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Fault Diagnosis Based on Likelihood Decomposition

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Abstract. A novel fault diagnosis method based on likelihood decomposition is proposed for linear stochastic systems described by autoregressive (AR) model. Assuming that at some time instant τ the fault of one of the following two types is occurs: innovation fault (actuator fault); and observation fault (sensor fault), the log-likelihood function is decomposed into two components based on the observations before and after τ, respectively, Then, the type of the fault is determined by comparing the log-likelihoods corresponding two types of faults. Numerical examples demonstrate the usefulness of the proposed diagnosis method.

Keywords. Fault diagnosis; likelihood; actuator fault; sensor fault; autoregressive model.

1. Introduction

Recently, design of detection system for abrupt changes or fault in dynamical systems has been receiving much attentions motivated by a wide variety of applications. A significant amount of statistical design techniques such as sequential probability ratio test (SPRT), generalized likelihood ratio (GLR) test, and their modifications are available for quick detection of system faults (see survey papers, [1], [4], [5], [8], [9], [12], and a book [2]). Once it is found that a fault has occurred, the next step is to determine what kind of fault actually has occurred, i.e., the stage of fault diagnosis. For this object, multiple model (MM) approach [12] and GLR approach [2] can be applied, for example. In this paper, we propose a novel fault diagnosis method based on likelihood decomposition for linear stochastic systems described by autoregressive (AR) model. We assume that at some time instant τ the fault of one of the following two types is occurs: innovation fault (actuator fault); and observation fault (sensor fault) [7]. We first decompose the log-likelihood function $\mathcal{L}$ into two components based on the observations before and after τ, respectively, Then, we determine that the fault is innovation fault or observation fault by comparing the log-likelihoods corresponding two types of faults. Numerical examples demonstrate the usefulness of the proposed diagnosis method.

2. Problem Statement

Consider a system described by an autoregressive (AR) model

$$x_t = \sum_{k=1}^{p} a_k x_{t-k} + v_t = \boldsymbol{a}^T \boldsymbol{x}_{t-1}^{t-p} + v_t \tag{1}$$

with observation system

$$y_t = x_t + w_t \tag{2}$$

where $\{v_t\}$ and $\{w_t\}$ are mutually independent normal random sequences with mean 0 and variance σ_{I0}^2 and σ_{O0}^2, respectively, and $\boldsymbol{a} = (a_1, a_2, \ldots, a_p)^T$ and $\boldsymbol{x}_{t-1}^{t-p} = (x_{t-1}, x_{t-2}, \ldots, x_{t-p})^T$. We assume that, at some time instant τ, a fault occurs either in the system (1) itself or in the observation system (2). The fault in the system (1) is characterized by the change of the mean and variance of innovations such that

$$v_t \sim N(\mu_I, \sigma_{I0}^2 + \sigma_{I1}^2), \quad \mu_I^2 + \sigma_{I1}^2 \neq 0, \quad t \geq \tau \qquad \text{(innovation fault (actuator fault))} \tag{3}$$

and the fault in the observation system (2) is characterized by a change of the mean of observation noise, i.e.,

$$w_t \sim N(\mu_O, \sigma_{O0}^2), \quad \mu_O \neq 0, \quad t \geq \tau \quad \text{(observation fault (sensor fault))} \tag{4}$$

The problem is to determine which type of fault actually occurred based on the observation sequences of $\{y_t, 1 \leq t \leq n\}$. Here we assume $1 \leq p < \tau < n$.

3. Decomposition of Likelihoods

Suppose that the time instant τ, when a fault occurred, is known. Let

$$\nu_t = y_t - \sum_{k=1}^{p} a_k y_{t-k}, \quad t \geq p \tag{5}$$

and

$$\sigma^2 = \sigma_{I0}^2 + (1 + \sum_{k=1}^{p} a_k^2)\sigma_O^2 \tag{6}$$

Let the covariance matrix of $\boldsymbol{y}_1^p = (y_1, y_2, \ldots, y_p)^T$ be $\sigma^2\Sigma(\boldsymbol{a})$, then we can obtain the following equation on the log-likelihood function $\mathcal{L}$ of $\boldsymbol{y}_1^p$ after some straightforward manipulations.

