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Decomposition of Category Mixture in a Pixel and its Application for Supervised Image Classification

Masao MATSUMOTO*, Kohei ARAI** and Takakazu ISHIMATSU*

* : Faculty of Engineering, Nagasaki University, Nagasaki 852 JAPAN.
**: Faculty of Science and Engineering, Saga University, Saga 840 JAPAN.

ABSTRACT

To make an accurate retrieval of the proportion of each category among mixed pixels (Mixel's) of a remotely sensed imagery, a maximum likelihood estimation method of category proportion is proposed. In this method, the observed multispectral vector is considered as probability variables along with the approximation that the supervised data of each category can be characterized by normal distribution. The results show that this method can retrieve accurate proportion of each category among Mixel's. And a index that can estimate the degree of error in each category is proposed. AS one of the application of the proportion estimation, a method for image classification based on category proportion estimation is proposed. In this method all pixel in a remotely sensed imagery are assumed to be Mixel's, and are classified to most dominant category. Among the Mixel's, there exists unconfidential pixels which should be categorized as unclassified pixels. In order to discriminate them, two types of criteria, Chi square and AIC, are proposed for fitness test on pure pixel hypothesis. Experimental result with a simulated dataset show an usefulness of proposed classification criterion compared to the conventional maximum likelihood criterion and applicability of the fitness tests based on Chi square and AIC.

Key words: Remote sensing, Proportion estimation, Mixel, Image Classification and Non-linear optimization.

INTRODUCTION

Generally, a pixel of remotely sensed imagery through the finite field of view contains the information from multiple categories, such pixel is called MIXEL (mixed pixel). The method to estimate the proportion of each category among MIXEL is called "*Category Decomposition*" (Inamura, 1987) and not a few works of category decomposition have been based on the generalized inversion matrix with the assumption which the supervised data of each category can be characterized by only the mean value (Hallum, 1972; Inamura, 1987; Ito and Fujimura, 1987) . However, the supervised data can be fluctuated by some reasons such as illumination condition, surface condition and so on, and such fluctuation makes the estimation error of the proportion. Moreover, in un-determine case the estimation error might be increased since this method does not explicitly considered the observation error.

To make the accurate retrieval of the proportion of each category among MIXEL, it is necessary to compensate the fluctuation of the supervised data and to consider the observation error. In this study, as a solution of such two problems, a category decomposition method based on maximum likelihood estimation is proposed. In this method, the observed pixel data is considered as a probability variables along with the approximations that the supervised data of each category and observation error are characterized by normal distribution.

As one of the application of the category decomposition method, the supervised image classification which is based on the criterion that **each MIXEL is classified into the maximum proportion category**, while the conventional image classification methods such as maximum likelihood classification are based on the assumption which each pixel is pure (Takagi and Shimoda, 1991). Also the unclassified limit to discriminate the ambiguous MIXEL from classifiable MIXEL on the basis of the fitness test of the pure pixel hypothesis is proposed.

MAXIMUM LIKELIHOOD ESTIMATION

The multispectral vector (number of band is M) of the Mixel which contain the information from N categories is shown in Eq. (1),

$$
\begin{aligned}
&I = A \cdot B + \varepsilon \\
&I = [I_1, I_2, \dots, I_M]^t, \\
&B = [B_1, B_2, \dots, B_N]^t, \\
&A = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & \vdots & \vdots \\ A_{M1} & \cdots & A_{MN} \end{bmatrix}, \\
&\varepsilon = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_M]^t,
\end{aligned}
\qquad (1)
$$

where I_i, B_j and A_{ij} are observed data in i-th band, category proportion of j-th category and supervised data of j-th category in i-th band, respectively and superscript t means transpose. In this study, on the basis of following approximations, the observed multispectral vector can be considered as a probability variable.

Assumptions:

(1) A_{ij} is expressed by normal distribution that average is $\bar{A}_{ij}$ and variable is σ^2_{ij}.

