'92 KACC 1992. 10. 19~21

ARW Method for Saturating Systems

CHONG-HO CHOI and JONG-KOO PARK
Dept. of Control and Instrumentation Eng.
ASRI and ERC-ACI by KOSEF
Seoul National Univ., Seoul 151-742
Korea
TEL: +82-2-880-7310, FAX: +82-2-885-6620

Abstract: This paper presents a compensator design method for multivariable feedback control systems with saturating actuators based on the concept of the equilibrium point. An explicit expression for the compensation matrix of the general anti-reset windup (ARW) scheme is derived by minimizing the distances between the equilibrium points. The resulting dynamics of the compensated controller exhibits the reduced model form of the unsaturated system which can be obtained by the *singular perturbational method.* The proposed method is applicable to any open-loop stable plants with saturating actuators whose controllers are determined by some design technique. An example is given to show the effectiveness of the proposed method.

1. Introduction

The actuator saturation not only deteriorates the performance of the control system, but can also lead to instability since the feedback loop is broken in such situations. When a linear controller shows *integral action*, the controller output will exceed the saturation level quickly. This results in serious performance degradation (large overshoots and large settling times) and the phenomenon is called the *reset windup*.

This paper deals with a two-step design procedure for saturating systems. That is, we assume that a linear controller has already been designed which gives satisfactory behavior for the multivariable linear system in the absence of saturating actuators. The objective is to provide an additional compensator that gives graceful performance degradation of the closed loop system under saturation. We address this objective based on the ARW scheme which was proposed by Åström and Wittenmark (1984) for the state space description of the system, since this configuration is somewhat standard for this problem.

The proposed method is motivated by engineering insight into the saturating systems. But it is closely related to the *singular perturbational model reduction method* which has been well known in control theory(Kokotovic *et al.*, 1976, 1986; Saksena *et al.*, 1984). The resulting compensated controller reflects the dynamics of closed-loop systems in the absence of saturating actuators. Moreover the compensation matrix in the controller is expressed in closed form with the plant and controller parameters. This method can be applied to control systems even when the high frequency gain matrix of the controller is not full rank whereas the conditioning technique cannot be directly applied to. The performance of the proposed method is satisfactory in view of the control objective that provides graceful performance degradation of the system under saturation.

2. Anti-reset Windup scheme

Generally, a controller is designed to have an integrator or relatively slow dynamics, *e.g.* PI controller, for the purpose of eliminating or reducing the steady state error. Hence a sudden change in the reference input or disturbances result in a large control input signal to the plant due to the *integral action* of the controller. At that moment, the control signal can exceed the saturation level and the output transient response can be undesirable. This phenomenon is called the *reset windup*. Controllers with relatively fast dynamics do not exhibit windup problems significantly, where the term relatively slow(fast) dynamics means that the dynamics of the controller is slow(fast) compared with the plant dynamics.

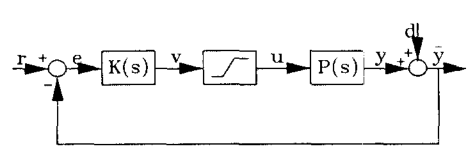

Figure 1. The control system with saturating actuators.

In this paper, we shall formulate and solve the problem using the state-space representation. The plant and the controller are expressed by the following minimal state space representations:

Plant:

$$\dot{x}_p(t) = Ax_p(t) + Bu(t), \quad (1)$$

$$y(t) = Cx_p(t) + Du(t), \quad (2)$$

$$u(t) = sat(v(t)), \quad (3)$$

Controller:

$$\dot{x}_c(t) = Fx_c(t) + Ge(t), \quad (4)$$

$$v(t) = Hx_c(t) + Le(t), \quad (5)$$

$$e(t) = r(t) - y(t) - d(t), \quad (6)$$

where $r(t) \in \Re^n$ is the reference input vector, $u(t) \in \Re^m$ is the control input vector, $y(t) \in \Re^n$ is the output vector, $v(t) \in \Re^m$ is the controller output vector, $d(t) \in \Re^n$ is the disturbance vector, $x_p(t) \in \Re^p$ is the state vector of the plant, and $x_c(t) \in \Re^q$ is the state vector of the controller. The dimensions of the constant matrices $A, B, C, D, F, G, H,$ and L are $p \times p, p \times m, n \times p, n \times m, q \times q, q \times n, m \times q,$ and $m \times n$ respectively.

