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APPROXIMATE ANALYSIS FOR PERFORMANCE EVALUATION OF SERIAL PRODUCTION LINE

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Abstract

This paper presents a decomposition method to evaluate the performance measures of transfer line with unreliable machine and finite buffers. The proposed method is to decompose the transfer line into a set of two machine lines for analysis. Nonhomogeneous lines are considered. In such lines, the machines may take the different lengths of time performing operations on parts. A simple transformation is employed in order to replace the line by a homogeneous line. The approximate transformation enables one to determine parameters of performance such as production rate and average buffer levels, and it also shows better applications for a large class of systems.

1. INTRODUCTION

The transfer line which consists of a series of machines separated by buffers is considered. In a transfer line, parts are processed sequentially by all machines. The performance of a transfer line is highly influenced by machine failures. When a machine breaks down, it cannot produce at all. Meanwhile, the number of parts in the downstream buffer decreases while the number of parts in the upstream buffer increases. If this condition persists, the downstream buffer becomes empty and the upstream buffer becomes full. Consequently, the downstream machines are starved and the upstream machines are blocked.

An extensive research has been done on the modeling and analysis of transfer lines. The models of discrete and continuous form have been widely investigated. The discrete model was first introduced by Buzacot[1], in which a basic assumption that machines can be broken down or repaired only at begining of a period was used. This model has been applied for two-machine lines[7]. The continuous model was originally proposed by Zimmern. In this case, the quantity of material in a buffer is a real number. When a machine is operational and is neither starved nor blocked, it transfers material from the upstream buffer to the downstream buffer continuously at a constant rate. This model has been applied for two-machine lines. Further work has been carried out to the analysis of longer transfer lines. An approximate method was described by Gershwin [4] for the analysis of the discrete model of long lines. The method was based on the decomposition of a line into a set of two-machine lines. The performance parameters can be efficiently computed using the algorithm proposed by Dallery et al.[3].

The present study is to determine a decomposition method for performance analysis of nonhomogeneous transfer lines with operation-dependent failures. The decompsition technique by Gershwin was emoloyed and was used to approximate the behavior of the line.

2. THE CONTINUOUS MODEL OF HOMOGENEOUS LINES

2.1 Continuous model

The flow of discrete parts in the transfer line was approximated by a continuous flow. Therefore, the quantity of material in each buffer B_i at any time t, $h_i(t)$, is a real number taking its value in the interval$[0,C_i]$. In our case the buffer state is the whole conveyor occupation description. The buffer level represents the total amount of material on the conveyor. The conveyor occupation may be expressed as a relative density function along the conveyor length. The behavior of the continuous model is defined as follows.

1) Each machine can be in two states: operational, not in a failure condition; or down, under repair. Let $\alpha_i(t)$ indicate the state of machine M_i at time t. $\alpha_i(t) = 1$ if M_i is operational and $\alpha_i(t) = 0$ if M_i is down.

2) A machine can be starved or blocked. Machine M_i is starved at time t if one of the upstream machines is down and all buffers in between this machine and machine M_i are

empty, i.e.,

$j < i$ such that $\alpha_i(t) = 0$ and $h_k(t) = 0$ for all $k = j,\ldots,i\text{-}1$.

Machine M_i is blocked at time t if one of the downstream machines is down and all buffers in between this machine and machine M_i are full, i.e.,

$j < i$ such that $\alpha_i(t) = 0$ and $h_k(t) = C_k$ for all $k = i,\ldots,j\text{-}1$.

Let $s_i(t)$ and $b_i(t)$ inicate the starvation and blocking conditions of machine M_i at time t: $s_i(t) = 0$ if M_i is starved and $s_i(t) = 1$ otherwise; $b_i(t) = 0$ if M_i is blocked and $b_i(t) = 1$ otherwise. A machine is idle if it is either starved or blocked.

3) A machine which is operational, and neither starved nor blocked, is working. When machine M_i is working, it transfers material from its upstream buffer B_{i-1} to its downstream buffer B_i at a continuous rate U. That is, a quantity of material Udt is transferred in time dt. The processing rate is the inverse of the processing time, i.e., $U = 1/T$.(Recall that for the time being, we only consider homogeneous lines.)