(2) The supervised data of each category is independent.

(3) ϵ_i is independent from the proportions and it is expressed by the normal distribution that average is 0 and variance is $\sigma^2_{\epsilon i}$.

(4) The observation error of each band is independent.

The correlation matrix R of each spectral band is expressed as follows,

$$R=\begin{bmatrix} 1 & \cdots & \rho_{1M} \\ \vdots & \vdots & \vdots \\ \rho_{1M} & \cdots & 1 \end{bmatrix}, \qquad \rho_{kl}=\frac{COV_{kl}}{\sqrt{VAR_k \cdot VAR_l}}, \quad (k,l=1,\ldots,M) \tag{2}$$

where ρ_{kl} is correlation coefficient between band k and l. And VAR and COV mean integrated variance shown in Eq. (3) and integrated covariance, respectively.

$$VAR_k = B^t \cdot S_k \cdot B, \qquad S_k = diag(\sigma^2_{k1}, \ldots, \sigma^2_{kM}) \tag{3}$$

From above consideration, the probability P(I; B) when the proportion vector **B** is occurred and multispectral vector **I** is observed is expressed as Eq. (4),

$$P(I;B)=\frac{1}{(2\pi)^{M/2}\sqrt{\det(Z')}}\exp\left[-\frac{1}{2}(I-\bar{A}B)^t Z'^{-1}(I-\bar{A}B)\right]$$
$$Z'=\begin{bmatrix} VAR_1 & \cdots & COV_{1M} \\ \vdots & \vdots & \vdots \\ COV_{1M} & \cdots & VAR_M \end{bmatrix} + diag[\sigma^2_{\epsilon 1}, \ldots, \sigma^2_{\epsilon M}]$$
$$\bar{A}=\begin{bmatrix} \bar{A}_{11} & \cdots & \bar{A}_{1N} \\ \vdots & \vdots & \vdots \\ \bar{A}_{M1} & \cdots & \bar{A}_{MN} \end{bmatrix} \tag{4}$$

The category proportion **B** has following constraints,

$$0 \le B_j \le 1, \quad (j=1,\ldots,N), \qquad \sum_{j=1}^{N} B_j = 1. \tag{5}$$

From the concept of maximum likelihood estimation (Rodgers, 1976), the proportion which maximize P(I; B) under the above constraints is the solution. This problem can be solved by constrained non-linear optimization technique (Konno and Yamashita 1978).

CLASSIFICATION THEORY

From the estimated proportion, the pixel is classified into the largest proportion category, it is called **Maximum Proportion Criterion (MPC)**. The conventional image classification methods have their own unclassified limits which is based on their own classification criterion. In this section, the unclassified limit based on the MPC is described. The meaning which a Mixel can be classified into a certain category is such Mixel can be described by the model that has much constraint. This is called **Pure Pixel Hypothesis**. To verify this hypothesis, the fitness test is applied. In this study the two kinds of fitness tests are proposed.

χ^2 Square Test

It is assumed that there are two models π_1 (degree of freedom is n_1) and π_2 (degree of freedom n_2) and π_1 is a special case of π_2 ($n_1 < n_2$). $P(\pi)$ is define as the likelihood of model π and the logarithmic ratio of likelihood χ^{*2} is defined as follows.

$$\chi^{*2} = -2\ln\left\{\frac{P(\pi_1^*)}{P(\pi_2^*)}\right\}, \tag{6}$$

where superscript * is the maximum likelihood estimate of the model. The χ^{*2} is asymptotically expressed by χ^2 distribution $\chi(n)$ which the degree of freedom is $n = n_2 - n_1$ (Takane, 1980). The fitness test of model π_1 is conducted by the comparison between χ^{*2} and the percentile value of the χ distribution $\chi(n; \alpha)$ with α of significant level.