The saturating actuators operate as

$$sat(v) = [sat(v_1) \;\; sat(v_2) \;\; \cdots \;\; sat(v_m)]^T, \quad (7)$$

where v_i, $i = 1, \cdots, m$, is the ith element of v and

$$sat(v_i) = \begin{cases} v_{i,l} & v_i < v_{i,l} \\ v_i & v_{i,l} \le v_i \le v_{i,h} \\ v_{i,h} & v_i > v_{i,h}, \end{cases} \quad (8)$$

for some constants $v_{i,l} (< 0)$ and $v_{i,h} (> 0)$.

The idea of the anti-reset windup is to provide a local arrangement around the controller with the difference between the controller output signal $v(t)$ and the saturated control signal $u(t)$(Åström and Wittenmark, 1984; Campo and Morari, 1990; Hanus *et al.*, 1987). The compensation scheme shown in Fig. 2 was presented in Åström and Wittenmark (1984) based on the observer technique. The dynamics of the compensated controller is the following:

Compensated controller:

$$\dot{x}_c(t) = Fx_c(t) + Ge(t) - M(v(t) - u(t))$$
$$= (F - MH)x_c(t) + (G - ML)e(t) + Mu(t), \quad (9)$$

$$v(t) = Hx_c(t) + Le(t), \quad (10)$$

where M is the compensation matrix with dimension $q \times m$. Introduction of the compensation matrix M does not change the behavior of the closed loop system when $u(t) = v(t)$. When $u(t) \neq v(t)$, however, the dynamics of the controller can be changed arbitrary by an adequate choice of M since (H, F) is observable by the minimality of the state space description.

The compensation matrix affects the performance of the closed loop system, but it is not known what is the best choice for M. If one has a reasonable method of selecting a proper M, then the two-step design procedure for saturating systems will be a powerful design tool. Hence many researchers are interested in finding a general method for selecting the compensation matrix M in recent years. For example, the conditioning technique is one of the major result in this area which has been developed based on a quite different viewpoint other than the observer technique. But it can be shown that this method is a special case of the ARW method in Fig. 2. The design object of this method is to ensure that the error $e(t)$ has no effect on the states of the controller in the event of saturation. And this is achieved by selecting $M = GL^{-1}$(Campo and Morari, 1990). Nevertheless, this method requires some modifications when L, the high frequency gain matrix of the controller, is not full rank. Moreover it does not reflect the plant dynamics in anyway. In the next section, we will develop a method to solve some of these problems, which is quite different from the conditioning technique in the compensation concept.

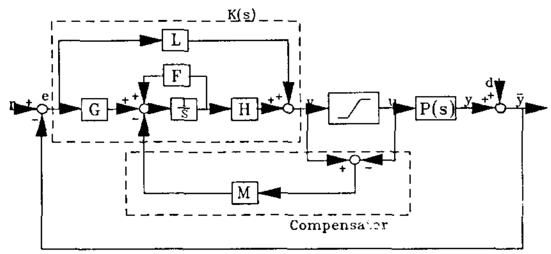

Figure 2. The system with an antireset windup compensation.

3. Derivation of the compensation matrix

In general, the performance degradation is caused by the fact that the states of the controller achieve different values from those in the absence of saturating actuators. This can result in improper control signals and consequently deteriorates the closed-loop performance.

