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A STATE-SPACE REALIZATION FORM OF MULTI-INPUT MULTI-OUTPUT TWO-DIMENSIONAL SYSTEMS

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ABSTRACT

In this paper, we propose a method for obtaining state-space realization form of two-dimensional transfer function matrices (2DTFM). It contains free parameters. And, we perform various consideration about it. Moreover, we present the conditions so that the state-space realization form exists.

1. INTRODUCTION

The transfer characteristics of two-dimensional digital systems, mixed lumped and distributed circuits, circuits containing variable parameters, delay-differential systems, and systems with time delays in multi-input multi-output system, can all be approximately expressed by two-dimensional transfer function matrices (2DTFM).

Many studies about the realization problem of this 2DTFM have been done [1]-[7], and they are very interesting. But, state-space realization form has not been reported.

Then, in this paper, we propose the state-space realization form which can be directly obtained from the coefficients of the given 2DTFM. This state-space realization form contains arbitrary constants, and by designating them properly, coefficients can be designated variously. By these free parameters, flexibility of circuit design can be extended, so that it is effective in low-sensitivity realization.

And, we perform various consideration about this state-space realization.

Moreover, we present the conditions so that the state-space realization exists.

2. A METHOD FOR OBTAINING STATE-SPACE REALIZATION

In this paper, we realize two-dimensional transfer function matrix of the type:

$$\mathbb{F}(s,z)=[g_{ij}(s,z)/f_{ij}(s,z)] \quad ((1,1)\leqq(i,j)\leqq(p,q)). \tag{1}$$

(i) We express (1) as

$$\mathbb{F}(s,z)=[\hat{g}_{ij}(s,z)]/f(s,z) \quad ((1,1)\leqq(i,j)\leqq(p,q)). \tag{2}$$

(ii) Consider the case of a separable-denominator 2DTFM, i.e.:

$$\hat{\mathbb{F}}(s,z)=[\hat{g}_{ij}(s,z)]/(\hat{f}(s)f_{n_1}(z))$$
$$((1,1)\leq(i,j)\leq(p,q)) \tag{3}$$

where $\hat{f}(s)$ in (3) is n_1-degree arbitrary polynomial and it is assumed that

$$\hat{f}(s)=\sum_{i=0}^{n_1-1}\hat{f}_i s^i+s^{n_1} \tag{4}$$

and $f_{n_1}(z)$ in (3) is coefficient polynomial of s^{n_1} in $f(s,z)$ in (2) and $f_{n_1}(z)$ is monic.

In (3), we regard s as the main variable and realize (3) by a controllable companion form with respect to s so that the following system is obtained:

$$\begin{cases}\mathbb{x}_S=\mathbb{A}\mathbb{w}_S+\mathbb{B}\mathbb{u}\\ \mathbb{y}=\mathbb{C}(z)\mathbb{w}_S+\mathbb{D}(z)\mathbb{u}\end{cases} \tag{5}$$
$$(\mathbb{x}_S=[x_{S1},x_{S2},\cdots\cdots,x_{S(n_1q)}]^T$$
$$\mathbb{w}_S=[w_{S1},w_{S2},\cdots\cdots,w_{S(n_1q)}]^T$$
$$x_{Si}=sw_{Si}\quad(i=1,2,\cdots\cdots,n_1q))$$

where $\mathbb{u}$ is input vector and $\mathbb{y}$ is output vector.

(iii) The following feedback transformation has the effect of transforming the system with separable-denominator 2DTFM (3) into the system with nonseparable-denominator 2DTFM (2):

$$\mathbb{u}=[(\hat{f}_0-f_0(z)/f_{n_1}(z))\mathbb{I}_q\cdots\cdots(\hat{f}_{n_1-1}$$
$$-f_{n_1-1}(z)/f_{n_1}(z))\mathbb{I}_q]\mathbb{w}_S+\mathbb{v} \tag{6}$$

where $f_i(z)$ $(i=0,1,\cdots,n_1)$ are coefficient polynomial of z^i $(i=0,1,\cdots,n_1)$ of $f(s,z)$ in (2) and $\mathbb{v}$ is control input vector.

