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An Approach for the Functional Extension of the Remote Measurement Method of Unsteady Flow Rate

(On an Interpolation Method)

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ABSTRACT

The paper describes an approach for estimating unsteady flow rate through oil hydraulic pipelines and components in real time. Recently we have proposed following three unsteady flow rate measurement approaches; RIFM, QIFM and TPFM, in which hydraulic pipeline dynamics are made use of. In this paper, we firstly propose new approaches, i.e, an interpolation and an extrapolation methods in combination with RIFM and TPFM. In the interpolation method, unsteady flow rate at the arbitrary internal location along the pipeline between two points for measuring the two point pressure can be estimated. In this paper, the accuracy and dynamic response of interpolation method are mainly experimentally investigated in detail.

1. INTRODUCTION

The measurement of unsteady flowrate through an arbitrary cross section of pipes and components is of vital importance to elucidate the dynamic characteristics of systems, to realize feedback control of flowrate in hydraulic power systems. There is also demand for a real time flow sensor of ability to measure unsteady flowrate with high accuracy and fast response under high pressure conditions.

Most recently we have been proposed three approaches, that is, (1) remote instantaneous flowrate measurement method (abbreviated as RIFM) [(1)], (2)quasi-remote instantaneous flowrate measurement method (abbreviated as QIFM) [(2)], (3) unsteady flowrate measurement using two point pressure measurement along the pipeline (abbreviated as TPFM) [(3)]. The validity of these approaches has been investigated by unsteady flowrate measurement and comparison with experimental results from a cylindrical choke - type instantaneous flowmeter (abbreviated as CCFM)[(4)] which is confirmed to respond to the change of flowrate at high frequencies above 400Hz.

In this paper, we firstly scheme the functional extension of measurement capability of these approaches proposed, then propose new approaches (i.e **interpolation and extrapolation method**) using only two points pressure measurements on the pipeline for estimating unsteady flowrate in real time passing through an arbitrary cross section along the pipeline. Under unsteady laminar flow condition, the estimated flowrate waveforms are compared with the results directly measured by CCFM.

2. PRINCIPLE OF UNSTEADY FLOWRATE MEASUREMENT

Consider the distributed parameter model of an axisymmetric, compressible, viscous, unsteady laminar liquid flow for a hydraulic pipeline as shown in **Fig.1a**. The dynamic relationship between pressure and flowrate at two distant cross sections (upstream and downstream section) along the pipeline is represented by the transfer matrix[(5)].

$$\begin{pmatrix} P_u(s) \\ Q_u(s) \end{pmatrix} = \begin{bmatrix} \cosh\{\lambda(s)L\} & Z_0(s)\sinh\{\lambda(s)L\} \\ \frac{1}{Z_0(s)}\sinh\{\lambda(s)L\} & \cosh\{\lambda(s)L\} \end{bmatrix} \begin{pmatrix} P_d(s) \\ Q_d(s) \end{pmatrix} \quad (1)$$

$P(s)$ and $Q(s)$ denote Laplace transforms of the pressure and flowrate derivation, and subscripts u and d refer to upstream and downstream section of the pipeline, respectively.

With consideration of frequency dependent viscous friction, propagation constant $\lambda(s)$ and characteristic impedance of the pipeline $Z_0(s)$ are given as follows:

$$\lambda(s) = \frac{s}{c}\{1 - \frac{2J_1(ja\sqrt{s/\nu})}{ja\sqrt{s/\nu}J_0(ja\sqrt{s/\nu})}\}^{-\frac{1}{2}}$$

$$Z_0(s) = \frac{\rho c}{\pi a^2}\{1 - \frac{2J_1(ja\sqrt{s/\nu})}{ja\sqrt{s/\nu}J_0(ja\sqrt{s/\nu})}\}^{-\frac{1}{2}}$$

where a is inner radius of the pipeline, c is sonic velocity in oil-filled pipeline, $J_n(x)$ is Bessel functions of first kind, $j = \sqrt{-1}$, L is the length between upstream and downstream along the pipeline, s is Laplace operator, ν is kinematic viscosity of the oil, ρ is density of the oil.

