

A Study on optimal tuning method of hybrid Controller

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Abstract

In the paper, an optimal tuning algorithm is presented to automatically improve the performance of a hybrid controller, using the simplified reasoning method and the proposed complex method. The method estimates automatically the optimal values of the parameters of a hybrid controller, according to the change rate and limitation condition of output. The controller is applied to plants with time-delay. Then, computer simulations are conducted at step input and the performances are evaluated in the ITAE.

1. Introduction

The aim of designing the controllers is to compensate the dynamic characteristics of the plant controlled. In this point of view, because of simplicity of the parameter tuning and the design, the PID controller is established very well and highlighted as one of widely used methods. But the conventional PID controller with linear relation to plants is sensitive to the control environment and the changes of the parameter, and it can not expect good results from the complex and nonlinear plants.

Because a fuzzy logic controller can use a large number of the linguistic control rules based on the experiences and the knowledge of human, it has been proved that the controller should be so designed as to well control not only the linear plants but also the

nonlinear plants. But it is difficult for the linguistic control rules to express perfectly the experience and knowledge of human. Furthermore, another difficult problem is to choose the parameters of the fuzzy logic controller, so that they may be to make the control performances better.

On the other hand, the PID controller is superior to the fuzzy logic controller in steady state and the fuzzy logic controller is superior to the PID controller in transient state. The hybrid controller that combines merits of fuzzy logic and PID controllers is considered. The control performance of the hybrid controller depends on its fuzzy parameters and PID parameters. Designing a hybrid controller, one of the most difficult problems is to estimate the parameters of the hybrid controller so that they may be to make the control performances better. Therefore, The new algorithm to make optimally autotuning the parameters like the parameters of the hybrid controller, through the response analysis of the plant, is required vigorously.

In this paper, an optimal tuning algorithm is presented to automatically improve the performance of a hybrid controller, using the simplified reasoning method and the proposed complex method. The algorithm estimates automatically the optimal values of the linguistic control rules, the scaling factors, the

membership functions and weighting coefficient of the hybrid controller, according to the change rate and limitation condition of output. The algorithm is introduced to hybrid(fuzzy PID + PID, etc.) and hybrid controller with Smith-predictor, and applied to 1 and 2 order plants with time - delay. Then, computer simulations are conducted at step input and the performances are evaluated in the ITAE.

## 2. Hybrid controller

The hybrid controller consists of a fuzzy PID controller and a PID controller. The principal elements are scaling factors, membership functions, weighting coefficient and PID coefficients. The block diagram of hybrid controller is shown in figure 1.

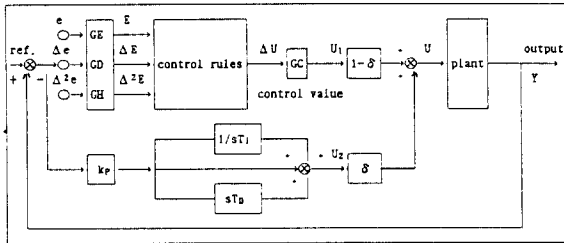


Fig.1 The scheme of hybrid controller

The fuzzy controller with linguistic control variables consists of the N control rules which are implemented by the fuzzy logic implications as eqn. (1).

$$R_k : \text{IF } E \text{ is } A_k, \Delta E \text{ is } B_k \text{ and } \Delta^2 E \text{ is } C_k. \\ \text{Then } \Delta U \text{ is } D_k \quad (1)$$

where  $R_k$  : k-th control rule,  $(k=1,2, \dots, N)$

$N$  : the number of control rules

$E$  : error

$\Delta E$  : change of error

$\Delta^2 E$  : change of variation error

$\Delta U$  : change of plant control input

$A_k, B_k, C_k$  and  $D_k$  : linguistic variables

If  $E^0, \Delta E^0$  and  $\Delta^2 E^0$  are substituted for the fuzzy variables  $E, \Delta E$  and  $\Delta^2 E$  of antecedent, true value of antecedent in each rule is like eqn. (2).

$$W_i = \min\{\mu_{A_i}(E^0), \mu_{B_i}(\Delta E^0), \mu_{C_i}(\Delta^2 E^0)\} \quad (2)$$

If the membership function  $D_i$  of consequent is not fuzzy set but singleton, the inferential value of eqn. (1) is simplified like eqn.(3), using the simplified reasoning method.

$$\Delta U = \frac{\sum_{i=0}^n W_i * D_i}{\sum_{i=0}^n W_i} \quad (3)$$

The PID controller consists of a conventional one with  $k_p, k_i$  and  $k_d$ . PID coefficients is tuned with parameters of fuzzy controller.