It can be expressed as

$$\mathbb{u}=\mathbb{F}(z)\mathbb{w}_S+\mathbb{v}. \tag{7}$$

This transformation will be used below to obtain a minimal realization for a nonseparable-denominator 2DTFM from that of a separable-denominator 2DTFM.

By substituting (7) into (5),

$$\begin{cases}\mathbb{x}_S=(\mathbb{A}+\mathbb{B}\mathbb{F}(z))\mathbb{w}_S+\mathbb{B}\mathbb{v}\\ \mathbb{y}=(\mathbb{C}(z)+\mathbb{D}(z)\mathbb{F}(z))\mathbb{w}_S+\mathbb{D}(z)\mathbb{v}.\end{cases} \tag{8}$$

(8) is a realization by a controllable companion form with respect to s, of (2). It can be understood that, by substituting (6) into (5), $\mathbb{F}(s,z)$ in (2) can be realized.

(iV) The output equation in (5) can be expressed as

$$\mathbb{y}=[\mathbb{C}(z)\,\vdots\,\mathbb{D}(z)]\,[\mathbb{w}_S{}^T\,\vdots\,\mathbb{u}^T]^T. \tag{9}$$

In (9),

$$[\mathbb{C}(z)\,\vdots\,\mathbb{D}(z)] \tag{10}$$

is $p\times(n_1+1)q$ rational function matrix. Then, if $p>(n_1+1)q$, we realize (10) by a controllable companion form, if $p<(n_1+1)q$, we realize (10) by an observable companion form and substitute it into (5).

By this realization method, realization dimension of (10) becomes lower.

The obtained system can be expressed as

$$\begin{cases}\mathbb{x}_z=\hat{\mathbb{A}}\mathbb{w}_z+[\hat{\mathbb{B}}_1\,\vdots\,\hat{\mathbb{B}}_2]\,[\mathbb{w}_S{}^T\,\vdots\,\mathbb{u}^T]^T\\ \begin{bmatrix}\mathbb{x}_S\\ \mathbb{y}\end{bmatrix}=\begin{bmatrix}\mathbb{0}\\ \hat{\mathbb{C}}\end{bmatrix}\mathbb{w}_z+\begin{bmatrix}\hat{\mathbb{D}}_{11}&\hat{\mathbb{D}}_{12}\\ \hat{\mathbb{D}}_{21}&\hat{\mathbb{D}}_{22}\end{bmatrix}\begin{bmatrix}\mathbb{w}_S\\ \mathbb{u}\end{bmatrix}\end{cases} \tag{11}$$
$$(\mathbb{x}_z=[x_{z1},x_{z2},\cdots\cdots,x_{z n_2\cdot\min\{p,(n_1+1)q\}}]^T$$
$$\mathbb{w}_z=[w_{z1},w_{z2},\cdots\cdots,w_{z n_2\cdot\min\{p,(n_1+1)q\}}]^T$$
$$x_{zj}=zw_{zj}\quad(j=1,2,\cdots\cdots,n_2\cdot\min\{p,(n_1+1)q\}))$$

where n_2 is the degree of $\mathbb{F}(s,z)$ in (2) with respect to z.

(V) As $\mathbb{F}(z)$ in (7) is $q \times n_1 q$ rational function matrix, we realize $\mathbb{F}(z)$ by an observable companion form and substitute it into (7).

The obtained system can be expressed as

$$\begin{cases} \hat{x}_z = \hat{\tilde{\mathbb{A}}}\hat{w}_z + \hat{\tilde{\mathbb{B}}}w_S \\ u = \hat{\tilde{\mathbb{C}}}\hat{w}_z + \hat{\tilde{\mathbb{D}}}w_S + v \end{cases} \tag{12}$$

$$\begin{aligned} (\hat{x}_z &= [\hat{x}_{z1}, \hat{x}_{z2}, \text{------}, \hat{x}_{zq}]^T \\ \hat{w}_z &= [\hat{w}_{z1}, \hat{w}_{z2}, \text{-----}, \hat{w}_{zq}]^T \\ \hat{x}_{zj} &= z\hat{w}_{zj} \quad (j=1,2,\text{------},q)). \end{aligned}$$