The basic principle of the remote measurement of instantaneous flowrate proposed previously is that one unknown among four variables $P_u(s)$, $Q_u(s)$, $P_d(s)$ and $Q_d(s)$ in equation (1) can be estimated by measuring

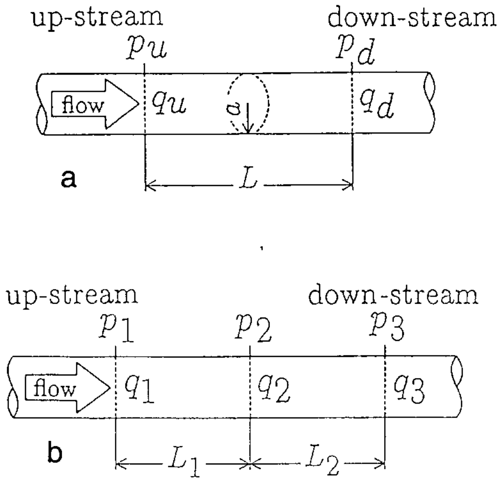

Fig.1 Hydraulic Pipeline systems

other two variables. Based on this principle, the following approaches have been proposed and experimentally confirmed.

(1) REMOTE INSTANTANEOUS FLOWRATE MEASUREMENT METHOD (**RIFM**) [1]

In **Fig.1a**, upstream flowrate $q_u(t)$ can be remotely estimated by measuring downstream pressure and flowrate $p_d(t)$, $q_d(t)$, or downstream flowrate $q_d(t)$ is estimated using upstream pressure and flowrate $p_u(t)$, $q_u(t)$.

(2)QUASI-REMOTE INSTANTANEOUS FLOWRATE MEASUREMENT METHOD (**QIFM**)[2]

Upstream flowrate $q_u(t)$ is estimated in real time by measured upstream pressure $p_u(t)$ and downstream flowrate $q_d(t)$. Conversely, downstream flowrate can be estimated from measured downstream pressure $p_d(t)$ and upstream flowrate $q_u(t)$ in similar manner.

(3)INSTANTANEOUS FLOWRATE MEASUREMENT USING TWO POINTS PRESSURE MEASUREMENT (**TPFM**) [3]

Upstream or downstream flowrate $q_u(t)$, $q_d(t)$ can be estimated in real time from measurements of upstream and downstream pressure $p_u(t)$, $p_d(t)$.

It is confirmed that these approaches are relatively robust to the change of kinematic viscosity so far as an unsteady viscous model (a dissipative model) of the pipeline is used [6].

The basic concept of the functional extension of these approaches is as follows: An instantaneous flowrate through an arbitrary cross section along the pipeline can be estimated using a remote instantaneous flowrate measurement method[1]. This method has a wide measurement capability if the condition that the point to which is desired to measure unsteady flowrate in hydraulic control systems are connected the same radius with the point of flowrate measurement as using input data is satisfied. Because the measured data of pressure and flowrate are used to estimate unsteady flowrate at the remote location in RIFM, an instantaneous flowmeter with high accuracy and fast response is needed, but an instantaneous flowmeter is not commercially available in the present situations.

To solve this problem as a means, we introduce the concept to make use of an instantaneous flowrate measurement method using two point pressure measurement instead of an instantaneous flowmeter in RIFM. In combination with TPFM and RIFM, unsteady flowrate through an arbitrary cross section along the pipeline can be estimated from measuring only two point pressures on the pipeline. By using this concept, the convenience and functional extension of measurement capability for estimating unsteady flowrate at an arbitrary cross section along the pipeline can be achieved without installing an instantaneous flowmeter.

Based on this principle, we propose **the interpolation** and **the extrapolation method** for estimating unsteady flowrate. **In the interpolation method**, unsteady flowrate at an arbitrary internal location along the pipeline between two points for pressure measurements $p_1(t)$, $p_3(t)$ in **Fig. 1b** can be estimated. RIFM has not measurement capability of real time but the interpolation method can be easily realized because the weighting functions in the time domain are obtained from the transfer functions. **In the extrapolation method**, unsteady flowrate at an arbitrary external portion between two pressure measuring points $p_1(t)$, $p_2(t)$ can be estimated using the measured values of the pressure $p_1(t)$, $p_2(t)$.