Consider the meaning of *equilibrium points.* The equilibrium point is a terminal state where any initial state moves to as time passes on if the control system is stable. In fact, the equilibrium point of the stable closed-loop system is unique in the absence of a saturating actuator. But the saturating actuator splits the unique equilibrium point into several *virtual* equilibrium points.

From this observation, we conjecture that the performance of the system will be improved as the distances between the equilibrium points of the saturated and unsaturated system become small. This seems to be unfamilar for the compensator design method, but it will be shown in the next section that this method has close relation with the singular perturbational model reduction method.

Now we will determine the compensation matrix M in the ARW scheme of Fig. 2 based on this idea. First, we will rewrite the dynamics of the plant (1) and the compensated controller

(9) depending on the absence or the presence of saturating actuators.

(i) The dynamics of the closed-loop system in the absence of saturating actuators:

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_p \end{bmatrix} = A_l \begin{bmatrix} x_c \\ x_p \end{bmatrix} + \begin{bmatrix} G[I - D(I+LD)^{-1}L] \\ B(I+LD)^{-1}L \end{bmatrix} (r-d), \quad (11)$$

where

$$A_l := \begin{bmatrix} F - GD(I+LD)^{-1}H & -G[I - D(I+LD)^{-1}L]C \\ B(I+LD)^{-1}H & A - B(I+LD)^{-1}LC \end{bmatrix}. \quad (12)$$

(ii) The dynamics of the closed-loop system in the presence of saturating actuators:

$$\begin{bmatrix} \dot{x}_{c,i} \\ \dot{x}_{p,i} \end{bmatrix} = A_{sat} \begin{bmatrix} x_{c,i} \\ x_{p,i} \end{bmatrix} + \begin{bmatrix} (G - ML)(r-d) + [M(I+LD) - GD]sat_i(v) \\ Bsat_i(v) \end{bmatrix} \quad (13)$$

$$A_{sat} := \begin{bmatrix} F - MH & -(G - ML)C \\ 0 & A \end{bmatrix}, \quad (14)$$

$$i = 1, \cdots, 3^m - 1,$$

where $sat_i(v)$ represents the output of the saturating actuators when the actuators operate in the ith mode among $3^m - 1$ possible modes. When the function $sat_i(\cdot)$ operates in the linear region, (13) becomes (11).

When the plant is open-loop unstable, the closed-loop system with saturating actuators can never be stabilized globally because there is always a state which cannot be made to converge to the desired state due to the limitation of the control inputs. Hence we will only consider open-loop stable plants. Further, we give some assumptions, which are necessary in determining the compensation matrix M.

Assumption 1.

(a) The plant is stable, *i.e.*, the system matrix A of (1) has stable eigenvalues.

(b) The controller provides acceptable nominal performance in the absence of saturating actuators.

(c) $A - B(I+LD)^{-1}LC$ is nonsingular.

Assumption 1.(a) and (b) are standard in this kind of works. Assumption 1.(c) is necessary for a technical reason and can be easily checked before designing the compensation matrix.

Choose M such that the distances between the equilibrium points of (11) and (13) are as small as possible in the sense of the *Euclidean norm.* This leads to an optimization problem. Let $(\bar{x}_c, \bar{x}_p)^T$ denote the equilibrium point of (11) and $(\bar{x}_{c,i}, \bar{x}_{p,i})^T$, $i = 1, \cdots, 3^m - 1$, denote the virtual equilibrium point of (13). Note that the distance between the equilibrium points of (11) and (13) can vary differently depending on the reference input r, the disturbance d, and satuaration levels.

Theorem 1.

Consider the general ARW scheme shown in Fig. 2. When Assumption 1 is satisfied, the solution M of

$$\min_M \left\{ J = \sum_{i=1}^{3^m-1} [(\bar{x}_c - \bar{x}_{c,i})^T(\bar{x}_c - \bar{x}_{c,i}) + (\bar{x}_p - \bar{x}_{p,i})^T(\bar{x}_p - \bar{x}_{p,i})]^{1/2} \right\} (15)$$

is uniquely determined by

$$M = G(D - CA^{-1}B)[(I+LD) - LCA^{-1}B]^{-1} \quad (16)$$

Proof.