The weighting coefficient  $\delta$  is assumed as a fuzzy variable.  $\delta$  is tuned with parameters of fuzzy controller and decides weight.

In the paper, all the parameters is automatically estimated and optimized by improved complex method. Optimal parameters is applied to hybrid controller.

## 3. Autotuning by improved complex method

Consider the optimal control to make the error minimum, using ISE or ITAE which shows an error characteristic of the control response to the step input and be a cost function to evaluate the optimal tuning state.

Hybrid controller also has an object to minimize ISE or ITAE as cost function. But, as we regulate the scaling factors, weighting coefficient, membership functions and PID coefficients in order to minimize ISE or ITAE, the cost function of hybrid controller has the nonlinear dynamic characteristics that can not be formulated. Also hybrid controller has a problem to apply general optimal techniques, because it is difficult to obtain the cost function and differential of ISE or ITAE.

In order to solve the problems, the autotuning algorithm using improved complex method, a kind of nonlinear program that abstract scaling factors, weighting coefficient, membership functions and PID coefficients for the minimum error, is suggested

The variables of cost function are given by scaling factors, membership functions, weighting coefficient and PID coefficients. After we select ITAE as cost function, we try to minimize the cost function at the step input. Since ISE is also a single optimal value, it can be chosen as the value of the cost function. But, even if ISE satisfies the minimum values, the optimal parameters of ISE are somewhat different from those of ITAE. Hence overshoot and reaching time etc. are a little different. When the difference is small, low-order plant can use ISE or ITAE as the cost function. However, as the difference is relatively big in the high-order plant, ISE or ITAE is chosen according to the object of control.

The scheme of system to autotune all the parameters is like figure 2. After the control output is calculated in off-line, ITAE is obtained. Such a series of values are repeatedly calculated by improved complex method until the standard deviation of ITAE is smaller than some prescribed small quantity. The parameters of optimal ITAE is stored as the new parameters of scaling factors, weighting coefficient, membership functions and PID coefficients.

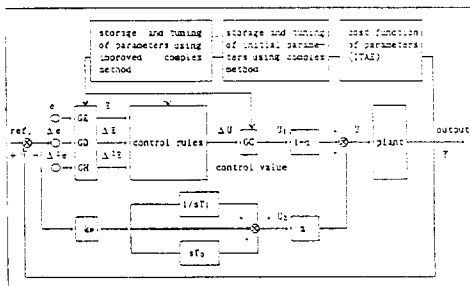


Fig.2 The scheme of autotuning hybrid controller

We realize the algorithm to expand the simplex concept - constrained optimization technique - to the complex method as follows.

<step 1>

The set of the initial values for the parameters is prepared more than the number of variables, arbitrarily. The parameters mean scaling factors,

weighting coefficient and membership functions. They are defined as  $X_k = (x_1^k, x_2^k, \dots, x_n^k; k = 1, 2, \dots, n, n+1, \dots, m)$  in  $n$  dimension space.

<step 2>

The initial values of  $\alpha$ ,  $\beta$  and  $\gamma$  is specified using the Reflection, Expansion and Contraction of simplex concept as follows:

$$i) \text{ Reflection : } X_r = X_o + \alpha(X_o - X_h) \quad (4)$$

$$ii) \text{ Expansion : } X_e = X_o + \gamma(X_r - X_o) \quad (5)$$

$$iii) \text{ Contraction : } X_c = X_o + \beta(X_h - X_o) \quad (6)$$

<step 3>

$X_h$  and  $X_l$  are the vertices corresponding to the maximum function value  $f(X_h)$  and the minimum function value  $f(X_l)$ .  $X_o$  is the centroid of all the points  $X_i$  except  $i=h$ . Reflected point  $X_r$  is given by  $X_r = X_o + \alpha(X_o - X_h)$ . If  $X_r$  may not satisfy all the constraints, a new point  $X_r$  is generated by  $X_r = (X_o + X_r)/2$ . This process is conducted repeatedly until  $X_r$  satisfies all the constraints. A new simplex is started.

<step 4>

If a reflection process gives a point  $X_r$  for which  $f(X_r) < f(X_l)$ , i.e. if the reflection produces a new minimum, we expand  $X_r$  to  $X_e$  by  $X_e = \gamma X_r + (1-\gamma)X_o$ . If  $X_e$  may not satisfy all the constraints, a new point  $X_e$  is generated by  $X_e = (X_o + X_e)/2$ . This process is conducted repeatedly until  $X_e$  satisfies all the constraints. If  $f(X_e) < f(X_l)$ , we replace the point  $X_h$  by  $X_e$  and restart the process of reflection. On the other hand, if  $f(X_e) > f(X_l)$ , we replace the point  $X_h$  by  $X_r$ , and start the reflection process again.