We firstly consider the interpolation method. From the transfer matrix equation (1), dynamic relationship between $p_1(t)$, $p_3(t)$ and the estimating unsteady flowrate $\hat{q}_2(t)$ in the Laplace domain is given as follows;

$$Q_2(s) = G_{ui}(s) \cdot P_1(s) - G_{di}(s) \cdot P_3(s) \qquad (2)$$

where transfer functions $G_{ui}(s)$ and $G_{di}(s)$ are given

$$G_{ui}(s) = \frac{\cosh\{\lambda(s) \cdot L_2\}}{Z_0(s) \cdot \sinh\{\lambda(s) \cdot (L_1 + L_2)\}} \qquad (3)$$

$$G_{di}(s) = \frac{\cosh\{\lambda(s) \cdot L_2\}}{Z_0(s) \cdot \tanh\{\lambda(s) \cdot (L_1 + L_2)\}} - \frac{\sinh\{\lambda(s) \cdot L_2\}}{Z_0(s)} \qquad (4)$$

For the extrapolation method, the relationship between $p_1(t)$, $p_2(t)$ and the estimating unsteady flowrate $\hat{q}_3(t)$ is given as follows;

$$Q_3(s) = G_{ue}(s) \cdot P_1(s) - G_{de}(s) \cdot P_2(s) \qquad (5)$$

where,

$$G_{ue}(s) = \frac{\cosh\{\lambda(s) \cdot L_2\}}{Z_0(s) \cdot \sinh\{\lambda(s) \cdot (L_1)\}} \qquad (6)$$

$$G_{de}(s) = \frac{\cosh\{\lambda(s) \cdot L_2\}}{Z_0(s) \cdot \tanh\{\lambda(s) \cdot (L_1)\}} + \frac{\sinh\{\lambda(s) \cdot L_2\}}{Z_0(s)} \qquad (7)$$

We, here, investigated into the interpolation method in detail.

Fig.2a,b show typical example of the frequency characteristics (solid line) of transfer functions $G_{ui}(s)$ and $G_{di}(s)$ in equations (3), (4). The ordinate represents gain and phase angle, the abscissa represents frequency.

By taking inverse Laplace transform of equation(2), the estimating unsteady flowrate $\hat{q}_2(t)$ in the time domain is determined by the following convolution integrals.

$$\hat{q}_2(t) = \int_{-\infty}^{t} g_{ui}(\tau) \cdot p_1(t-\tau)d\tau + \int_{-\infty}^{t} g_{di}(\tau) \cdot p_3(t-\tau)d\tau \quad (8)$$

where, $g_{ui}(t)$ and $g_{di}(t)$ represent the weighting functions in the time domain corresponding to transfer functions $G_{ui}(s)$ and $G_{di}(s)$ in equations (3), (4). If the weighting functions are obtained, estimation of unsteady flowrate in the time domain is possible, and is achieved in real time.

Equation (8) is discretized with respect to time to estimate unsteady flow rate in real time by implementing the convolutions using a microcomputer system, so that the difference equation is described by the following discrete convolution summations.

$$\hat{q}_u(n\Delta T) = \sum_{i=0}^{N-1} g_{ui}(i\Delta T) \cdot p_1(n\Delta T - i\Delta T)\Delta T + \sum_{i=0}^{N-1} g_{di}(i\Delta T) \cdot p_3(n\Delta T - i\Delta T)\Delta T \quad (9)$$

where ΔT is sampling time interval, N is the number of convolution terms, and implementing time of discrete convolution T_0 is given by $\Delta T \times N$.

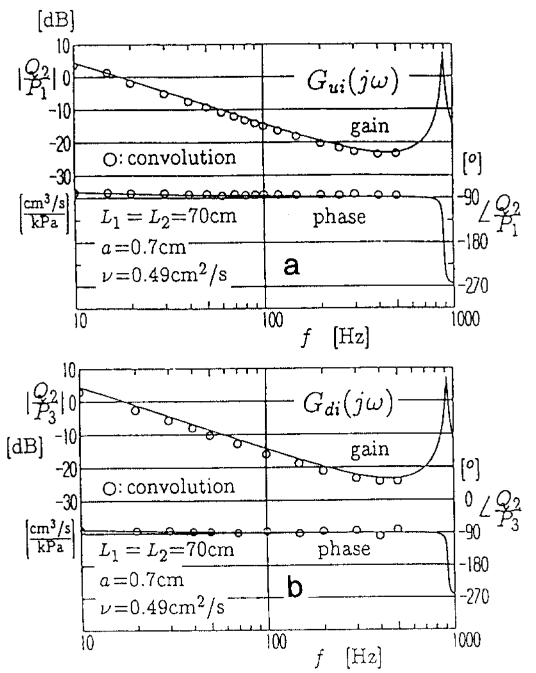

Fig.2 Typical example of the frequency characteristics of transfer functions $G_{ui}(s)$ and $G_{di}(s)$