Omitted for brevity.

Remarks:

(i) M of (16) is independent of the reference input r, the disturbance d, and the saturation levels, although the distance between the saturated and unsaturated equilibrium points can vary differently depending on them. Hence, in the two-step design procedure, once a linear controller is designed *a priori*, the resulting M is determined directly.

(ii) The proposed M depends explicitly on the plant and controller dynamics whereas the conditioning scheme depends only on the controller dynamics. Furthermore, the proposed method is meaningful even when the high frequency gain of the controller is 0, *i.e.*, $L = 0$ in (5), though the conditioning technique can not be applied directly.

(iii) When the directionality of control input affects the closed-loop performance substantially, it is insufficient to compensate the saturated system only with the proposed method. It was suggested to preserve the direction of the control input when one of the input signals is saturated(Campo and Morari, 1990). This can be well achieved by inserting an additional block in the loop.

4. Theoretical verification of the proposed method

The proposed compensation scheme will be shown to have some relation with the *singular perturbational model reduction method* which is a popular method for obtaining reduced-order representations of linear systems(Kokotovic *et al.*, 1976, 1986; Saksena *et al.*, 1984). To begin with, the principal contents of the perturbed method will be reviewed.

Consider the linear time-invariant singularly perturbed system.

$$\dot{x} = A_{11}x + A_{12}z + B_1u \quad (17)$$

$$\mu\dot{z} = A_{21}x + A_{22}z + B_2u \quad (18)$$

and

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

where x, z, and u are n, m, and r dimensional column vectors respectively, and μ represents a small positive parameter. All the elements of A_{ij} and B_i have comparable magnitudes. When A_{22} is nonsingular, one can obtain a reduced order system by setting $\mu = 0$ and

$$\bar{z} = -A_{22}^{-1}A_{21}\bar{x} - A_{22}^{-1}B_2\bar{u}. \tag{19}$$

The reduced order system is

$$\dot{\bar{x}} = A_0\bar{x} + B_0\bar{u} \tag{20}$$

where

$$A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}, \quad B_0 = B_1 - A_{12}A_{22}^{-1}B_2. \tag{21}$$

The reduced-order state matrix A_0 is the *Schur complement of A_{22} in A* and is denoted by (A/A_{22})(Cottle, 1974). The Schur complement of a partitioned matrix appears frequently in the model reduction problems and other various areas.

This method is valid in any ordinary state equation such as

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u \tag{22}$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u \tag{23}$$

if it satisfies the condition that the dynamics of x_2 is sufficiently faster than that of x_1, and the resulting reduced model, by setting $\dot{x}_2 = 0$, is (Liu and Anderson, 1989)

$$\dot{\bar{x}}_1 = (A_{11} - A_{12}A_{22}^{-1}A_{21})\bar{x}_1 + (B_1 - A_{12}A_{22}^{-1}B_2)\bar{u}. \tag{24}$$

The perturbational method not only reduces the model order, but also approximates the original states as the following(Kokotovic *et al.*, 1986): If $Re\ \lambda(A_{22}) < 0$, then the solution x and z of the original system (17), (18) is approximated, for μ sufficiently small, by

$$x(t) \approx \bar{x}(t) \tag{25}$$

$$z(t) \approx -A_{22}^{-1}A_{21}\bar{x}(t) + z_f(\frac{t-t_0}{\mu}) \tag{26}$$

where $z_f := z - \bar{z}$.

Now consider systems containing saturating actuators. As pointed out in Section 2, the reset windup phenomenon is basically caused by the relatively slow dynamics of the controller. In such systems, the trajectory of the controller states evolves differently from the trajectory of the states in the absence of saturating actuators. It is thus desirable to retain the trajectory of the controller states to the trajectory of the linear system as closely as possible.