4) When a machine is working, it may fail. The time to failure of machine M_i is exponentially distributed with rate λ_i. That is,

$$prob[\alpha_i(t+dt)=0/\alpha_i(t)=1,s_i(t)1,b_i(t)=1]= \mu_i dt+O(dt^2).$$

When a machine failes, it is under repair. The time to repair of machine M_i is exponentially distributed with rate μ_i. That is

$$prob[\alpha_i(t+dt)=1/\alpha_i(t)=0]=\mu_i dt+O(dt^2).$$

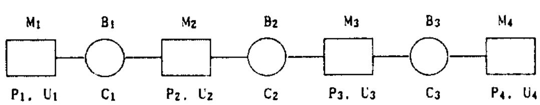

Fig. 1. A trasnsfer line with four machines.

2.2 Some basic equations

Some basic relationships for steady state performance parameters of the continuous model of the transfer line L was established. The following quantities related to machine Mi are introduced:

$$I_i = \lambda_i/\mu_i \text{ and } e_i = 1/(1+I_i) \quad (1)$$

where e_i is the isolated efficiency of machine M_i. As only two parameters of λ_i, μ_i, I_i, e_i are independent, we choose I_i and μ_i as the elementary parameters of a machine, and the other parameters can be obtained from equations (1).

The following performance parameters of the continuous model in steady state are defined:

E_i: probability of machine M_i being busy (also called efficiency),

ps_i: probability of machine M_i being starved,

pb_i: probability of machine M_i being blocked.

Some relationships can be established between performance parameters. The relation is related to the conservation of material flow:

$$E_i = E_1 \quad \text{for all } i=2,\ldots,K \quad (2)$$

By the definition of efficiency,

$$E_i = prob[\alpha_i(t)=1/s_i(t)=1,b_i(t)=1]*prob[s_i(t)=1,b_i(t)=1].$$

$prob[\alpha_i(t)=1/s_i(t)=1,b_i(t)=1]$ is equal to the isolated efficiency of M_i, e_i. In the continuous model, the probability of a machine being starved and blocked simultaneously is 0. thus, we have:

$$prob[s_i(t)=1,b_i(t)=1]=1\text{-}ps_i\text{-}pb_i$$

which implys :

$$E_i=e_i(1\text{-}ps_i\text{-}pb_i) \quad \text{for all } i=1,\ldots,K \quad (3)$$

3. ANALYSIS OF THE DECOMPOSITION

3.1 Decomposition

An approximate method which decomposes the K-machine line L into a set of K-1 two-machine lines L(i); $i=1,\ldots,K\text{-}1$(figure 2) is proposed. Each line L(i) is composed of an upstream machine, $M_u(i)$, and a downstream machine, $M_d(i)$, separated by a buffer B(i). Machine $M_u(i)$ represents the portion of line L upstream of B_i, and machine $M_d(i)$ the portion of line L downstream from B_i.

The principle of the decomposition is that the behavior of the material flow in the buffer B(i) closely matches that of the flow in buffer B_i of line L. It seems natural to choose the processing rates of the machines in line L(i) to be equal to 1. Furthermore, the capacity of buffer B(i) is chosen to be equal to that of B_i, i.e. C_i. Now the unknown parameters of each line L(i) are the failure and repair rates of the upstream and downstream machines : $\lambda_u(t)$, $\mu_u(t)$, $\lambda_d(t)$, $\mu_d(t)$, respectively. The object of the approximate method is to determine these parameters.

For the upstream machine of line L(i), we define the following quantities

$$I_u(i)=\lambda_u(i)/\mu_u(i)$$
$$e_u(i)=\mu_u(i)/(\lambda_u(i)+\mu_u(i)) \quad (4)$$

Similary, for the downstream machine, we define

$$I_d(i)=\lambda_d(i)/\mu_d(i)$$
$$e_d(i)=\mu_d(i)/(\lambda_d(i)+\mu_d(i)) \quad (5)$$

For each line L(i), we define the following performance parameters.

E(i) : Efficiency of line L(i). This is the proportion of time machine $M_d(i)$ is working.

