Magnetic Levitation Control by Attracrtive Force Compensation

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ABSTRACT

This paper presents a procedure to design a real time control system for a magnetic levitation system based on the state space approach by adopting a control method compensating attractive force according to load variation of maglev vehile.Also the paper has realized a robust control algorithm for the change of self-inductance parameters and the disturbance such as the change of mass of Maglev vehicles.The theoretical results are applied to the gap control preblem of an attractive-type-magnetic levitation system and the effectiveness is proved by the implementation of digital control using 16 bits microcomputer.

1. Introduction

Maglev vehicles are under consideration for rapid transit as well as for high speed transportation in several countries.Due to the properties of electromagnets,the guidance and support magnets must be controlled because the system, without control, is unstable. Since some of the magnet parameters may be uncertain,and there may also occur input disturbances, e.g. irregularities of the elevated guideway, an insensitive control system with respect to uncertainties is required between the support magnet and the guideway.
The control of attractive magnetic levitation system has been studied by T.Hoshino et al.(1979),M.Morishita and T.Ide(1983), and Yamamura et al.(1974,1979). A single magnet has mainly been considered in their theoretical studies. T.Chikada and K.Furuta et al.(1982) treated two magnets system to levitate two iron plates connected by acrylic acid resin bar and they achieved the control object by using a digital computer based on CAD package. S.B.Kim and N.S.Jeong(1992) have applied a concept of the robust control of LQ state feedback regulators with poles in a specified region in the presence of system uncertainty to an attractive -type-magnetic levitation system.

This paper presents a procedure to design a real time control system for a magnetic levitation system based on the state space approach by adopting a control method compensating attractive force according to load variation of maglev vehicle. We can see from experiment results that a robust control for the change of self-inductance parameters and the disturbance such as the mass change of maglev vehicles has been realized.

2. System Description and Mathematical Model

2.1 System Description

A schematic diagram of the electro-magnetic levitation system is described as shown in Fig.1 where an iron bar is fixed on the acrylic acid resin plate.The objective of the control is to design a robust controller such that the iron amature follows the gap reference under the presence of unmeasurable disturbance. In reality,it is very difficilt to carry out the experiment making use of the actual magnetic levitation system. Accordingly, this study is to carry out the experiment by making a model as shown in Fig.1;

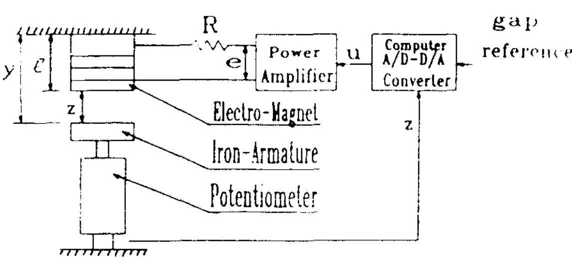

Fig.1 A schematic diagram of electro-magnetic levitation system

2.2 Mathematical Model

Using the parameter symbols shown in Fig.1, the mathematical model of the magnetic levitation system is given as follows(S.B.Kim et al,1992);

$$m\frac{d^2z}{dt^2} = mg - f_m + \tau_L \tag{1}$$

$$f_m = \alpha\left(\frac{i}{z}\right)^2 \tag{2}$$

$$e = Ri + \frac{d}{dt}(L(z)\, i) \tag{3}$$

$$e = -k_o u \tag{4}$$

where m : mass
f_m: magnetic force
α: exchange coefficient between current and attractive force
k_o: gain of preamplifier
e : coil voltage
u : control input of power amplifier
$L(z)$: self inductance
τ_L : Load variation