The closed-loop dynamics of the saturated system in (13) can be rewritten as

$$\dot{x}_{c,i} = (F - MH)x_{c,i} - (G - ML)Cx_{p,i}$$
$$+(G - ML)(r - d) + [M(I + LD) - GD]sat_i(v) \tag{27}$$

$$\dot{x}_{p,i} = Ax_{p,i} + Bsat_i(v) \tag{28}$$

When the compensation matrix M is selected as (16), it can be easily seen that, by direct substitution, the following is true:

$$\dot{x}_{c,i} = (P - QS^{-1}R)x_{c,i} + (T - QS^{-1}U)(r-d)$$
$$+QS^{-1}(Ax_{p,i} + Bsat_i(v)) \tag{29}$$

$$= (P - QS^{-1}R)x_{c,i} + (T - QS^{-1}U)(r-d) + QS^{-1}\dot{x}_{p,i}, \tag{30}$$

where

$$P := F - GD(I + LD)^{-1}H, \tag{31}$$

$$Q := -G[I - D(I + LD)^{-1}L]C, \tag{32}$$

$$R := B(I + LD)^{-1}H, \tag{33}$$

$$S := A - B(I + LD)^{-1}LC, \tag{34}$$

$$T := G[I - D(I + LD)^{-1}L], \tag{35}$$

$$U := B(I + LD)^{-1}. \tag{36}$$

Note that, in the singular perturbational method, the reduced model is obtained by neglecting the fast dynamics of the system, which corresponds to the plant dynamics $x_{p,i}$ in this case. Thus

$$\dot{x}_{c,i} \approx (P - QS^{-1}R)x_{c,i} + (T - QS^{-1}U)(r-d). \tag{37}$$

Remarks:

(*iv*) If M is chosen as in (16), then the resulting dynamics of the compensated controller has the form of the reduced model of the unsaturated closed-loop system obtained by the singular perturbational model reduction method. Therefore, the compensated controller states, $x_{c,i}$, follow x_c in the event of saturation.

(*v*) In the proposed method, the virtual equilibrium point $\bar{x}_{c,i}$ of the saturated controller is the same as the equilibrium point $\bar{x}_c$ of the unsaturated controller. In the singular perturbational reduced model, $\bar{x}_{c,i}$ is only approximately equal to $\bar{x}_c$. Thus the additional term in (30), $QS^{-1}\dot{x}_{p,i}$, can be regarded as a compensation term for this.

5. Stability analysis of the system

We have focused on the performance improvement of saturated systems, but the main concern is whether the system will remain stable under saturation. Although the saturation is rather a simple nonlinearity, it is difficult to show the asymptotic stability of the control system. Thus it is customary to treat the stability in the sense of L_{2e}(Campo and Morari, 1990; Kapasouris *et al.*, 1988). L_{2e} stable systems have the property that inputs of bounded energy give rise to outputs of bounded energy. This section also addresses the L_{2e} stability.

Theorem 2.

Suppose that systems with saturating actuators satisfy Assumption 1. Then the overall system is stable if the system matrix of the compensated controller, $F - MH$, is Hurwitz.

Proof.

Note that (13) includes the case of the absence of saturating actuators (11) and $sat_i(\cdot)$ is a bounded function. Therefore the theorem follows with A_{sat} stable. ■

As observed in Section 4, $F - MH$ is the Schur complement of S in A_l, *i.e.*, (A_l/S). Thus the given system is stable under saturation provided that the reduced model of the linear system is meaningful in the singular perturbational model reduction method.

As previously stated, the behavior of the linear representation in (11) is well suited for the perturbational methods. That is, the controller dynamics, x_c, is relatively slow compared with the plant dynamics, x_p. Hence the eigenvalues of $F - MH$, when M is chosen as proposed, can approximate the first q eigenvalues of (11) provided the matrix S is Hurwitz. This is always true when the controller has zero transmission at infinite frequency.