For the unclassified limit of MPC, π_1 is the pure pixel which contains the information from the largest proportion category (degree of freedom is 0) and π_2 is the Mixel (degree of freedom is N - 1, N is the number of the categories). The unclassified pixel is defined as follows.

$$\begin{cases} \chi^2(N-1;\alpha) \le \chi^{*2} & \textit{Unclassified,} \\ \chi^2(N-1;\alpha) > \chi^{*2} & \textit{Classified.} \end{cases} \tag{7}$$

The value of χ is large when $P(\pi_1)$ is small, so the number of unclassified pixel become large when α is large.

AIC test

The value of AIC (Akaike's Information Criterion) of model π: $AIC(\pi)$ is defined as follows.

$$AIC(\pi) = 2n - 2\ln[P(\pi)] \tag{8}$$

The model π which minimize the $AIC(\pi)$ is the best model to describe.

For the unclassified limit of MPC, the unclassified pixel is defined in the case which the value of AIC of the pure pixel is larger than that of Mixel.

VERIFICATION

The above methods are verified as following manner.

Simulated data

The Supervised Dataset As the supervised dataset, the average and variance of 5 categories in 2 bands, and in this case each band assumed to be independent, so the integrated variance - covariance matrix Z' is diagonal. The supervised dataset is shown in Table 1.

Simulated Category Proportion By using uniform random number from 0 to 1, 100 of proportion of each category is generated as in Eq. (9),

$$B_k=\frac{r_k}{\sum_{i=1}^{N} r_i}, \quad (k=1,...,N). \tag{9}$$

where r_k is uniform random number from 0 to 1. This dataset is used as the truth data.

Simulated Pure Pixel Data Since the fluctuation of pure pixel data is allowed in this method, the pure pixel data is generated based on random number of multivariational normal distribution (Takane, 1980). In the generation, the variance of the supervised data is scaled by 0.2, 0.4, 0.6, 0.8 and 1.0 to estimate the effect of the variance of supervised dataset.

Observation Error Generation To estimate the degree of observation error, the observation error is generated random number of normal distribution which average is 0 and standard deviation is 0, 2, 4, 6, 8 and 10 [Count].

Mixel Dataset Generation From the previous simulated dataset, 30 types of Mixel datasets are generated based on Eq. (1). Each dataset contains 100 of Mixel data and the category proportion is the same as in any dataset.

Extraction of True Pure Pixel The proposed unclassified limits are affected by the variance of supervised data and that of observation error, in each condition, based on each unclassified limit, the pure pixel is extracted from each dataset. This data is used in the verification of the classification criterion.

Estimation of the Category Proportion

As a conventional category proportion method, the generalized inversion method (Inamura, 1987) is used in this study to compare with the proposed method. In this case the number of category (5 categories) is greater than the number of band (2 bands), so this problem is non-determine. And this method, the non-negative constraint is not considered.

Comparison between χ^2 Test and AIC Test

To make a comparison between 2 kinds of unclassified limit, The number of true pure pixel and ratio of correctly classified limit is calculated. As a significant level of χ^2 test, 1, 5 and 10 [%] are selected.

Comparison between Maximum likelihood Criterion and Maximum Proportion Criterion

To compared the proposed criterion with conventional one, The ratio of correctly classified pixel from the proposed criterion (MPC) and maximum likelihood criterion (MLC) calculated. As an unclassified limit of MLC, -20, -15 and -10 of logarithmic likelihood is adopted.

RESULTS

Estimation of the Category Proportion

In this study, category proportion is estimated based on maximum likelihood estimation (Matsumoto, Terayama and Arai, 1992). This method is sensitive to the variance of the supervised data and that of observation error, The root mean square error of the estimated proportion (RMSE) is calculated in each Mixel dataset, and to express the effect of variances, mean variance AVG[σ] is calculated as follows.

$$AVG[\sigma]=trace[Z'] \tag{10}$$

The RMSE's are shown in Fig. 1. The result is enough level in the condition which the number of unknown variable is 5 and the number of known variable is 2. It is clarified that when the mean variance become larger (the variance of the supervised data and / or that of observation error become larger) the RMSE become larger.