Linealizing eqs. (1)-(4) at the equilibrium point of $z = z_o$, $i = i_o$ and using the following equation for the self inductance L(x) (T.Hoshino et al.1979)

$$L(z) = L_a + \frac{L_b}{z} \quad (L_a, L_b : \text{constant values}) \tag{5}$$

we have the linearized model equation :

$$f_m = \tau_L - m\frac{d^2z}{dt^2} \tag{6}$$

$$i = \frac{f_m}{\beta} + \gamma z \tag{7}$$

$$e = Ri + L_o\frac{di}{dt} - (L_o - L_a)\,\gamma\frac{dz}{dt} \tag{8}$$

$$e = -k_o u \tag{9}$$

where, $\beta = \dfrac{2f_{mo}}{i_o}$

$\gamma = \dfrac{i_o}{x_o}$

We can express eqs. (6)-(9) as a 3rd order differential equation;

$$\frac{d^3z}{dt^3} = \frac{R\gamma\beta}{mL_o}z + \frac{\beta L_a\gamma}{mL_o}\frac{dz}{dt} - \frac{R}{L_o}\frac{d^2z}{dt^2} + \left(\frac{R}{mL_o}\tau_L + \frac{1}{m}\frac{d\tau_L}{dt}\right) + \frac{\beta k_o}{mL_o}u \tag{10}$$

If the samling time is significantly short comparing to the load change τ_L, we may assume:

$$\frac{d\tau_L}{dt} = 0 \tag{11}$$

Then, we can obtain the following state equation:

$$\frac{dx}{dt} = Ax + Bu \tag{12}$$

$$y = Cx$$

$$x = \begin{bmatrix} z \\ \frac{dz}{dt} \\ \frac{d^2z}{dt^2} \\ \tau_L \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{Rr\beta}{mL_o} & \frac{\beta L_a r}{mL_o} & -\frac{R}{L_o} & \frac{R}{mL_o} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{\beta k_o}{mL_o} \\ 0 \end{bmatrix} \qquad C = [\,1 \;\; 0 \;\; 0 \;\; 0\,]$$

The system parameters of this experimental unit are as shown in Table 1.

Table 1 Parameters of electro-magnetic levitation system

Symbols	Description	Measured values
m	mass of iron amature and potentiometer core	0.085 [Kg]
R	resistance	3 [Ω]
z_o	gap position at equilibrium point	0.4 [cm]
i_o	current at equilibrium point	1.05 [A]
ℓ	length of magnetic iron bar	9.6 [cm]
L_a	coefficient of L(z)	0.00112[H]
L_b	coefficient of L(z)	0.193[H·cm]
L_o	self inductance at equilibrium point	0.483[H]
k_o	gain of pre-amplifier	2.94

3. Control System Design

3.1 Observer design for estimation of load change

Since load change is not measurable, we need an observer for estimating it. So, according to the state equation, it can be estimated by the minimal order state observer using Gopinath's design method. (K.Furuta et al,1988)
In the minimal order state observer, its state equation may be written as

$$\dot{m} = \hat{A}m + \hat{B}y + \hat{J}u\,, \quad m(0) = 0 \tag{13}$$

$$\hat{x} = [\,\hat{C}\;\;\hat{D}\,]\begin{bmatrix} m \\ y \end{bmatrix}$$

where $\hat{A} \in R^{(n-p)*(n-p)}$, $\hat{B} \in R^{(n-p)*p}$, $\hat{J} \in R^{(n-p)*n}$.
It is well known that the n-p dimensional observer (14) is the minimal order state observer if and only if it satisfies the following conditions (K.Fututa et al,1988);
① there exist a $U \in R^{(n-p)*n}$ satisfying

$$UA - \hat{A}U = \hat{B}C$$
$$\hat{J} = UB$$
$$\hat{C}U + \hat{D}C = JU$$

② All the eigenvalues of $\hat{A}$ have negative real parts.