X(i) : Production rate of line L(i). This is the production of rate of machine $M_d(i)$.

ps(i) : Probability of machine $M_d(i)$ being starved in line L(i).

pb(i) : Probability of machine $M_u(i)$ being blocked in line L(i).

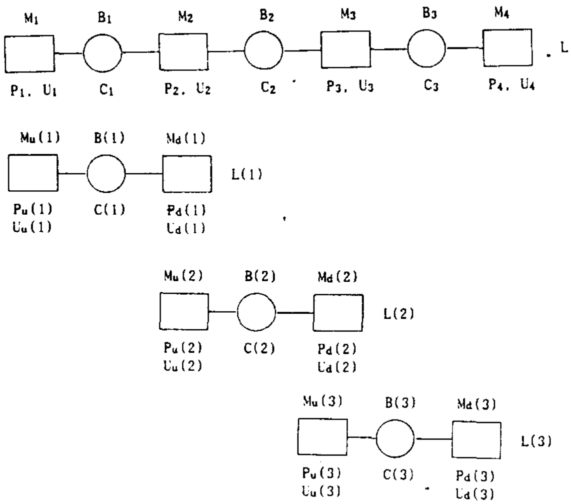

Fig. 2. Decomposition of the four-machine line into three two-machine lines

The performance parameters are functions of the parameters of the upstream and downstream machines of L(i). The efficiency and production rate of line L(i) are again related

$$X(i)=UE(i) \qquad \text{for any } i=1,\dots,K-1. \quad (6)$$

Now, by similar arguments as for the derivation of equation (3), we can obtain

$$E(i) = e_u(i)(1-pb(i)) \qquad \text{for any } i=1,\dots,K-1 \quad (7)$$

$$E(i) = e_d(i)(1-ps(i)) \qquad \text{for any } i=1,\dots,K-1. \quad (8)$$

As stated earlier, the principle of the decomposition is to determine the characteristics of the machines of each line L(i) such that the behavior of material flow through buffer B(i) closely matches that of the flow in buffer B_i of line L. Especially, the throughput of line L(i) should be equal to that of machine M_{i+1} in line L; the probability of machine $M_d(i)$ being starved in line L(i) should be equal to that of machine M_{i+1} in line L; and the probability of machine $M_u(i)$ being blocked in line L(i) should be equal to that of machine M_i in line L. that is

$$X(i) = X_{i+1} \qquad \text{for any } i=1,\dots,K-1 \quad (9)$$

$$ps(i) = ps_{i+1} \qquad \text{for any } i=1,\dots,K-1 \quad (10)$$

$$pb(i) = pb_i \qquad \text{for any } i=1,\dots,K-1. \quad (11)$$

(9) can equivalently be written

$$E(i) = E_{i+1} \qquad \text{for any } i=1,\dots,K-1. \quad (12)$$

From these relations, a first set of conditions can be derived. From (12) and the conservation of flow in line L(2), we obtain

$$E(1)=E(2)=\dots=E(i)=\dots=E(k-1). \quad (13)$$

Using (9),(10), and (11), (3) yields

$$E(i-1) = e_i(1-ps(i-1)-pb(i)) \qquad \text{for any } i=2,\dots,K. \quad (14)$$

Now using (7),(8), and (13), after some manipulation, (14) can be written

$$1/e_d(i-1)+1/e_u(i) = 1/E(i-1)+1/e_i \quad \text{for any } i=2,\dots,K-1 \quad (15)$$

Let us now consider the failure-repair mechanism of the upstream machine of line L(i). Machine $M_u(i)$ models the part of line L upstream of buffer B_i. Therefore, a failure of $M_u(i)$ represents either a failure or a starvation of machine M_i. A starvation of machine M_i is a consequence of either a failure or a starvation of machine M_{i-1}. Now a failure or a starvation of machine M_{i-1} is represented by a failure of machine $M_u(i-1)$. There, a failure of machine $M_u(i)$ results from either a failure of machine M_i or a failure of machine $M_u(i-1)$. Then we have