<step 5>

If the reflection process gives a point  $X_r$  for which  $f(X_r) > f(X_l)$ , for all  $i$  except  $i=h$ , and  $f(X_r) < f(X_h)$ , then we replace the point  $X_h$  by  $X_r$ . In this case, we contract the simplex as follows:  $X_c = \beta X_h + (1-\beta)X_o$ . If  $f(X_r) > f(X_h)$ , we will use  $X_c = \beta X_h + (1-\beta)X_o$  without changing the previous point  $X_h$ . If  $X_c$  may not satisfy all the constraints, a new point  $X_e$  is

generated by  $X_o = (X_0 + X_o)/2$ . This process is conducted repeatedly until  $X_o$  satisfies all the constraints. If the contraction process produces a point  $X_c$  for which  $f(X_c) < \min[f(X_h), f(X_r)]$ , we replace the point  $X_h$  by  $X_c$  and proceed with the reflection again. On the other hand, if  $f(X_c) = \min[f(X_h), f(X_r)]$ , we replace all  $X_i$  by  $(X_i + X_1)/2$ , and start the reflection process again.

(step 6)

The method is assumed to have converged whenever the standard deviation of the function at the vertices of the current simplex is smaller than some prescribed small quantity  $\epsilon$  like eqn.(7).

$$Q = \left\{ \frac{\sum_{i=1}^{n+1} [f(X_i) - f(X_o)]^2}{n+1} \right\}^{1/2} \leq \epsilon \quad (7)$$

If Q may not satisfy eqn.(7), we go to step 3.

#### 4. Computer simulation and results

To evaluate the performances and characteristics of hybrid controller with optimal autotuning algorithm, the plants with time-delay is given in eqn. (8)-(9). Computer simulation is conducted at the step input of sampling time 0.5[s]. We analyze the various cases of fuzzy PID and hybrid controllers in examples. Table 1 and figure 3 are initial linguistic control rule and membership functions using in examples.

$$\text{PLANT 1 : } \begin{aligned} Y(s) &= \frac{e^{-2s}}{s+1} \\ U(s) & \end{aligned} \quad (8)$$

$$\text{PLANT 2 : } \begin{aligned} Y(s) &= \frac{e^{-0.8s}}{(s+1)(s+2)} \\ U(s) & \end{aligned} \quad (9)$$

Table 1. linguistic control rules for 3-fuzzy variables

(a) $\Delta^2 E = N$				(b) $\Delta^2 E = Z$				(c) $\Delta^2 E = P$			
	N	$\Delta E$	P		N	$\Delta E$	P		N	$\Delta E$	P
N	NB	NB	NM	N	NB	NM	NS	N	NM	NS	ZE
E	Z	NM	NS	ZE	E	Z	PS	E	Z	PS	PM
P	ZE	PS	PM	P	PS	PM	PB	P	PM	PB	PM

The results are like figures 4-5.

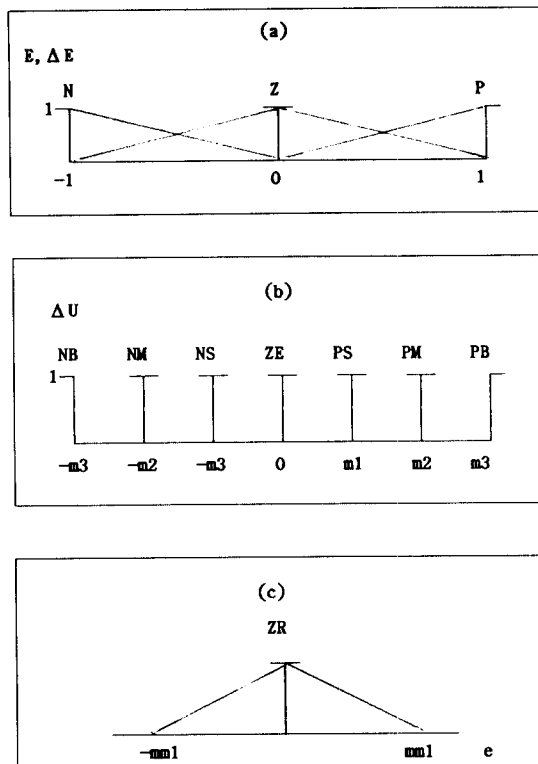


Fig.3 (a),(b) : Membership functions of linguistic control rule  
(c) : Membership function of weighting coefficient

