DESIGN METHOD OF COMPUTER-GENERATED CONTROLLER FOR LINEAR TIME-PERIODIC SYSTEMS

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ABSTRACT

The purpose of this project is the presentation of new method for selection of a scalar control of linear time-periodic system. The approach has been proposed by Radziszewski and Zaleski [4] and utilizes the quadratic form of Lyapunov function. The system under consideration is assigned either in closed-loop state or in modal variables as in Calico, Wiesel [1]. The case of scalar control is considered, the gain matrix being assumed to be at worst periodic with the system period T, each element being represented by a Fourier series. As the optimal gain matrix we consider the matrix ensuring the minimum value of the larger real part of the two Poincare exponents of the system. The method, based on two-step optimization procedure, allows to find the approximate optimal gain matrix. At present state of art determination of the gain matrix for this case has been done by systematic numerical search procedure, at each step of which the Floquet solution must be found.

1. INTRODUCTION

$$\dot{x} = P(t)\,,\ x(t_0) = x_0,\ x \in R^2,\ t \in (t_0, \infty) \qquad (1)$$

The time-periodic system under consideration is of the form (1) where P(t) is periodic with the period T.

The closed loop state equation of the controlled system is of the form:

$$\dot{x} = [A(t) + G(t)K(t)]x \qquad (2)$$

where 1 x 2 matrix G(t) describes the distribution of control in the system and 2 x 1 matrix K(t) is the gain matrix (both matrices are assumed to be periodic with the same period T). We assumed the elements of the gain matrix to be represented by truncated Fourier series,

$$G(t) = [g_1(t), g_2(t)]^T$$

$$K(t) = [K_1(t), K_2(t)]$$

$$K_i(t) = K_i(0) + \sum_{n=1}^{m} [AK_i(n) \sin\frac{2\pi nt}{T} + BK_i(n) \cos\frac{2\pi nt}{T}]$$

2. LYAPUNOV FUNCTION

Introduce the quadratic form of Lyapunov function

$$V(x) = x^T S x \tag{3}$$

where S is the constant symmetric, positive-definite matrix. Introduce also the generalized norm $\|x\|$ induced by the scalar product $x^T S x$. The Lyapunov's derivative of (3) along solutions of (1) is of the form

$$\dot{V} = x^T (P^T S + SP) x$$

Introduce now the auxiliary matrix:

$$C(t) = S^{-1} P^T S + P \tag{4}$$

Due to the periodicity of P(t) the matrix P(t) the matrix C(t) is periodic. The eigenvalues of the matrix C(t) are the same as these the symmetric matrix $(P^T S + SP)$ and thus they are real. Denoting by $L_{max}(t)$ (also periodic) the maximal eigenvalue of the matrix C(t), the estimation holds

$$V(x(t)) \le V(x_0) \exp \left[\int_0^T L_{max}(t) dt \right] \tag{5}$$

Denoting

$$\Lambda = \frac{1}{T} \int_0^T L_{max}(t) dt$$

we get the estimation of the induced norm at the solution

$$\|x(T)\|_s \le \|x(0)\|_s \exp[\Lambda]$$

Thus Λ will serve as the estimation of the real part of the maximal Poincare exponent.

3. ANALYSIS

The maximum eigenvalue of the matrix C(t) is of the form

$$L_{max}(t) = \frac{TrC}{2} + \sqrt{\left(\frac{TrC}{2}\right)^2 + \det C}$$

By direct computation one can see that the trace TrC of the matrix C(t) depends only on the coefficients of the Fourier series, while the determinant detC depends as on these coefficients as on the elements of the matrix S

$$TrC = TrC\,(t, K_i(0), AK_i(1), , AK_i(m), BK_i(1), , BK_i(m)), \quad i = 1,2$$

$$detC = detC\,(t, K_i(0), AK_i(1), , AK_i(m), BK_i(1), , BK_1(m), S_{11}, S_{12}, S_{21}, S_{22}), \quad i = 1,2$$

This fact implies the two-sub-step procedure of optimization of Λ. At the first sub-step the quantity Λ is minimized while varying the values of the elements of the matrix S and freezing the values of the Fourier series coefficients. At the second sub-step Λ is minimized while varying the values of the Fourier series coefficients and freezing the previously chosen values of the elements of the matrix S. This procedure is continued till the improvement of the value of Λ becomes negligibly small. The simplification of the problem may be achieved by using the Cauchy-Schwartz inequality to approximate the integral in the expression for Λ.

