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Optimal Design of Finger Phalanges

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ABSTRACT: An optimal design method to determine the lengths of finger phalanges is proposed especially for anthropomorphic design. The quality of designs are quantified by several measures of global isotropy for design. Also, for an example, optimal design of two fingers is performed and the results are compared with the anatomical data.

1 Introduction

The enhanced performance of robot hands may be obtained by more completely emulating human hand as it is the optimal model developed in centuries. The design of robot hands should cover a number of important features. These are its kinematic design which establishes its dexterity; drive mechanisms and scale which determines its strength, power and size of objects; the degree of freedom and the method of control which determine its usefulness[4-10]. Although there are a lot of researches about multi-fingered hand, most of them are focused on manipulation and control. Thus, it is hard to find the approach for the basic kinematic design of the hands. From purely engineering point of view, Salisbury[6] determined the type and degree of freedom of his fingers by using the number and type synthesis procedure. Moreover, the condition number was introduced to search for the isotropic point in the finger workspace. He developed a design concept for each finger but did not extend it to that of the whole hand. Yun[10] developed a five fingered hand and proposed a unified design approach, but the design method was limited to only thumb and index finger.

In this paper, based on the anthropomorphic model of the hand, we tried to set up optimal design principles for multi-fingered hand. The centerpart of the relation between anatomy and engineering is the modeling of facts, that is, how to realistically model human motion or structure and apply to the optimization function. Actualy, it does not completely describe the realities of human hand because only fingertip motions are taken into consideration. However, the proposed design method and optimization function can be a good tool to guide the kinematic design of the hand.

2 The Human Hand

The human hand is a marvelous product of biological evolution and adaptation. Anatomically, the hand is comprised of specially shaped bones, each having a specific function. The bones of the hand are illustrated in Fig. 1. The main parts of human motion are performed by the thumb and index finger and the others play complementary roles. For proper finger-thumb cooperation, two conditions must be met: (1) *correct kinematic relationships between lengths of fingers and length of thumb*, and (2)*thumb rotation which leads to opposition.* Napier has researched and classified primates throughout the world for many years[1]. He established the definitions of geometric relationships and a summary of his results suggests that kinematic relationships within each species of primate remain relatively constant.

In general, the ratios of phalanges may be derived by dividing the length of a given finger into three phalanges. For the ratio of phalanges, a couple of works were performed, Hogarth[2] claimed that each phalanx is 3/2 longer than the previous phalanx and Youm[3] suggested this ratio follows the Fibonacci sequences (e. g. $1:1:2:3:5\cdots$) from proximal to distal. Placement of the thumb relative to the fingers is another critical area for overall hand performance. From a functional perspective, the thumb should keep finger opposition primarily but there is no reported result. Another observation is that, in general, the distal and middle phalanges of a given finger, move together in a given ratio which means that the distal phalanx travels at an absolute determined angular rate faster than the middle phalanx. This ratio is to be constant in almost, if not all human hands.

3 Kinematic Design Concept of an Anthropomorphic Hand

Kinematic design procedures can be regarded as a kind of procedure of kinematic synthesis. Kinematic synthesis consists of type synthesis, number synthesis and dimension synthesis. As for the number and types of the hand, all the primates including human are the same. However, for the actual dimensions, we can not find consistent results[1–3]. The basic prepositions which should be accepted to design anthropomorphic hand are the degrees of freedom of each finger, joint types and range of motions. Thus, the remaining one is the actual dimensions of each fingers, that is, link length of individual fingers and their relative locations on the palm. Now, it will be discussed on how to optimally determine the length of phalanges and relative locations of the fingers. Here, we apply our robotic engineering knowledge to determining the dimension of fingers optimaly.

3.1 Phalangeal Lengths of Individual Finger

While determining the lengths of phalanges, conventional approaches use the criterion functions which represent dexterity and accuracy[6,13–14], in a local sense. However, these may not represent optimal ones rigorously, since the obtained values are evaluated at some specific point in the workspace and may not satisfy global optimal property over the workspace. It may have meanings to design the hand satisfying the specifications at given operating points or to set an operating point at the most optimized point. If so, we do not have to design complicated multi–fingered hand but only specific gripper will be successful. Therefore, the design of multi–fingered hand should begin with the generalization of desired properties over the workspace and it may agree to anthropomorphism.

