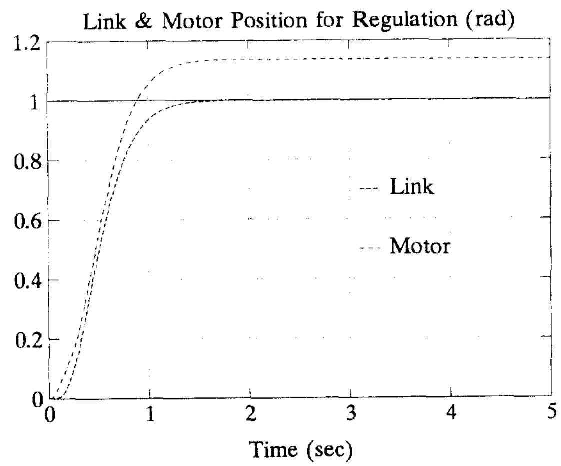

Figure 1: Position for Link angle and Motor angle: Regulation

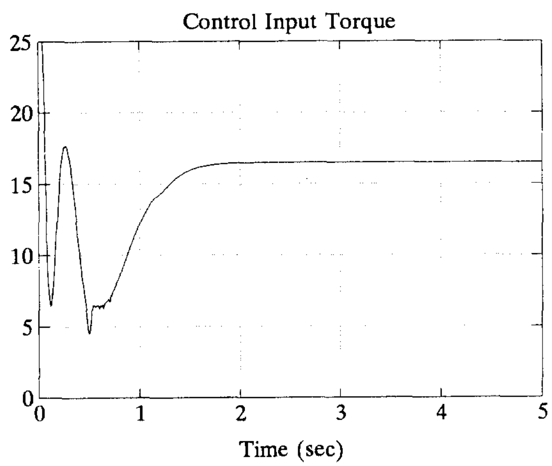

Figure 2: Control Input Torque: Regulation

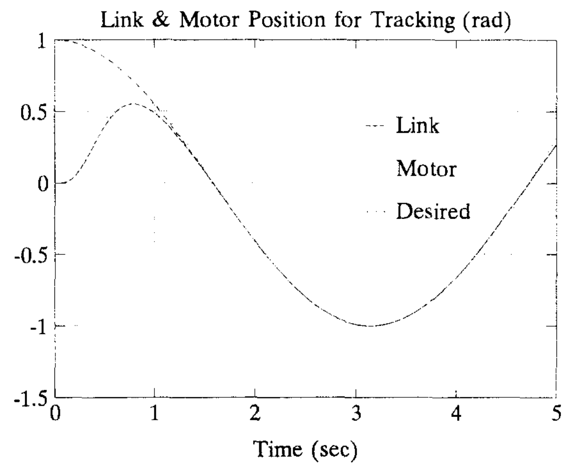

Figure 3: Position for Link angle and Motor angle: Tracking

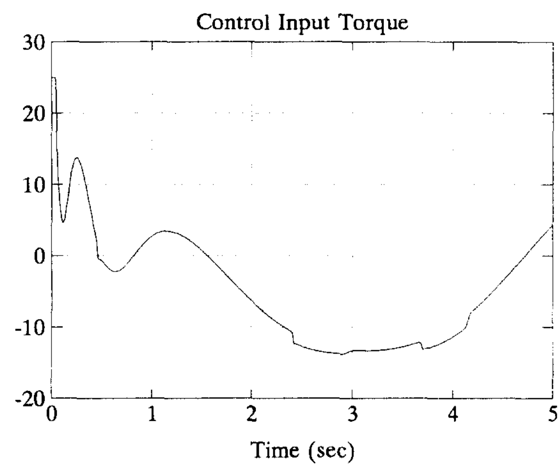

Figure 4: Control Input Torque: Tracking