Two possible representations of the matrix S were considered. The first one utilized the relation

$$S = M^T D M \tag{6}$$

where M is an orthogonal matrix, D is diagonal one, such that the element $d_{11}=1$ and $d_{22}>0$. For the second representation

$$S = \begin{bmatrix} \alpha & \beta \\ \beta & 1 \end{bmatrix} \qquad (7)$$

the positive definiteness condition is given directly

$$\alpha - \beta^2 > 0 \qquad (8)$$

The form (7) of the matrix S was selected for programming. For the first sub-step the program require the constrained minimization method. The Box's Complex Method was selected for this purpose. For the second sub-step the Simplex Method was applied.

4. MAIN RESULTS

The following examples are the practical applications and results that are derived from the numerical optimization procedures.

Example 1

The system of the form (2) was selected with the goal to determine the gain matrix minimizing the larger part of the Poincare exponent.

$$A = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$$

$$G(t) = [sint \quad -sint]$$

$$K_i(t) = K_i(0) + \sum_{n=1}^{5} [AK_i(n)\sin\frac{2\pi nt}{T} + BK_i(n)\cos\frac{2\pi nt}{T}]$$

$$i = 1,2$$

The initial gain matrix was chosen as having all the coefficients equal to zero. For these values $\Lambda=+0.1$ which shows instability of the system. Table 1 summarizes the values of gains from the optimization procedure for example 1. The resulting Poincare exponent estimate attains the value $\Lambda=-0.10999806<0$ showing that the system is stabilized.

i	1	2
$K_i(0)$	0.0918	0.2195
$AK_i(1)$	-1.0440	0.0129
$BK_i(1)$	0.0475	0.4887
$AK_i(2)$	0.1236	-0.0049
$BK_i(2)$	0.4224	0.4305
$AK_i(3)$	0.0562	-0.0138
$BK_i(3)$	0.3383	0.4781
$AK_i(4)$	0.1055	-0.0047
$BK_i(4)$	0.2163	0.4349
$AK_i(5)$	0.7579	0.2270
$BK_i(5)$	0.0411	0.2440

Table 1. Output Data for Example 1

Example 2

The system quoted after [3] is of the form (1)

$$P(t) = \begin{bmatrix} -1 + a\cos^2 t & 1 - a\,sint\,cost \\ -1 - a\,sint\,cost & -1 + a\sin^2 t \end{bmatrix} \qquad (9)$$

A closed-form solution of the system is known and is given by

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} e^{(a-1)t}cost & e^{-t}sint \\ -e^{(a-1)t}sint & e^{-t}cost \end{bmatrix} \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix} \qquad (10)$$

for all values of a. The solution (10) shows that asymptotic stability requires that $a<1$ and that for all values of $a\leq 0$ the value of Λ may not be less than -1. Rewriting the system in form (2) gives

$$\dot{x} = [A(t) + G(t)K(t)]x,$$

$$P(t) = A(t) + G(t)K(t)$$

$$= \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} + \begin{bmatrix} cost \\ -sint \end{bmatrix} [acost \quad -asint]$$

For selected series of initial values of a as shown at the Table 2 the results of procedure converge with the error of 0.09% to the expected value of $\Lambda=-1$.

Initial value of a	Λ
-2	-0.9999996
-5	-0.9999994
0	-0.9998086
2	-0.9991928
5	-0.9993884

Table 2. Output Data for Example 2

5. CONCLUSIONS

The active control of linear, time-periodic system has been considered and pole placement techniques have been developed by using Lyapunov quadratic function. Example 2 shows that if converges to the lower limit of the exact value of the system Poincare exponent. The close to optimal method that is introduced in this project will enable to control the linear time-periodic system and scalar control of two modes resulting in easier and simpler control system. Finally, analysis of the gain function is provided estimating their influence about the objective time-periodic systems. The conclusion of this project gives general approaching way using computational method for design controller of linear time-periodic systems.

6. REFERENCES

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