(VI) Combining (11)-(12), the state-space realization of $\mathbb{F}(s,z)$ in (2) is given by

$$\begin{cases} \begin{bmatrix} x_S \\ x_z \\ \hat{x}_z \end{bmatrix} = \begin{bmatrix} \mathbb{A}_{11} & 0 & \mathbb{A}_{13} \\ \mathbb{A}_{21} & \mathbb{A}_{22} & \mathbb{A}_{23} \\ \mathbb{A}_{31} & 0 & \mathbb{A}_{33} \end{bmatrix} \begin{bmatrix} w_S \\ w_z \\ \hat{w}_z \end{bmatrix} + \begin{bmatrix} \mathbb{B}_1 \\ \mathbb{B}_2 \\ 0 \end{bmatrix} v \\ y = [\mathbb{C}_1 \mid \mathbb{C}_2 \mid \mathbb{C}_3]\,[w_S^T \mid w_z^T \mid \hat{w}_z^T]^T + \mathbb{D}v \end{cases} \tag{13}$$

where

$$\begin{aligned} &\mathbb{A}_{11} = \hat{\mathbb{D}}_{11} + \hat{\mathbb{D}}_{12}\hat{\tilde{\mathbb{D}}}, \quad \mathbb{A}_{13} = \hat{\mathbb{D}}_{12}\hat{\tilde{\mathbb{C}}}, \quad \mathbb{A}_{21} = \hat{\mathbb{B}}_1 + \hat{\mathbb{B}}_2\hat{\tilde{\mathbb{D}}} \\ &\mathbb{A}_{22} = \hat{\mathbb{A}}, \quad \mathbb{A}_{23} = \hat{\mathbb{B}}_2\hat{\tilde{\mathbb{C}}}, \quad \mathbb{A}_{31} = \hat{\tilde{\mathbb{B}}}, \quad \mathbb{A}_{33} = \hat{\tilde{\mathbb{A}}} \\ &\mathbb{B}_1 = \hat{\mathbb{D}}_{12}, \quad \mathbb{B}_2 = \hat{\mathbb{B}}_2, \quad \mathbb{C}_1 = \hat{\mathbb{D}}_{21} + \hat{\mathbb{D}}_{22}\hat{\tilde{\mathbb{D}}}, \quad \mathbb{C}_2 = \hat{\mathbb{C}} \\ &\mathbb{C}_3 = \hat{\mathbb{D}}_{22}\hat{\tilde{\mathbb{C}}}, \quad \mathbb{D} = \hat{\mathbb{D}}_{22}. \end{aligned}$$

3. VARIOUS CONSIDERATION

(i) The dimension of state-space realization in (13) is

$$(n_1+n_2)q + n_2 \cdot \min(p, (n_1+1)q) \tag{14}$$

at most.

And, in the denominator of (3), let the polynomial of z be arbitrary, and we regard z as the main variable and realize (3) with respect to z, and perform the similar step, so that the dimension of the obtained state-space realization is

$$(n_1+n_2)q + n_1 \cdot \min(p, (n_2+1)q) \tag{15}$$

at most.

Consequently, in order that the dimension of the state-space realization becomes lower, in case that $n_1 > n_2$, we should realize $\hat{\mathbb{F}}(s,z)$ in (3) with respect to s formerly, in case that $n_1 < n_2$, we should realize it with respect to z formerly.

(ii) In case that we realize $\hat{\mathbb{F}}(s,z)$ in (3) by an observable companion form with respect to s or z, the feedback transformation is fairly complicated and state-space realization can't be obtained easily.

(iii) (8), i.e., realization by a controllable companion form with respect to s, of $\mathbb{F}(s,z)$ in (2), can be expressed as

$$\begin{bmatrix} x_S \\ y \end{bmatrix} = \begin{bmatrix} \mathbb{A}_C(z) & \mathbb{B}_C \\ \mathbb{C}_C(z) & \mathbb{D}_C(z) \end{bmatrix} \begin{bmatrix} w_S \\ v \end{bmatrix}. \tag{16}$$

In case that we realize (16) in order to obtain state-space form, realization dimension is

$$(n_1+2n_2)q + 2n_2 \cdot \min(p, n_1 q) \tag{17}$$

, (17) is much higher than (14).

By dividing (8) into (5) and (6), the dimension of the state-space realization can be reduced fairly.

Therefore, to begin with, we realize (3), and perform (6) on it.