Hitherto this paper has dealt with the two-step design approach, and has focused on the second stage, *i.e.*, the problem of performance improvement. But it is required, in the first stage, that the controller is designed so that (A_l/S) is Hurwitz to guarantee the stability of the system.

6. An illustrative example

In the process of LQG/LTR design, integrators are usually added to facilitate the loop shaping procedure and eliminate the zero steady-state errors. For the loop transfer recovery, the low control weighting for the linear quadratic regulation problem is widely used. When saturating actuators exist, the reset windup phenomenon can occur since the control input can become very large as the control weighting parameter approaches zero.

Consider a linearized state representation for a F-8 aircraft(Kapasouris *et al.*, 1988).

$$\dot{x}_p(t) = Ax_p(t) + Bu(t), \quad (38)$$

$$y(t) = Cx_p(t), \quad (39)$$

$$u(t) = sat(v(t)), \quad (40)$$

where

$$A = \begin{bmatrix} -0.8 & -0.0006 & -12 & 0 \\ 0 & -0.014 & -16.64 & -32.2 \\ 1 & -0.0001 & -1.5 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -19 & -3 \\ -0.66 & -0.5 \\ -0.16 & -0.5 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \quad (41)$$

The LQG/LTR controller for the augmented plant can be computed as follows.

$$K_{LQG}(s) = G_{cg}(sI - A_a - B_aG_{cg} - H_{fg}C_a)^{-1}H_{fg} \quad (42)$$

where

$$A_a = \begin{bmatrix} 0 & 0 \\ B & A \end{bmatrix}, \quad B_a = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad C_a = \begin{bmatrix} 0 & C \end{bmatrix} \quad (43)$$

and H_{fg} and G_{cg} represent the filter gain matrix and the control gain matrix, respectively, and are given by(Kapasouris *et al.*, 1988)

$$H_{fg} = \begin{bmatrix} -0.844 & 0.819 \\ -11.54 & 13.47 \\ -0.86 & 0.25 \\ -47.4 & 15 \\ 4.68 & -4.8 \\ 4.82 & 0.14 \end{bmatrix}, \quad (44)$$

$$G_{cg} = \begin{bmatrix} 52.23 & 3.36 & -73.1 & 0.0006 & 94.3 & -1072 \\ 3.36 & 29.7 & 2.19 & 0.006 & -908.9 & 921 \end{bmatrix}. \quad (45)$$

Hence the actual controller including the integrators is

$$K(s) = \frac{I}{s}K_{LQG}(s). \quad (46)$$

And the state realization of (46) can be described by

$$\dot{x}_c(t) = Fx_c(t) + Ge(t), \quad (47)$$

$$v(t) = Hx_c(t), \quad (48)$$

$$e(t) = r(t) - y(t), \quad (49)$$

where

$$F = \begin{bmatrix} A_a - B_aG_{cg} - H_{fg}C_a & 0 \\ G_{cg} & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} H_{fg} \\ 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & I \end{bmatrix}. \quad (50)$$

The control inputs are limited by ± 25. Simulations are performed with the reference inputs $r(t) = (10 \quad 10)^T$. Note that the responses of the saturated system without compensation scheme (Fig. 4) are degraded very much with respect to those of the linear system (Fig. 3). Now we adopt the proposed method to improve the performance. The compensation

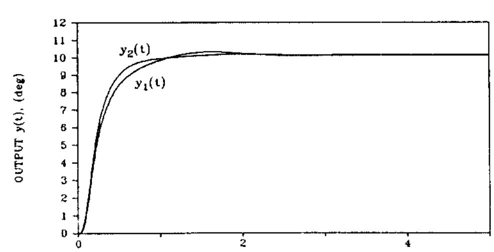

Figure 3. Output responses of the linear system.