3.2 Relative Locations on the Palm

The relative location of each finger can be determined depending on the length and location of metacarpals anatomically and may be determined based on the consideration that the cooperative property of each finger should be maximized globally. As mentioned before, the cooperation is mainly performed between the thumb and the other fingers, especially index finger. Therefore, first, the optimal location between the thumb and index finger is determined and then, the other fingers can be located one by one according to the hierarchy of fingers.

4 A New Measure of Global Isotropy for Design

Various attempts have been made for devising methods to evaluate the performance of robotic manipulators[6,12–14]. Most of the criterion functions assess the behavior of the manipulator locally and the Jacobian matrix $\boldsymbol{J}$ is the core of the measure. The most well known one, the *manipulability* is not adequate for the design as it is not bounded and independent of the operating point, that is *translation–invariance*[14]. In this paper, we focus on the isotropy of fingertip. The isotropy means the amount of the directional eveness of motion and accuracy of static force exertion. For this isotropic design, we propose several measures for the design of hands.

4.1 Local Measures of Isotropy

The condition number of the Jacobian matrix can be written as:[6]

$$C = \frac{\sigma_{\max}}{\sigma_{\min}}, \tag{1}$$

$$= \|\boldsymbol{J}\| \cdot \|\boldsymbol{J}^{-1}\| \tag{2}$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the largest and the smallest singular values of the Jacobian matrix, respectively and $\|\cdot\|$ denotes norm. The condition number has been used for two purposes: first, as a measure of proximity of singularity and second, as a measure of accuracy of force exertion. The condition number is independent of the scale of a manipulator and lower bounded as 1. However, it has disadvantages that it cannot be expressed analytically as a function of joint angles and unbounded in upper limits. As an another local measure of Isotropy, the measure of isotropy is defined[13]. In general, the measure of manipulability(simply manipulability)[12] is defined by

$$W = \sqrt{\det \boldsymbol{J}\boldsymbol{J}^T}, \tag{3}$$

and can also be represented as

$$W = \sqrt{\lambda_1\lambda_2\cdots\lambda_m}, \tag{4}$$

$$= \sigma_1\sigma_2\cdots\sigma_m \tag{5}$$

where m is the rank of task space and $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_m \geq 0$ are eigenvalues of $\boldsymbol{J}\boldsymbol{J}^T$ and $\sigma_1 \geq \sigma_2 \geq \cdots \geq 0$ are singular values of the Jacobian matrix. To remove the order dependency of manipulability, the manipulability can be modified as follows[13],

$$M = \sqrt[m]{\det \boldsymbol{J}\boldsymbol{J}^T}, \tag{6}$$

where M has a dimension of eigenvalue = $[\text{length}]^2$. It is a *geometric mean* of eigenvalues and represents an average measure of the easiness to move the end effector locally. Also, the measure Ψ, represents the *arithmetic mean* of eigenvalue of $\boldsymbol{J}\boldsymbol{J}^T$ and defined as follows,

$$\Psi = \frac{\text{trace}(\boldsymbol{J}\boldsymbol{J}^T)}{m}, \quad = \frac{\lambda_1 + \lambda_2\cdots\lambda_m}{m}. \tag{7}$$

The modified manipulability M was defined as the *geometric* mean of the eigenvalues of $\boldsymbol{J}\boldsymbol{J}^T$ and the measure Ψ as their *arithmetic* mean. Thus Ψ is always greater than M, and equal to M when all eigenvalues are the same. The equality of all eigenvalues implies isotropy of the m–dimensional ellipsoid(that is, m–dimensional sphere). A new measure of isotropy Δ is defined as[13]

$$\Delta = \frac{M}{\Psi} \tag{8}$$

which has an upper bound of 1 and lower bound 0. A larger Δ implies a more isotropic ellipsoid. On the contrary, a smaller condition number implies a more isotropic ellipsoid. The *minimum* value of C is one while the *maximum* value of Δ is one. If both the condition number and the measure of isotropy are equal to one (the optimal value), then the resulting designs become the same for any m-dimensional task space. Two advantages of the measure of isotropy over the condition number are:

- the measure of isotropy Δ can be expressed analytically as a function of joint angles.
- the condition number does not take into consideration the middle axis of the ellipsoid of $\boldsymbol{J}\boldsymbol{J}^T$ for a 3 or higher dimensional task space while the measure of isotropy becomes optimal when the length of the middle axis is equal to the arithmetic mean of the lengths of the major and the minor axis. For $m = 2$, the two measure have the same results.

The condition number and measure of isotropy only represent the local property and they are functions of joint variable $\boldsymbol{\theta}$. To evaluate the globally isotropic condition, two measures, *Integration of Isotropy(II)* and *Average Isotropy(AI)*, are proposed. Both of them are based on the measure of isotropy. The condition number can not be used as an adequate global design criterion function as it is impossible to express analytically and there is no upper bound and decreasing property for better isotropy condition.

4.2 Integration of Isotropy(II)

The measure of integration of isotropy evaluates the volumetric workspace weighted with the measure of isotropy. When we maximize the integration of isotropy over the workspace, we can achieves the maximization of isotropy in the whole workspace globally. It is defined as follows:

$$II = \int_R \Delta \ dV . \tag{9}$$

where R represents finger workspace and dV represents infinitesimal volume in the workspace. As the workspace or the measure of isotropy become larger II becomes larger. Its dimension is $[\text{length}]^m$.

4.3 Average of Isotropy(AI)

The measure of average of isotropy is defined as follows:

$$AI = \frac{\int_R \Delta \ dV}{\int_R \ dV}. \tag{10}$$

It is dimensionless measure and evaluates average isotropic condition over the workspace. Although the measure of isotropy is a function of joint variable $\boldsymbol{\theta}$, II and AI should be integrated over the Cartesian space. If the Cartesian space and joint space have the same dimension $m = n$, there is no problem but in redundant cases ($m < n$), mapping from joint to Cartesian space is not one to one anymore. Now, assuming that we always manipulate the fingers in the best isotropic condition for the given Cartesian space, one to one mapping can be achieved using the following condition.

$$\Delta(\boldsymbol{x}) = \max_{\boldsymbol{x}} \quad \Delta(\boldsymbol{\theta}) \tag{11}$$

The above relation makes reverse mapping from task space to joint space as one to one and AI, II can be evaluated over the workspace.

4.4 Cooperative Measure of Isotropy(CMI)

The aforementioned measures are useful for single finger. The cooperative property of multi-fingers is represented using formerly proposed global measures as follows;

$$CMI_{II} = \int_{\tilde{R}} \Delta_1 \Delta_2 \cdots \Delta_p \ dV , \tag{12}$$

$$CMI_{AI} = \frac{\int_{\tilde{R}} \Delta_1 \Delta_2 \cdots \Delta_p \ dV}{\int_{\tilde{R}} \ dV} , \tag{13}$$

$$\tilde{R} = R_1 \cap R_2 \cap \cdots R_p \tag{14}$$

where p denotes the number of fingers. Two measures for evaluating cooperative capability of multi-fingers are meaningful as the measure of isotropy is bounded from 0 to 1 and scale independent increasing function. The product of the isotropy measure of each finger over the Cartesian space shows common feature of isotropy of all the fingers and also, upper and lower bounded.

4.5 Taskspace Measure of Isotropy(TMI)

As extended form of design criterion function, *Taskspace Measure of Isotropy* is suggested. These measures represent global property over the given task space. Therefore, with this criterion function, the optimized length of phalanges can be obtained to satisfy optimal isotropy condition over the given workspace. The proposed taskspace cooperative measure of isotropy can be defined as

$$TMI_{II} = \int_T \Delta_1 \Delta_2 \cdots \Delta_p \ dV , \tag{15}$$

$$TMI_{AI} = \frac{\int_T \Delta_1 \Delta_2 \cdots \Delta_p \ dV}{\int_T \ dV} \tag{16}$$

where T denotes given task space.