The Number of True Pure Pixel

In Fig. 2, The numbers of true pure pixel calculated based on the unclassified limits. When the mean variance becomes large, the number of true pure pixel become large. This is because that The likelihood in Eq. (4) or (6) become broad function. In χ^2 test, the number is slightly affected by the value of significant level, and decreases when the significant level increases. This means that in χ^2 test, the users can control the number of classified pixel by significant level. The number from AIC test is almost the same as in the case of significant level is 10 [%] of χ^2 test.

The Ratio of Correctly Classified Pixel In Fig. 3, the ratio of correctly classified pixel (the number of correctly classified pixel / the number of selected pure pixel) based on the unclassified limits. The result shows that the 2 unclassified limits are almost the same but in the χ^2 test which the significant level is small the ratio become small because of many selected pure pixel.

Comparison between the Criteria

Fig. 4 shows the ratio of correctly classified pixel from MLC and MPC. In MLC, when the unclassified limit is large (−10), the ratio is always zero because there are no selected pure pixel, and the other 2 cases of MLC show the ratio which is up to 40 [%]. On the other hand, the ratio from MPC is about 10 [%] higher than that from MLC. This shows the higher classification accuracy of MPC than MLC because of the consideration of the Mixel.

CONCLUSIONS

The previous results lead to following conclusions. The observed multispectral vector is considered as probability variables along with the approximation that the supervised data of each category can be characterized by normal distribution. The results from simulated Mixel data show that this method can retrieve more accurate proportion. And by considering the Mixel, the classification criterion which each pixel is classified into the maximum proportion category can classify the remotely sensed imagery more accurately than maximum likelihood criterion. And as the unclassified limit of this criterion, the fitness test of pure pixel hypothesis is suitable.

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Table 1 Average and Variance of the Supervised Data

		1	2	3	4	5
A	PC1	97.8	162.4	127.3	60.9	107.8
V	PC2	62.2	135.1	162.0	100.9	187.7
V	PC1	160.4	841.1	185.7	94.0	178.2
R	PC2	309.9	681.3	430.4	329.3	586.2