$$t_u(i) = \alpha r_u(i-1) + (1-\alpha)t_i \quad (16)$$

where $t_u(i)$ and t_i are the mean times to repair (MTTR) of machines $M_u(i)$ and M_i, respectively, and $r_u(i-1)$ is the average residual repair time of machine $M_u(i-1)$ when the starvation of machine M_i occurs. As a result, the quantity is given by

$$\alpha = \frac{Ps(i-1)\ t_u(i)}{r_u(i-1)\ pf_u(i)} \quad (17)$$

After some manipulations, (16) can finally be expressed as

$$\mu_u(i) = X\mu_u(i-1) + (1-X)\mu_i \qquad \text{for any } i=2,\dots,K-1. \quad (18)$$

$$\text{where } X = \frac{ps(i-1)}{l_u(i)E(i)}$$

A similar analysis of the failure-repair mechanism of machine $M_d(i)$ yields the following equation:

$$\mu_d(i) = Y\mu_d(i+1) + (1-Y)\mu_{i+1} \qquad \text{for any } i=1,\dots,K-2. \quad (19)$$

$$\text{where } Y = \frac{pb(i+1)}{l_d(i)E(i)}$$

Finally, there are boundary conditions.

$$\lambda_u(1) = \lambda_i$$
$$\mu_u(i) = \mu_i \quad (20)$$
$$\lambda_d(K-1) = \lambda_k$$
$$\mu_d(K-1) = \mu_k$$

There is a total of 4(K-1) equations among (13),(15),(18), (19), and (20) in 4(K-1) unknowns: $\lambda_u(i)$,$\mu_u(i)$,and $\lambda_d(i)$, $\mu_d(i)$,for i 1,...,K-1.

3.2 NONHOMOGENEOUS LINES

A decomposition method has been presnted for the approximate analysis of the continuous model of a transfer line. However, this technique is restricted to the analysis of homogeneous lines. Now

nonhomogeneous lines are considered. In such lines, machines may take different lengths of time at machine M_i. The nonhomogeneous lines are initially transformed into homogeneous line. The transformation replace each machine of the original line by an equivalent single machine. All equivalent machines have the same processing time T, which is equal to the processing time of the fastest machine of the original line. Let λ_i^e and μ_i^e be the parameters of the equivalent machine of machine M_i. These parameters must be chosen such that the behavior of the homogeneous line is close to the behavior of the original line. For two unknown parameters, we need two equations, the first equation is obtained by prescribing that the equivalent machine has the same isolated production rate as the original machine. Let x_i and x_i^e be the isolated production rate of the original and the equivalent machines, respectively. They are given by

$$x_i = \frac{1}{T_i}\frac{\mu_i}{\lambda_i+\mu_i} \quad (21)$$

$$x_i^e = \frac{1}{T}\frac{\mu_i^e}{\lambda_i^e+\mu_i^e} \quad (22)$$

Therefore, the first condition leads to

$$\frac{\mu_i^e}{\lambda_i^e+\mu_i^e} = \frac{T}{T_i}\frac{\mu_i}{\lambda_i+\mu_i} \quad (23)$$

The second condition is related to the repair rate. Since the repair mechanism of a machine is not dependent on the behavior of the rest of the line, both machines should have the same repair rate, i.e.,

$$\mu_i^e = \mu_i \quad (24)$$

From (23) and (24), we get

$$\lambda_i^e = \lambda_i+(\lambda_i+\mu_i)(T_i-T)/T \quad (25)$$

Thus, in the homogenization transformation, the failure rate of the machine is adapted to the new speed of the machine (25). Since $T \le T_i$, we have $\lambda_i^e \ge \lambda_i$. That was expected, since the equivalent machine is faster than the original one, its failure rate must be higher in order to keep the isolated production rate equal.

4. NUMERICAL RESULTS

In the case of nonhomogeneous lines, several examples are considered which are refered to as lines L1 to L3. These examples are taken in part from [8]. Lines L1 and L2 have K=6 machines while line L3 has K=4 machines. The data and results are given in Tables 1 - 3, respectively. Tables 1 - 3 show that decomposition method works well for nearly homogeneous lines, but for highly unbalanced lines, the results are not satisfactory. These may be due to continuous flow approximation and decompositon technique.