$$-2\mathcal{L} = -2\mathcal{L}_I - 2\mathcal{L}_{II}, \tag{7}$$

where

$$\begin{aligned} -2\mathcal{L}_I &= \log|\Sigma(\boldsymbol{a})| + \sigma^{-2}\boldsymbol{y}_1^{pT}\Sigma^{-1}(\boldsymbol{a})\boldsymbol{y}_1^p. \\ &\quad + 2(\tau-1)\log\sigma + \sigma^{-2}\sum_{t=p+1}^{\tau}\nu_t^2 \\ -2\mathcal{L}_{II} &= (n-\tau+1)\log(\sigma^2+\sigma_{I1}^2) \\ &\quad + (\sigma^2+\sigma_{I1}^2)^{-2}\sum_{t=\tau}^{n}(\nu_t - (\mu_I + \kappa_t\mu_O))^2 \end{aligned} \tag{8}$$

with

$$\kappa_t = \begin{cases} 1 - \sum_{k=1}^{t-\tau} a_k & \tau \leq t \leq \tau + p - 1 \\ 1 - \sum_{k=1}^{p} a_k & \tau + p \leq t \leq n \end{cases} \tag{9}$$

Furthermore, $-2\mathcal{L}_{II}$ can be decomposed into

$$-2\mathcal{L}_{II} = -2\mathcal{L}_{IIa} - 2\mathcal{L}_{IIb} \tag{10}$$

where

$$\begin{aligned} -2\mathcal{L}_{IIa} &= (\sigma^2+\sigma_{I1}^2)^{-2}[(n-\tau+1)(\mu_I + \kappa_t\mu_O - \bar{\nu}_t)^2 \\ &\quad + S_{\kappa\kappa}(\mu_O - \frac{S_{\nu\kappa}}{S_{\kappa\kappa}})^2] \\ -2\mathcal{L}_{IIb} &= (\sigma^2+\sigma_{I1}^2)^{-2}(S_{\nu\nu} - \frac{S_{\nu\kappa}^2}{S_{\kappa\kappa}})) \\ &\quad + (n-\tau+1)\log(\sigma^2+\sigma_v^2) \end{aligned} \tag{11}$$

with

$$\begin{aligned} \bar{\nu}_t &= \frac{1}{n-\tau+1}\sum_{t=\tau}^{n}\nu_t \\ \bar{\kappa}_t &= \frac{1}{n-\tau+1}\sum_{t=\tau}^{n}\kappa_t \\ S_{\nu\nu} &= \sum_{t=\tau}^{n}(\nu_t - \bar{\nu}_t)^2 \\ S_{\nu\kappa} &= \sum_{t=\tau}^{n}(\nu_t - \bar{\nu}_t)(\kappa_t - \bar{\kappa}_t) \\ S_{\kappa\kappa} &= \sum_{t=\tau}^{n}(\kappa_t - \bar{\kappa}_t)^2 \end{aligned} \tag{12}$$

Since the statistic

$$\mathcal{N}_t = \nu_t/\sigma \tag{13}$$

obeys a standard normal law under the hypothesis H_0 : $\mu_I = 0$, $\sigma_{I1}^2 = 0$, $\mu_O = 0$, i.e., no faults occur, we can test whether a fault occurs or not by monitoring the statistic $\mathcal{N}_t$.

Remark 1. Even if the variance σ^2 is not known, we can carry out a similar test by substituting its estimate into (13) and by using the fact that the distribution of the statistic $\mathcal{N}_t$ is t-distribution.

Remark 2. The maximum likelihood estimates $(\hat{a}_k, \hat{\sigma}_{I0}^2, \hat{\sigma}_{I1}^2, \hat{\sigma}_O^2)$ of $(a_k, \sigma_{I0}^2, \sigma_{I1}^2, \sigma_O^2)$ do not depend μ_I nor μ_O, since the likelihood has been decomposed into a term $\mathcal{L}_I + \mathcal{L}_{IIb}$ involving only $(a_k, \sigma_{I0}^2, \sigma_{I1}^2, \sigma_O^2)$ and a term $\mathcal{L}_{IIa}$ which becomes zero when μ_I and μ_O attain their maximum likelihood estimates $\hat{\mu}_I$ and $\hat{\mu}_O$, respectively.

4. Likelihood Ratio Decision Rule

Supposing that the fault is a single type, i.e., arising either the system fault or the observation fault at time τ, and that the values of the variances σ_{I0}^2, σ_{I1}^2 and σ_O^2 are known, we consider the decision rule for determining which type of fault occurs.