(IV) In [7], $\mathbb{F}(s,z)$ in (2) is realized by an observable companion form with respect to s. It can be expressed as

$$\begin{pmatrix} x_S \\ y \end{pmatrix} = \left(\begin{array}{c|c} \mathbb{A}(z) & \mathbb{B}(z) \\ \hline \mathbb{C} & \mathbb{D}(z) \end{array}\right) \begin{pmatrix} w_S \\ u \end{pmatrix}. \quad (18)$$

In case that we realize (18) in order to obtain state-space form, realization dimension is

$$(n_1+2n_2)p+2n_2\cdot\min(q,n_1p) \quad (19)$$

, (19) is also much higher than (14).

(V) As the state-space realization in (13) contains arbitrary constants $\hat{f}_k$ (k=0,1,-----,n_1-1), free parameters exist in the state-space realization.

4. EXISTENCE OF STATE-SPACE REALIZATION

(In case that s(z) is a dynamical variable)

We calculate backward 2DTFM from (13).

$$\hat{\mathbb{F}}(s,z)=(f_{n_1}(z)/f_{n_1}(z))\mathbb{F}(s,z) \quad (20)$$

That is to say, $\mathbb{F}(s,z)$ in (2) can be realized with common factor $f_{n_1}(z)$ between denominator and numerator of each entry. Therefore, if $f_{n_1}(z)$ in (2) is not a stable polynomial, the realization system is unstable.

In case that we realize (3) with respect to z formerly in order to obtain state-space form, we express the denominator of (2) as

$$\sum_{j=0}^{n_2}\hat{f}_j(s)z^j. \quad (21)$$

It is necessary that $\hat{f}_{n_2}(s)$ in (21) is a stable polynomial.

(In case that s(z) is a statical variable)

From $\mathbb{A}_{11}$, $\mathbb{A}_{22}$, $\mathbb{A}_{33}$ in (13), it can be understood that if $f_{n_1}(z)$ in (2) and $\hat{f}_{n_2}(s)$ in (21) satisfy the following [Condition 1], loops containing positive coefficients without dynamical elements exist in the circuit expressing (13), so that (13) is unstable.

[Condition 1]

In case that variable range of s(z) is positive, all coefficients of $\bar{f}_{n_i}(w)$ (expressing $f_{n_1}(z)$ or $\hat{f}_{n_2}(s)$ (w expresses s or z)) have the same sign.

In case that variable range of s(z) is negative, it is assumed that

$$\bar{f}_{n_i}(w)=\sum_{k=0}^{n_j-1}\bar{f}_{n_i}^{(k)}w^k+w^{n_j}. \quad (22)$$

If n_j (j=1,2) is odd number,

$$\bar{f}_{n_i}^{(k)} \quad (k=0,2,-----,n_j-1)<0 \quad (23)$$

and

$$\bar{f}_{n_i}^{(k)} \quad (k=1,3,-----,n_j-2)>0. \quad (24)$$

If n_j (j=1,2) is even number,

$$\bar{f}_{n_i}^{(k)} \quad (k=0,2,------,n_j-2)>0 \quad (25)$$

and

$$\overline{f}^{(k)}_{n_i} \quad (k=1,3,\cdots,n_j-1) < 0. \qquad (26)$$

Because, $\mathbb{A}_{11}$, $\mathbb{A}_{22}$, $\mathbb{A}_{33}$ in (13) are companion matrix, the circuit expressing (13) has feedback loop containing only several potentiometers $1/\omega$, $-\overline{f}^{(k)}_{n_i}$. If [Condition 1] is not satisfied, feedback loop whose product of potentiometers is positive yields, then, the above-mentioned loops exist.

In case that [Condition 1] is not satisfied, it is possible to perform the following variable transformation

$$\omega=(c\hat{\omega}+d)/(a\hat{\omega}+b) \qquad (27)$$

on $\mathbb{F}(s,z)$ in (2), so that [Condition 1] is satisfied.

5. CONCLUSIONS

We presented a state-space realization form of multi-input multi-output two-dimensional system. Arbitrary constants $\hat{f}_k$ $(k=0,1,\cdots,n_1-1)$ in it, can be designated freely. By them, flexibility of elements in the circuit increase and it is effective in low-sensitivity realization.

Moreover, we obtained the conditions so that the state-space realization exists.

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