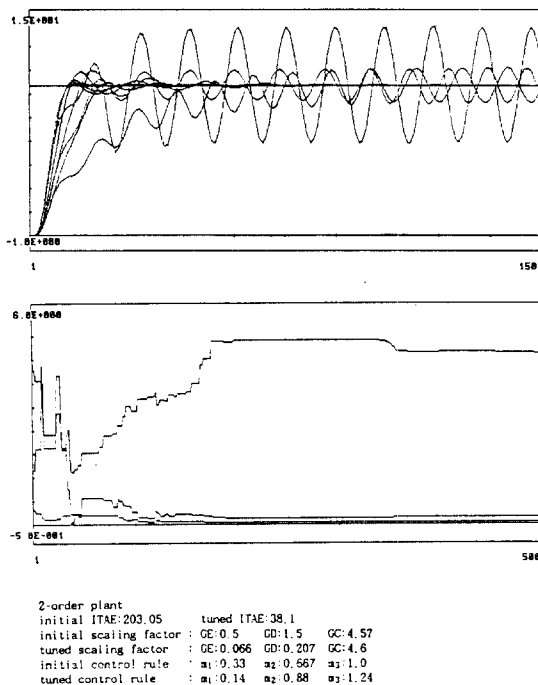
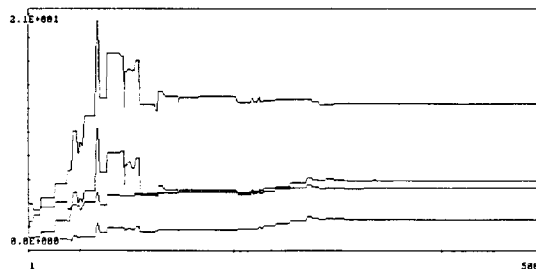
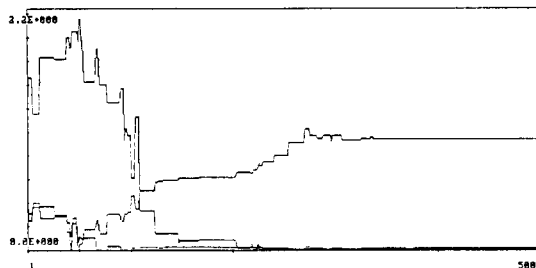
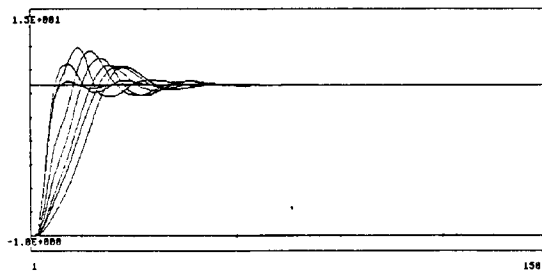


Fig. 4 Output of plant 2 with fuzzy PID controller



2-order plant  
 initial ITAE: 84.5951    tuned ITAE: 32.215  
 initial scaling factor : GE: 0.41    GD: 0.45    GH: 4.51    GC: 1.6  
                                       Kp: 0.1    Ki: 0.145    Kd: 1.15  
 tuned scaling factor : GE: 2.7125    GD: 5.4644    GH: 6.1250    GC: 12.8407  
                                       Kp: 1.0137    Ki: 0.0154    Kd: 0.0193  
 initial control rule : a1: 0.33    a2: 0.88    a3: 1.0  
 tuned control rule : a1: 0.0861    a2: 0.6068    a3: 1.0642

Fig. 5 Output of plant 2 with hybrid controller

### 5. Conclusions

This paper presents an optimal algorithm to autotune the scaling factors, membership functions, PID coefficients and weighting coefficient, using the proposed complex method. The method is applied to the time-delay plants of hybrid controller.

Some results are drawn from computer simulation as follows:

1. The scaling factors converge to the optimal values, according to adjusting the scaling factors through the proposed method, iteratively.
2. It is easy to autotune the linguistic control rules

factors as the initial values.

3. Because the optimal parameters are tuned automatically, under the change rate and limitation condition of output, the proposed algorithm be applied to the real plant.

4. The optimal parameters are obtained by not only the determination of the initial parameters (as Chien Hrones Reswrk and Cohen Coon methods), but also the choice of the initial ill-condition, using the proposed algorithm.

5. The response of plants with time delay is more than the conventional PID at the step input. Especially, the hybrid controller with Smith-predictor is excellent.

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