It can be shown that the maximum of $\mathcal{L}_{II}$ with respect to μ_I under $\mu_O = 0$ (the system fault case) is given by

$$\begin{aligned} R_S &= -2\mathcal{L}_{II}(\mu_I = \hat{\mu}_I, \mu_O = 0) \\ &= (\sigma^2+\sigma_{I1}^2)^{-2}S_{\nu\nu} + (n-\tau+1)\log(\sigma^2+\sigma_{I1}^2) \end{aligned} \tag{14}$$

with $\hat{\mu}_I = \bar{\nu}_t$.

On the other hand, the maximum of $\mathcal{L}_{II}$ with respect to μ_O under $\mu_I = 0$ and $\sigma_{I1}^2 = 0$ (the observation fault case) is given by

$$R_O = -2\mathcal{L}_{II}(\mu_I = 0,\ \sigma_{I1}^2 = 0,\ \mu_O = \hat{\mu}_O)$$
$$= \sigma^{-2}\Big\{\sum_{t=\tau}^{n}\nu_t^2 - \frac{\Big(\sum_{t=\tau}^{n}\kappa_t\nu_t\Big)^2}{\sum_{t=\tau}^{n}\kappa_t^2}\Big\} \tag{15}$$
$$- 2(n-\tau+1)\log\sigma$$

with $\hat{\mu}_O = \sum_{t=\tau}^{n}\kappa_t\nu_t / \sum_{t=\tau}^{n}\kappa_t^2$.

Then, the likelihood ratio decision rule, selecting the fault type with the larger likelihood, is given as follows:

Decision Rule

If $R_S/R_O > K(>1)$, then decide the fault is the system fault, and

If $R_O/R_S > K(>1)$, then decide the fault is the observation fault,

with a suitably chosen constant K.

5. Numerical Examples

We apply the proposed decision rule to a system modeled by an autoregressive model. The model corresponding to the normal mode is described

$$x_t = 0.5x_{t-1} - 0.2x_{t-2} + v_t$$
$$y_t = x_t$$

where $\{v_t\}$ is an independent normal random number sequences with mean zero and variance 0.01, and a fault occurs at $\tau = 101$. The fault modes considered are:

Case 1. System fault, i.e., system equation changes to

$$x_t = 0.5x_{t-1} - 0.2x_{t-2} + v_t$$

where the mean of $\{v_t\}$ is 0.5.

Case 2. Observation fault, i.e., observation equation changes to

$$y_t = x_t + 0.5$$

In Figs.1, 2 and 3, the behaviors of $\{x_t\}$, $\{y_t\}$ and $\{\mathcal{N}_t/\sigma\}$ processes are shown. The time instant when the fault occurred be estimated by monitoring $\{\mathcal{N}_t\}$. Furthermore, we can recognize the fault source for each case by monitoring the behavior of R_S/R_O and R_O/R_S as shown in Fig.4.

6. Conclusions

A likelihood ratio rule is proposed for recognizing the source of faults arising in the system described by autoregressive (AR) models. Numerical examples illustrate the usefulness of the proposed decision rule. Though we considered here only a simple case, this approach can be extended to more general situations such as cases of unknown AR parameters and noise characteristics. Suitable choice of the threshold constant K should be investigated further.

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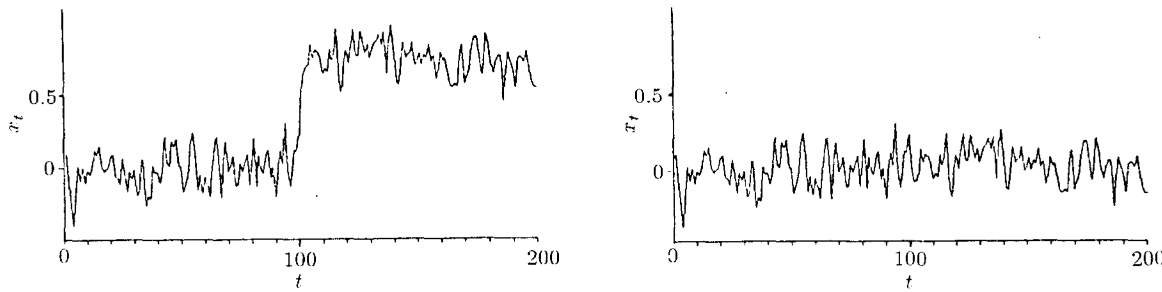
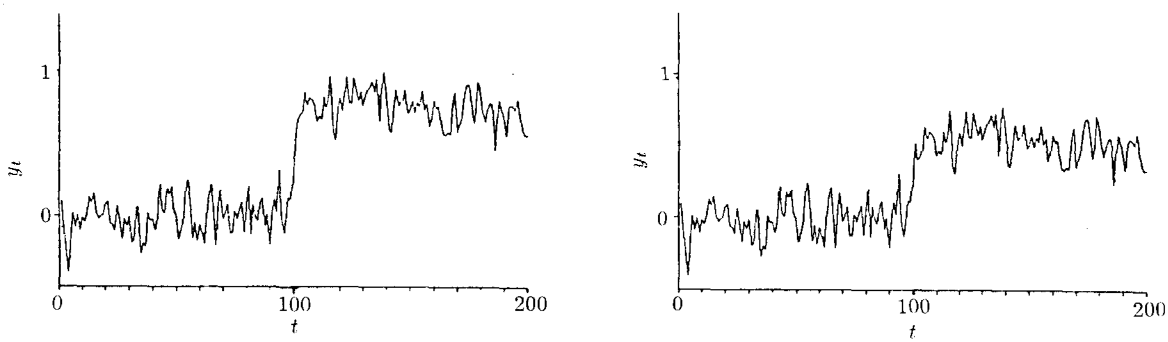