3.2 Servo compensator design (S.B.Kim et al,1991)

Consider a time-invariant continuous system:

$$\dot{x}(t) = Ax(t) + Bu(t) + \delta(t), \quad x \in R^n, u \in R^m \tag{14}$$

$$y(t) = C\,x(t), \quad y \in R^p$$

where $\delta(t)$ is a disturbance vector satisfying

$$P_d(D)\,\delta(t) = 0 \tag{15}$$

where D is a differential operator meaning $D = \frac{d}{dt}$

For the reference input $y_r(t)$ satisfying

$$P_r(D)y_r(t) = 0 \quad (16)$$

define the following error vector:

$$e(t) = y(t) - y_r(t) \quad (17)$$

In order to consider the problem such that the error vector becomes zero for $t \to \infty$, define the least common multiple polynomial of the form:

$$P(D) = \ell.c.m\,(P_d(D), P_r(D))$$
$$= D^{(q)} + \alpha_{q-1}D^{(q-1)} + \cdots + \alpha_o \quad (18)$$

Since the reference input $y_r(t)$ satisfies :

$$P(D)y_r(t) = 0 \quad (19)$$

operating P(D) to the error vector (20), we can obtain :

$$e^{(q)} + \alpha_{(q-1)}e^{(q-1)} + \cdots + \alpha_o e$$
$$= CP(q^{-1})x(t) \quad (20)$$

which has the state equation:

$$\dot{z}(t) = Nz(t) + MP(D)x(t) \quad (21)$$

where $z = [\ e\ e^1 \cdots e^{(q-1)}\ ]^T$

$$N = \begin{bmatrix} 0 & 1 & 0 & \cdots\cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & & & & \\ 0 & & & 0 & 1 \\ -\alpha_o & -\alpha_1 & \cdots & -\alpha_{q-2} & -\alpha_{q-1} \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C \end{bmatrix} \quad (21)$$

Also, operating P(D) to eq.(14) and using $P(D)\delta(t) = 0$, we can obtain:

$$P(D)\dot{x}(t) = AP(D)x(t) + BP(D)u(t) \quad (22)$$

Then, the extended state equation for eqs.(20) and (22) can be written as :

$$\dot{x}_e(t) = A_e x_e(t) + B_e v(t) \quad (23)$$
$$e(t) = C_e x_e(t)$$

where, $x_e(t) = [P(D)x^T(t)\ z^T]^T$, $v = P(D)u$

$$\Phi_e = \begin{bmatrix} A & 0 \\ M & N \end{bmatrix}, \quad \Gamma_e = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad C_e = [0\ [1\ 0 \cdots 0\ ]]$$

Assume that the control law v(t) of eq.(23) is given by

$$V = P(D)u(t) \quad (24)$$

Defining the servo compensator variable as the following :

$$\zeta(t) = P(D)z(t) \quad (25)$$

then the control law can be given by

$$u(t) = F[x^T\ \zeta^T]^T = [F_1\ F_2][x^T\ \zeta^T]^T \quad (26)$$

where the feedback matrix F can be chosen by an appropriate design method, and the resultant compensator is thus :

$$\dot{\zeta}(t) = N\zeta(t) + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} e(t) \quad (27)$$

The closed-loop system with the control law (26) and servo compensator (27) is shown in Fig.2.

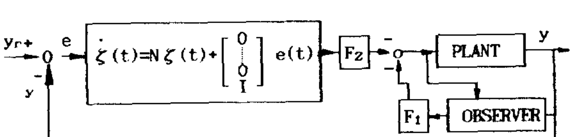

Fig.2 Block Diagram of servo control system

The resultant closed-loop system based on the servo compensator and the minimal order state observer can be composed by the block diagram shown in Fig.3

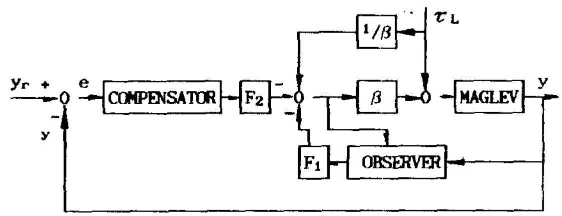

Fig.3 Servo control system with compensation of attractive force variation

4. Experimental Result

(1) Servo Compensator

The extended state equation (12) at the point of equilibrium (z_o=0.4[cm], i_o=1.05[A]) can be obtained as follows:

$$\frac{dx_e}{dt} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 30434.8 & 12.2 & -6.211 & 0.07 \\ 1 & 0 & 0 & 0 \end{bmatrix} x_e + \begin{bmatrix} 0 \\ 0 \\ 9584 \\ 0 \end{bmatrix} v \quad (28)$$