3. DERIVATION OF THE WEIGHTING FUNCTIONS

The weighting functions $g_{ui}(t)$ and $g_{di}(t)$ corresponding to the transfer functions $G_{ui}(s)$ and $G_{di}(s)$ are obtained by using inverse fast Fourier transform (abbreviated as IFFT). The functions $g_{ui}(t)$ and $g_{di}(t)$ are obtained as follows.

$$g_{ui}(k\Delta T) = \Delta f \cdot \sum_{n=0}^{N_s-1} G^*_{ui}(j2\pi n\Delta f) \times \exp(-j2\pi nk/N_s) \quad (10)$$

$$g_{di}(k\Delta T) = \Delta f \cdot \sum_{n=0}^{N_s-1} G^*_{di}(j2\pi n\Delta f) \times \exp(-j2\pi nk/N_s) \quad (11)$$

$$k = 0, 1, 2, 3, \cdots, N_s - 1$$

where $G^*_{ui}(j2\pi n\Delta f)$, $G^*_{di}(j2\pi n\Delta f)$ are complex conjugates of $G_{ui}(j2\pi n\Delta f)$, $G_{di}(j2\pi n\Delta f)$, respectively. ΔT is sampling time interval, N_s is total number of samples.

Fig 3a,b show typical examples of the weighting functions $g_{ui}(t)$ and $g_{di}(t)$ for the length of the pipeline $L_1 = L_2 = 70$cm, kinematic viscosity ν=0.49 cm²/s and sampling frequency of 4.5kHz.

For carrying out high speed operations in equation (9), it is obliged to use the values of the weighting functions within a finite time interval. As shown in **Fig. 3a, b**, the consecutive amplitudes of the weighting functions obtained here become sufficiently small over the time period t=300ms. The truncation time of the weighting functions (i.e implementing time of convolutions) is approximately equal to 300ms in this case, so that the number of convolution terms N=1350. The validity of the weighting functions obtained here are also investigated in comparing theoretical values of frequency response characteristics of $G_{ui}(j\omega)$, $G_{di}(j\omega)$ with the results of convolution between the weighting functions and sinusoidal input for various frequencies. The results are plotted in **Fig. 2a,b**. In **Fig. 2a,b**, the solid lines represent the gain and phase angle calculated from equations (3), (4). The results of convolutions are indicated by symbols ○, respectively. In comparison, experimental gains and phase angles are in good agreement with the theoretical values up to about 500Hz in the frequency range, and show the validity and accuracy of weighting functions obtained here.

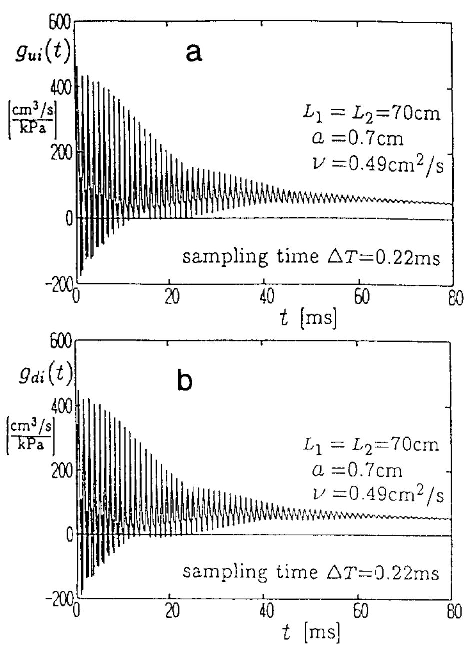

Fig 3 Typical examples of the weighting functions $g_{ui}(t)$ and $g_{di}(t)$

4. EXPERIMENT

In the experiment, the estimated flowrate waveforms $\hat{q}_2(t)$ by the interpolation method using the measured pressure $p_1(t)$ and pressure $p_3(t)$ as input data are compared with the results by the remote instantaneous flowrate measurement method using measured pressure $p_3(t)$ and flowrate $q_3(t)$.

Fig. 4 shows a schematic diagram of experimental apparatus used in this study. The straight circular copper pipelines with inner diameter of 14cm are used for the test pipeline, the semiconductor type pressure transducers are installed at the location of upstream, midstream and downstream section along the pipeline in order to measure pressure $p_1(t)$, $p_2(t)$ and $p_3(t)$.