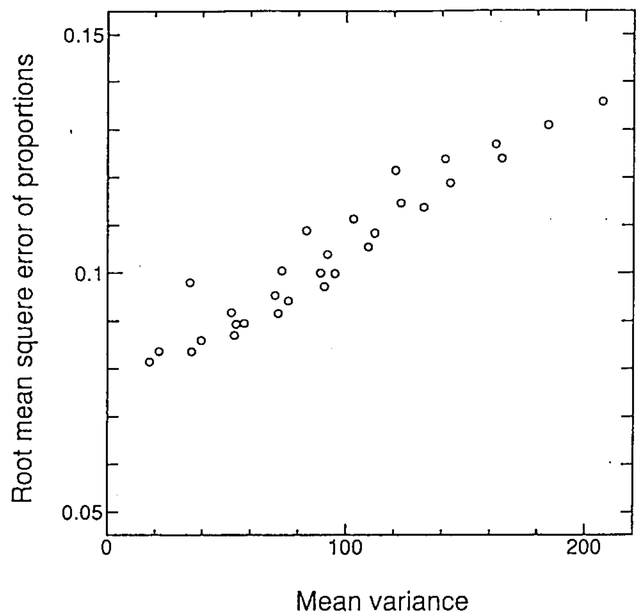

Fig. 1 RMSE of Estimated Category Proportion

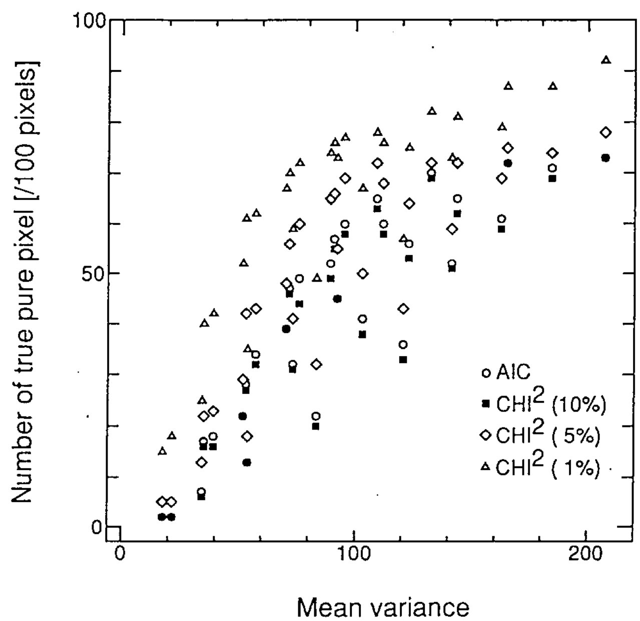

Fig.2 Number of True Pure Pixel

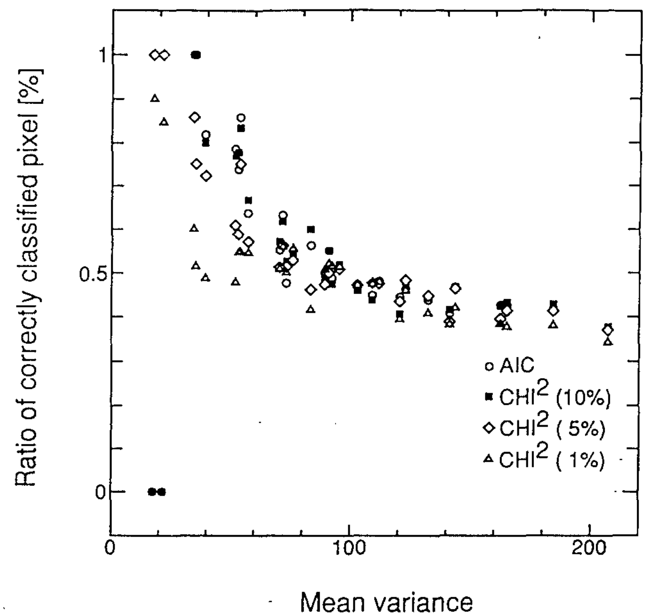

Fig. 3 Ratio of Correctly Classified Pixel (from various unclassified limit)

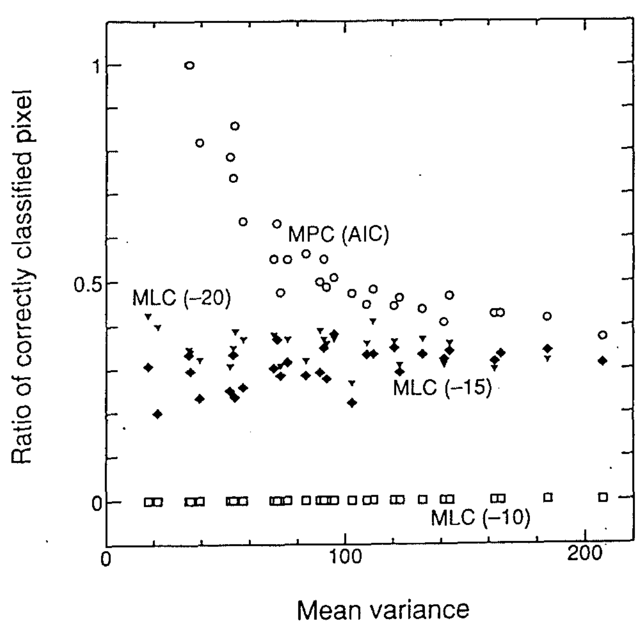

Fig. 4 Ratio of Correctly Classified Pixel (from various criteria)