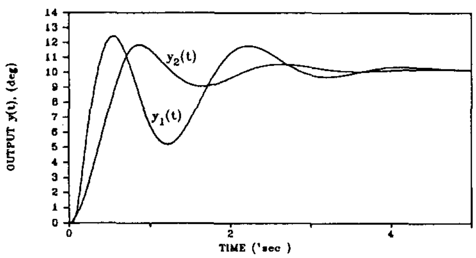

Figure 4. Output responses of the saturated system without compensation.

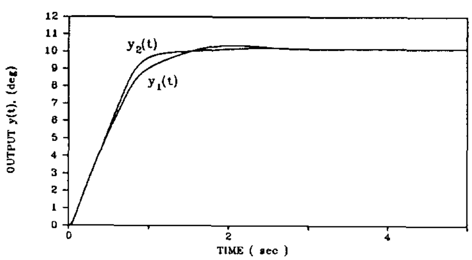

Figure 5. Output responses of the saturated system with compensation.

matrix M is

$$M = -GCA^{-1}B = \begin{bmatrix} 5.8108 & -0.0540 \\ 12.6106 & 4.1657 \\ 23.1185 & -1.3166 \\ 1238.3 & -69.9317 \\ -24.6118 & -0.2590 \\ -174.9144 & 10.7056 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (51)$$

The stability of the system is guaranteed since $F - MH$ is Hurwitz. And the performance is improved quite a lot by the compensation matrix M of (51) as can be seen in Fig. 5.

7. Concluding Remarks

In this paper, we have proposed a design technique of the compensation matrix which results in effective saturation compensation for MIMO control systems with saturating actuators. The compensation matrix of the general ARW scheme is derived explicitly by minimizing the distances between the equilibrium points of the saturated and unsaturated system.

The resulting dynamics of the compensated controller exhibits the reduced model form of the unsaturated system in the sense of the singular perturbational method. Thus the states of the compensated controller can be approximately restored to the states of the unsaturated controller. This improves the performance of the closed-loop system when the actuator saturates.

The proposed method is meaningful even when the high frequency gain of the controller is not full rank though the conditioning technique can not be applied directly. The design procedure is simple in constrast to other compensation schemes and the simulation example shows the effectiveness of the proposed method.

References

Åström, K. J. and B. Wittenmark (1984). *Computer Controlled Systems: Theory and Design.* Englewood Cliffs, N.J.: Prentice-Hall.

Campo, P. J. and M. Morari (1990). Robust control of processes subject to saturation nonlinearities, *Computers and Chem. Engng.*, **14**, 4/5, 343-358.

Cottle, R. W. (1974). Manifestations of the Schur complement, *Lin. Alg. and Appl.*, **8**, 189-211.

Glattfelder, A. H. and W. Schaufelberger (1983). Stability analysis of single loop control systems with saturation and antireset-windup circuits, *IEEE Trans. Automatic Control*, **28**, 12, 1074-1081.

Hanus, R., M. Kinnaert, and J. L. Henrotte (1987). Conditioning technique, a general anti-windup and bumpless transfer method, *Automatica*, **23**, 6, 729-739.

Kapasouris, P., M. Athans, and G. Stein (1988). Design of feedback control systems for stable plants with saturating actuators, *27th IEEE Conf. on Decision and Control*, 469-479.

Kokotovic, P. V., H. K. Khalil, and J. O'Reilly (1986). *Singular perturbation methods in control: Analysis and design.* London: Academic Press.

Kokotovic, P. V., R. E. O'Malley, Jr, and P. Sannuti (1976). Singular perturbations and order reduction in control theory - An overview, *Automatica*, **12**, 3, 123-132.

Liu, Y. and B. D. O. Anderson (1989). Singular perturbation approximation of balanced systems, *Int. J. Control*, **50**, 4, 1379-1405.

Saksena, V. R., J. O'Reilly, and P. V. Kokotovic (1984). Singular perturbations and time-scale methods in control theory: Survey 1976 - 1983, *Automatica*, **20**, 5, 273-293.