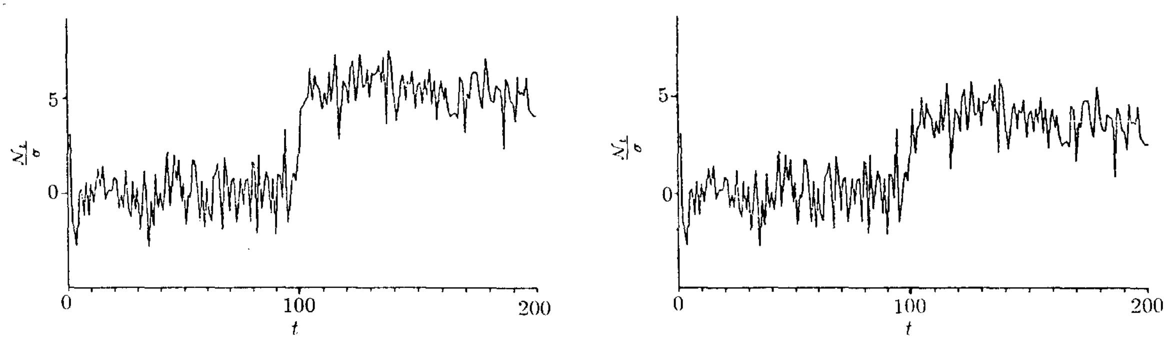
(a) Case 1 (b) Case 2

Fig.1 Behaviors of $\{x_t\}$ process



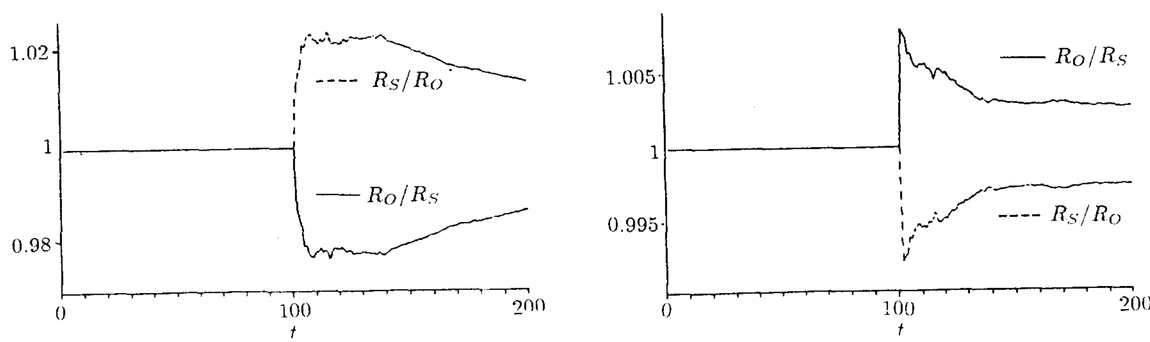
(a) Case 1 (b) Case 2

Fig.2 Behaviors of $\{y_t\}$ process



(a) Case 1 (b) Case 2

Fig.3 Behaviors of $\{\mathcal{N}_t/\sigma\}$ process



(a) Case 1 (b) Case 2

Fig.4 Behaviors of test statistics R_S/R_O and R_O/R_S