Using the well-known pole assignment method, we can obtain the following feedback law:

$$F = [\ -7.161304{+}D0\ \ -0.023466{+}D0\ \ -0.009263{+}D0\ \ -1.2519219{+}D1\ ] \quad (29)$$

where, the given closed-loop poles are as follows:

$$\lambda_i : [\ -2.93782 \pm j15.59822 \quad -4.567665 \quad -6.98903D{+}01\ ] \quad (30)$$

(2) Design of Observer

We can obtain the minimal order state observer using Gopinath's design method as the following:

$$\frac{dm(t)}{dt} = \hat{A}m(t) + \hat{B}y(t) + \hat{J}u(t) \quad (31)$$
$$\hat{x}(t) = \hat{C}m(t) + \hat{D}y(t) \quad (32)$$

(a) in the case of poles : $-35 \pm j5$, $-10{+}j0$

$$\hat{A} = \begin{bmatrix} -63.789 & 1 & 0 \\ -853.80 & -6.211 & 0.07 \\ -178571.4 & 0 & 0 \end{bmatrix} \quad \hat{B} = \begin{bmatrix} -3203.03 \\ -29407.4307 \\ -13176607.7 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \hat{D} = \begin{bmatrix} 1 \\ 63.789 \\ 866.006521 \\ 178571.429 \end{bmatrix} \quad \hat{J} = \begin{bmatrix} 0 \\ \\ 6733.5 \\ 0 \end{bmatrix}$$

(b) in the case of poles : $-30 \pm j10$, $10{+}j0$

$$\hat{A} = \begin{bmatrix} -53.789 & 1 & 0 \\ -665.9165 & -6.211 & 0.07 \\ -142857.14 & 0 & 0 \end{bmatrix} \quad \hat{B} = \begin{bmatrix} -2215.14 \\ -9595.96546 \\ -9112714.29 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \hat{D} = \begin{bmatrix} 1 \\ 53.789 \\ 678.116521 \\ 142857.143 \end{bmatrix} \quad \hat{J} = \begin{bmatrix} 0 \\ \\ 6733.5 \\ 0 \end{bmatrix}$$

In order to implement the digital control, digitalizing the observer of eqs.(31) and (32) ,we have the following discrete observer :

$$\mathbf{m}(k+1)=\hat{A}_D\mathbf{m}(k)+\hat{B}_Dy(k)+\hat{J}_Du(k) \qquad (33)$$

$$\hat{x}(k)=\hat{C}_D\mathbf{m}(k)+\hat{D}_Dy(k) \qquad (34)$$

(c) in the case of poles : $-35\pm j10$, $-10+j0$

$$\hat{A}=\begin{bmatrix} 0.7183 & 0.00419 & 7.66\text{-D7} \\ -3.5832 & -0.96001 & 3.42\text{-D4} \\ -742.03 & -1.95426 & 9.99\text{-D1} \end{bmatrix} \qquad \hat{B}=\begin{bmatrix} -13.99147 \\ -113.8568 \\ -57837.45 \end{bmatrix}$$

$$\hat{C}=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad \hat{D}=\begin{bmatrix} 1 \\ 63.789 \\ 866.0065 \\ 178571.42 \end{bmatrix} \qquad \hat{J}=\begin{bmatrix} 0.0749 \\ \\ 33.0411 \\ -32.279 \end{bmatrix}$$