Table I

LINE L1 Nonhomogeneous Line

	1	2	3	4	5	6
T_i	0.356	0.28	0.28	0.28	0.28	0.347
$1/\lambda_i$	9.24	45	45	45	45	9.24
$1/\mu_i$	0.76	5	5	5	5	0.76
C_i	4	2	2	2	2	4
	Q_1	Q_2	Q_3	Q_4	Q_5	X
Simula.	1.4	0.9	0.88	0.88	2.26	2.116
Decomp.	1.69	0.89	0.91	0.93	2.03	2.165

Table II

LINE L2 Nonhomogeneous Line

	1	2	3	4	5	6
T_i	0.35	0.25	0.30	0.32	0.30	0.31
$1/\lambda_i$	30	14	35	40	36	14
$1/\mu_i$	4	6.5	10	8.5	12	3.5
C_i	100	100	150	250	250	
	Q_1	Q_2	Q_3	Q_4	Q_5	X
Simula.	55	54	61	89	52	2.3
Decomp.	57.1	56.1	69.2	97.3	66.3	2.24

Table III

LINE L3 Nonhomogeneous Line

	1	2	3	4
T_i	1.5	1	0.8	1.6
$1/\lambda_i$	100	140	190	250
$1/\mu_i$	12	18	35	12.5
C_i	20	20	15	
	Q_1	Q_2	Q_3	X
Simula.	6.1	11.5	11.8	0.556
Decomp.	7.7	9.2	9.8	0.515

Tables 4 and 5 are the results of real production line. Although the performance of a transfer line is highly influnced by blocking and starvation, the results using decomposition method are an approximate estimation of those of original lines.

Table IV

LINE L4 Nonhomogeneous Line

	1	2	3	4	5	6
T_i	14.5	14.3	14.3	16.8	14.4	14.6
$1/\lambda_i$	1317	130	451	645	744	427
$1/\mu_i$	20	25	23	24	42	30
C_i	3	3	3	3	3	3
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Simula.	2.93	2.49	2.85	2.51	2.77	2.77
Decomp.	2.92	2.53	2.78	2.40	2.71	2.70
	7	8	9	10	11	12
T_i	15.8	14.5	15.4	16.6	16.1	18.4
$1/\lambda_i$	172	134	175	161	104	265
$1/\mu_i$	48	34	20	29	32	20
C_i	3	3	3	3	3	3
	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	X
Simula.	1.52	1.48	2.03	1.63	0.90	0.044
Decomp.	1.75	1.66	1.93	1.61	0.98	0.041

Table V

LINE L5 Nonhomogeneous Line

	1	2	3	4	5	6
T_i	7.0	3.0	3.5	12.8	11.0	4.8
$1/\lambda_i$	1317	130	451	645	744	427
$1/\mu_i$	20	25	23	24	42	30
C_i	3	3	3	3	3	3
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Simula.	2.73	2.80	2.83	2.39	2.10	2.64
Decomp.	2.68	2.76	2.84	2.11	1.83	2.33
	7	8	9	10	11	12
T_i	11.0	10.0	10.0	15.0	15.0	6.0
$1/\lambda_i$	172	134	175	161	104	265
$1/\mu_i$	48	34	20	29	32	20
C_i	3	3	3	3	3	3
	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	X
Simula.	2.22	2.04	2.21	1.36	0.13	0.044
Decomp.	2.06	2.07	2.14	1.32	0.27	0.036

5. CONCLUSION

In this paper, we have presented an approximate technique for the analysis of transfer lines with unreliable machines and finite buffers. The behavior of the line was approximated by a continuous model and analyzed by means of a decomposition technique. One advantage of using the continuous model is that the equations involved in the decomposition are readily derived and accurately satisfied. This paper is to present a simple way of analyzing nonhomogeneous lines by introducing the simple transformation. The transformation has enabled us to replace the original line by a homogeneous line whose performance is close to that of the original line. The results obtained showed that this transformation is very satisfactory. We obtained the result for extending the decomposition technique to real nonhomogeneous lines. Such an extention would be useful for analizing lines with very different processing times in which cases the results are further to be improved.

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