The cylindrical choke type instantaneous flowmeter (abbreviated as CCFM) are used to compare with the estimated results by this method, and to obtain the downstream flowrate $q_3(t)$ as input data in RIFM. The cylindrical choke with inner diameter d = 2.55mm and length l = 15mm is used.

The upstream end of the hydraulic pipeline system is connected to an accumulator (10 l), and a servo valve is set at downstream end in order to generate the unsteady

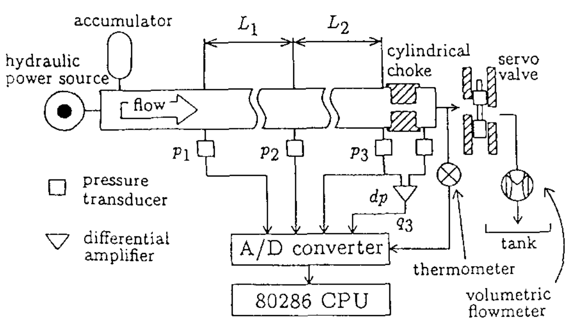

Fig. 4 Schematic diagram of experimental apparatus

flow conditions in hydraulic pipeline system. Oil temperature is measured by a thermistor-type thermometer.

Sampled data are fed into microcomputer through a 12 bit A/D converter at the sampling frequency of 4.5kHz.

5. EXPERIMENTAL RESULTS

In the experiment, oil temperature is maintained about 31±1°C, and the kinematic viscosity of oil is 0.49cm²/s at that temperature, and operating pressure at the upstream end of the pipeline is set at 3.0MPa.

The accuracy and response of the estimated flow rate $\hat{q}_2(t)$ by the interpolation method using $p_1(t)$ and $p_3(t)$ are compared with the results by the RIFM using measured pressure $p_3(t)$ and flowrate $q_3(t)$ under unsteady laminar oil flow conditions which are generated by a servo valve.

Fig. 5 describes typical examples of recorded pressure and flowrate waveforms for rectangular wave input 10Hz, the length of the pipeline $L_1 = L_2$ = 70cm. The ordinate represents pressure and flowrate, the abscissa denotes time. The top and 2nd and 3rd waveforms are indicated the measured upstream, midstream and down-

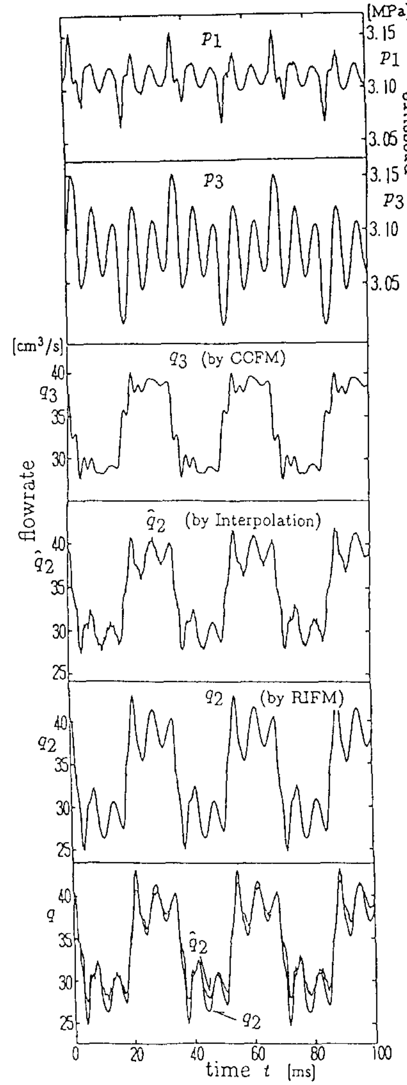

Fig. 5 Typical examples of recorded pressure and flowrate waveforms for rectangular wave input 30Hz

stream pressure $p_1(t)$, $p_2(t)$ and $p_3(t)$ and the 4th one shows the measured flowrate waveforms $q_3(t)$ by CCFM. The 5th one is the estimated flowrate waveform $\hat{q}_2(t)$ by RIFM and the 6th shows the estimated flowrate waveform $\hat{q}_2(t)$ by the interpolation method proposed here. The 7th one is represented the estimated flowrate waveforms $\hat{q}_2(t)$ by RIFM and the interpolation method at the same coordinate. Although the flowrate ripple of high frequencies are come out at the rise period when it is subjected to rectangular wave input 10Hz, the estimated flow rate waveforms $\hat{q}_2(t)$ by interpolation method and by RIFM are in good agreement on the whole. It is considered that this difference at the transient periods may be cause from obtaining the weighting functions approximately by IFFT in the interpolation method. In practice, it is also confirmed that the estimated results by using RIFM in off line which the convolution integrals are implemented in the frequency domain are not come out this difference.