(d) in the case of poles : $-30\pm j10$, $-10+j0$

$$\hat{A}=\begin{bmatrix} 0.7572 & 0.00430 & 7.78\text{-D7} \\ -2.86460 & -0.96196 & 3.43\text{-D4} \\ -608.513 & -1.58919 & 9.99\text{-D1} \end{bmatrix} \qquad \hat{B}=\begin{bmatrix} -9.793230 \\ -30.42873 \\ -40859.24 \end{bmatrix}$$

$$\hat{C}=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad \hat{D}=\begin{bmatrix} 1 \\ 53.789 \\ 678.1165 \\ 142857.14 \end{bmatrix} \qquad \hat{J}=\begin{bmatrix} 0.0761 \\ \\ 33.0640 \\ -26.141 \end{bmatrix}$$

(3) Experimental Results

Fig.4,Fig.5 and Fig.6 show the exprimental results of the magnetic levitation system by applying the control algorithm of section 3.

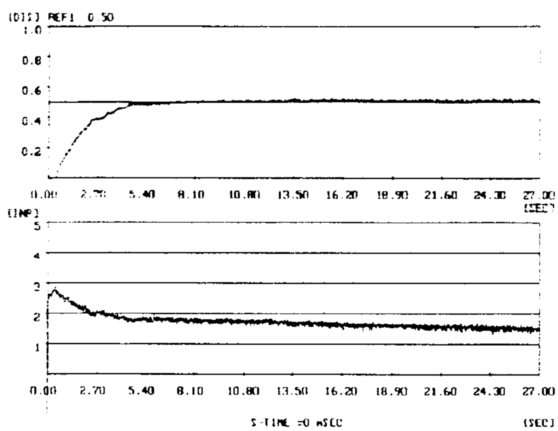

Fig.4 Experimental result for step reference

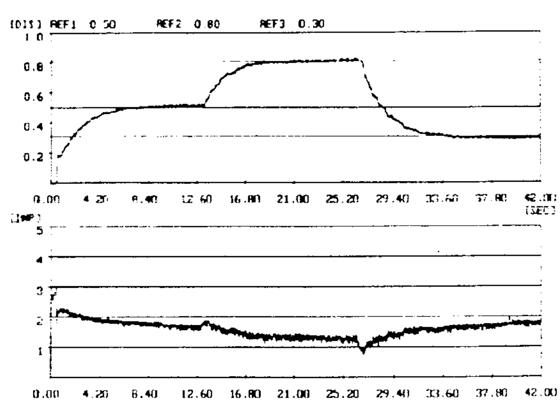

Fig.5 Experimental result for disturbance

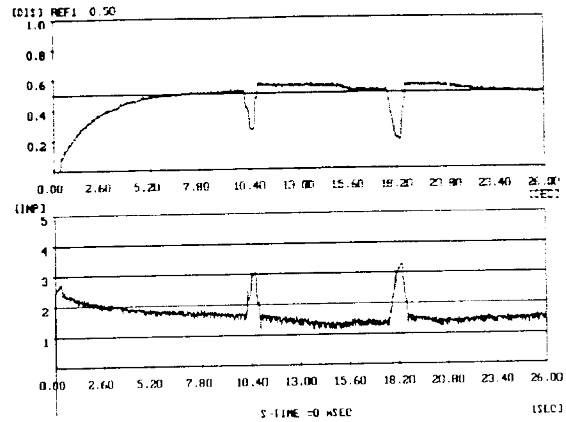

Fig.6 Experimental results for exchange reference

It can be observed from the exprerimental results that the magnetic levitation system is robust for the parameter perturbation, reference change and disturbance such as arbitrary load change.

5. Conclusion

The paper presents a procedure to design a real time control system for a magnetic levitation system based on the state space approach by adopting a control method compensating attractive force according to load variation of maglev vehicle. The adopted control method is implemented using 16-bits microcomputer and from the exprerimental results we can see that the magnetic levition system is robust for parameter perturbation, reference change and arbitrary disturbance.

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