5 Design of the Thumb and Index Finger

The optimization problem is to find the best combinations of phalanges of several fingers and their relative locations. From the basic assumptions, Jacobian of each finger can be easily formulated. It is almost impossible to get fully optimized solutions for all the design variables simultaneously or if possible, it takes enormous time.

Thus, we reside on the hierarchical optimization procedure. First the most frequently used finger(we think the index finger is the most useful one) is designed and the other fingers are designed with reference to this finger according to hierarchy of usefulness. Also, the distance of the other fingers are treated as another phalanges of the fingers and they are optimized with the phalanges of each finger simultaneously. For unit total length of the index finger, the lengths and locations of other finger are determined as a ratio. Therefore scale independent design parameters can be obtained. The problem is formulated in the standard form of **parametric optimal design** and it is solved using gradient projection method with constraints[15].

As an example, the thumb and the index finger design are performed according to the proposed design criteria. In this work, we model them as two fingers with planar motion neglecting radial ulna deviation motion as shown in Fig. 2. Design parameters to be determined are $\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6$ where the distance between two fingers are also modeled as another phalanx ℓ_4 and thus, the problem is how to determine the 6 finger phalanges of two cooperative 3 d.o.f planar fingers. Here, basic assumptions, heuristic constraint derived from anthropomorphism and optimization problem will be discussed.

5.1 Basic Assumptions

To simplify the formulation of the problem, the following basic assumptions are made.

- rigid-body models with point contacts with friction
- linearized kinematics(infinitesimal motion)
- quasi-static analysis (no inertial or viscous terms)
- no sliding or rolling of the fingertips.
- neglected fingertip geometry

5.2 Heuristic Constraints

We use a heuristic constraint wherein the proximal phalanges are always longer than or equal to the distal phalanges. This constraint is based on the following observations:

- the distal phalanx is more appropriate for fine motion and the proximal phalanx for gross motion
- according to the anatomical data about the range of joint motion, the distal phalanx is shorter than the proximal phalanx to avoid self-collision during wrapping of fingers.

The joint ranges are set as follows, which are based on the anatomical measurements.

- $0 \le \theta_i \le \pi$ for $i = 1, 2, 3$
- $\theta_4 = \pi$
- $\pi/2 \le \theta_5 \le \pi$
- $3/2\pi \le \theta_6 \le 2\pi$

5.3 Results & Discussions

The optimization was performed for the two finger model as shown in Fig. 2. First, the phalangeal ratio of the index finger are determined for the unit total length. Then the ratio of total length of the thumb to the index finger and the ratios of each phalanx are obtained. As shown in Table 1, the phalangeal ratio of individual fingers by Youm[3] is 2 : 2.82 : 4.89 which follows almost Fibonacci sequence and the ratio is 2 : 3 : 4.5 in the case of Hogarth[2]. Although the phalangeal ratios of both cases are a little different, they nearly follow the most natural forms and the trends are similar to each other. The obtained optimized results included in Table 1 show that the middle and distal phalanges are the same and the sum of these two lengths is equal to that of proximal phalanx. It is the initial part of Fibonacci sequence(1 : 1 : 2 : 3 : $\cdots$) similar to those of Youm and Hogarth. This result comes from the maximization effect of workspace in the criterion function. Another point of the results is the ratio between the total length of index finger and the thumb, which is called the opposition index[1]. According to Napier[1], the opposition index is 0.65 in the case of humans while the lower primates exhibit mean indices as low as 0.45. The opposition indices from optimization procedures are 0.73 for CMI_{II} and 0.45 for CMI_{AI}. It shows that the opposition index of the optimized design is very similar to that of anatomical observations. Moreover, it can be suggested that the total length of the thumb is less than that of the index finger and actually, the opposition index be set approximately from 0.4 to 0.75 in finger design. As for the relative location of the fingers, we can get some useful information which has not been reported anatomically and it was hard to determine in actual engineering design. Figs. 3 and 4 show the workspaces of the thumb and index finger with optimized phalangeal lengths for the measures of CMI_{II} and CMI_{AI}. The results of CMI_{AI} suggest narrower workspace than those of CMI_{II}. The design based on CMI_{AI} can give more isotropic one in a smaller workspace but the design based on CMI_{II} gives broader workspace with modest isotropy. In the taskspace design, the optimized design for the specified workspace was performed(here, it was given as a circle with radius 0.2 and center $(-0.05, 0.5)$ for this example). Table 2 shows the results of taskspace design and it may be a useful design method to get optimized designs for user defined workspaces.