Fig. 6 depicts the comparison the estimated results for rectangular wave input 10Hz and the length of the pipeline L_1=84cm, L_2=140cm. In similar manner as shown in **Fig.6**, the top and 2nd and 3rd waveforms are indicated the measured upstream, midstream and downstream pressure $p_1(t)$, $p_2(t)$ and $p_3(t)$ and the 4th one shows the measured flowrate waveforms $q_3(t)$, respectively. The 5th one is the estimated flowrate waveform $\hat{q}_2(t)$ by RIFM and the 6th shows the estimated flowrate waveform $\hat{q}_2(t)$ by the interpolation method proposed here. The 7th one is represented the estimated flowrate waveforms $\hat{q}_2(t)$ by RIFM and interpolation method at the same coordinate. In comparison, the estimated flow rate waveforms $\hat{q}_2(t)$ by the interpolation method and by RIFM are in good agreement on the whole.

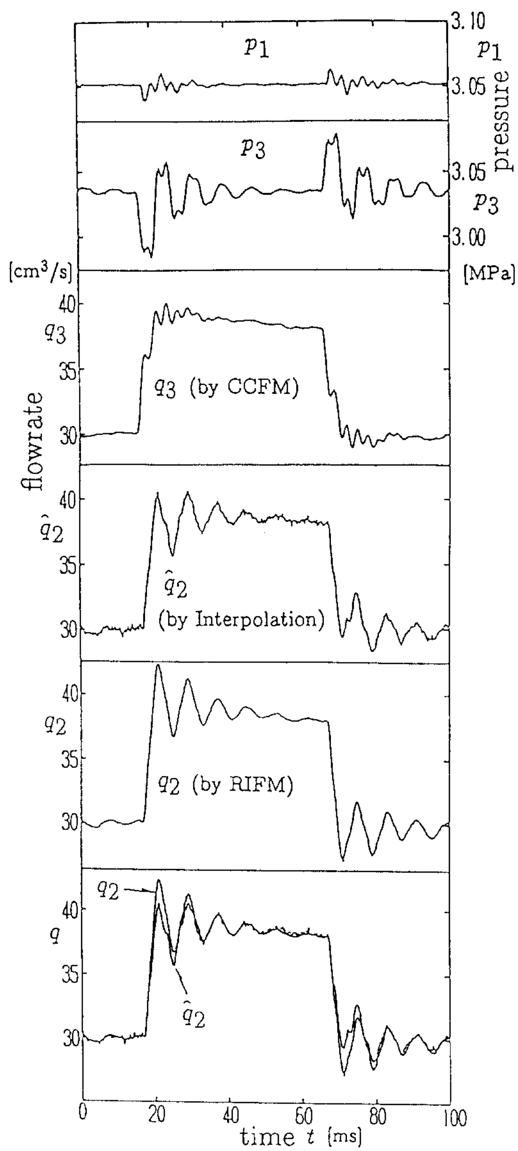

Fig. 6 Typical examples of recorded pressure and flowrate waveforms for rectangular wave input 10Hz

The real time measurement of the interpolation method proposed here can be achieved by using the high speed numerical operations of discrete convolutions such as a similar technique of reference (2),(3).

6. CONCLUSIONS

In combination with RIFM and TPFM in the remote instantaneous flowrate measurement methods proposed previously, new approaches, **the interpolation and the extrapolation method**, for estimating unsteady flow rate in real time are proposed. Particularly, the accuracy and the dynamic response of the interpolation method are experimentally investigated in detail. The results show good agreements between the estimated and measured flow rate waveforms under unsteady laminar oil flow conditions. The method proposed here is useful in estimating unsteady flow rate through an arbitrary cross section in the hydraulic pipelines and components without installing an instantaneous flowmeter at the desired location to get an instantaneous flowrate information. We also intend to present the extrapolation method having more practical use in the near future.

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