In fact, the human motion is the combination of several grasp patterns and thus, the more design factors and constraints are included in design procedure, the more similar results may be obtained to the anatomical data.

6 Conclusion

A design concept of anthropomorphic hand is proposed. Several global isotropic measures were proposed for the design of hands and the results show that the link lengths of the index finger follows the initial part of the Fibonacci sequence. The opposition index was calculated and it shows similarity to those of anatomical data. Moreover, we could suggest how to determine the relative location between fingers.

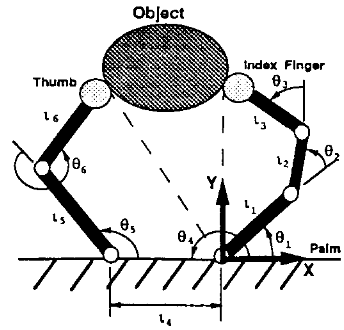

Fig. 2 The design model of the thumb and the index finger.

Table. 1 Anatomical data and design of finger phalanges based on the measure of CMI

	Youm	Hogarth	CMI_{II}	CMI_{AI}
ℓ_1	0.504	0.474	0.45	0.5
ℓ_2	0.290	0.316	0.275	0.25
ℓ_3	0.206	0.21	0.275	0.25
ℓ_4	·	·	0.22	0.5
ℓ_5	·	·	0.37	0.25
ℓ_6	·	·	0.36	0.2

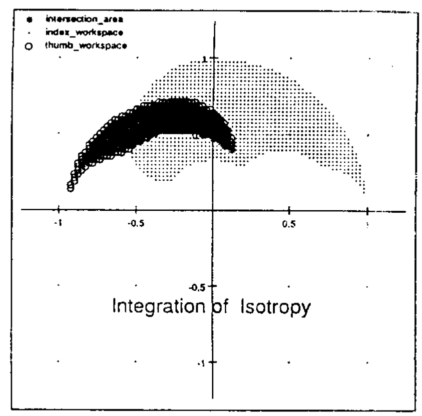

Fig. 3 The workspaces of the thumb and the index finger by the measure of CMI_{II}.

Table. 2 Design of finger phalanges based on the measure of TMI

	TMI_{II}	TMI_{AI}
ℓ_1	0.475	0.525
ℓ_2	0.263	0.238
ℓ_3	0.263	0.238
ℓ_4	0.220	0.588
ℓ_5	0.400	0.363
ℓ_6	0.360	0.363

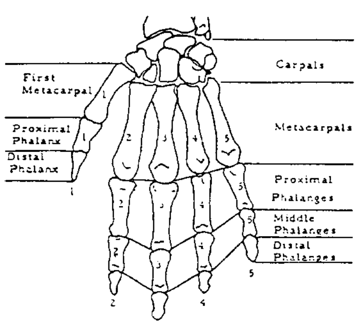

Fig. 1 The bones of human hand.

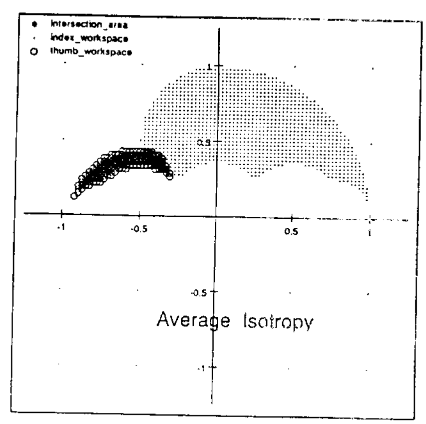

Fig. 4 The workspaces of the thumb and the index finger by the measure CMI_{AI}.

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