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Receding Horizon LQG Controller with FIR Filter

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Abstract

When there exist parameter uncertainty, modelling errors and nonminimum phase zeros in control object system, the stability robustness of conventional LQG and LQG/LTR methods are not satisfactory[2,8]. Since these methods are performed on the infinite horizon, it is very hard to establish exact design parameters and thus they have lots of problems to be applied to real systems. So in this paper we propose RHLQG/FIRF optimal controller which has robust stabilty against parameter uncertainty, nonminimum phase zeros and modelling errors. This method uses only the information around at present and therefore shows good performance even when we do not know exact design parameters. We here compare LQG and LQG/LTR method with RHLQG/FIRF controller and exemplify that RHLQG/FIRF controller has better robust stability performance via simulations.

1. Introduction

The LQ controller is known to be robust against parameter variations and model uncertainty because of its excellent gain and phase margin [1]. However, these excellent properties are not guaranteed in the corresponding LQG control design. The LQG controller may become unrobust against parameter variation and model uncertainty in plant [2],[8]. To improve the robustness of the LQG controller, an asymptotic recovery method, known as LQG/LTR(Linear Quadratic Gaussian with Loop Transfer Recovery), has been introduced, which recover stability margins of the corresponding LQ controller or Kalman filter. It has been recognized that in spite of its remarkable robustness the LQG/LTR method may suffer from poor stability property for the parameter variation, nonminimum phase system and time-varying systems[8]. Also remarkable results have been known that the LQ controller may acquire far off unstable modes for small variations in the plant parameter[2]. These findings cast some doubts practically on the asymptotic recovery, even in the minimum phase case, since the recovered stability margins cannot guarantee the LQG design robustness for all possible quadratic performance criteria.

So in this paper we propose a new control method, named RHLQG/FIRF(Receding Horizon LQG with FIR Filter), for enhancing the stability robustness to the system parameter uncertainty and modelling error. The conventional LQG and LQG/LTR method are performed on the infinite horizon so it is very difficult to estabilish exact design parameters and thus they have much problems to be applied to real applications. Hence there are many designer's interested in solving the problems proposed here. In this paper we proposed the RHLQG/FIRF optimal controller which has robust stability against parameter variation and modelling errors. This control method uses only the information around at present therefore shows nice performance even when we do not know exact design parameters.

Here we analytically shows that the RHC method [4] is robuster than LQ control method and the FIR filter [3] is robuster than Kalman filter, respectively, using the FARE (Fake Algebraic Riccati Equation) [5]. Finally, we here compare LQG and LQG/LTR with proposed control method and exemplify via simulation that RHLQG/FIR controller has better stability robustness.

This paper is organized as follows: In section 2 we derive RHLQG/FIRF algorithm. In section 3 we show that the RHC and FIR filter have better robustness than the LQ controller and Kalman filter. In section 4 we exemplify the robustness of RHLQG/FIRF algorithm via computer simulation and in section 5 the conclusion will be given.

2. RHLQG/FIRF Algorithm

In this section we shall begin examining design methods based on optimal control theory . The particular theory we shall be concerned with here is that of the so-called RHLQG/FIRF problem. The problem addressed is the following: Suppose that we have a plant model in time varying state-space form

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t)$$
$$y(t) = Cx(t) + v(t) \qquad (2.1)$$

where $x(t)$ is a $n\times 1$ state vector, $u(t)$ is a $m\times 1$ system input vector , $y(t)$ is a $l\times 1$ system output vector, A, B and C are time varying matrices with appropriate dimensions, and w and v are zero-mean white Gaussian noises with covariances

$$E[w(t)w^T(s)] = Q\,\delta(t-s)$$
$$E[v(t)v^T(s)] = I\,\delta(t-s) \qquad (2.2)$$

It is assumed here that w and v are uncorrelated with each other. The problem is then to devise a feedback-control law which minimizes the cost function

$$J = E\{x^T(t+T_c)Fx(t+T_c) + \int_t^{t+T_c} [y^T(\tau)Q_y y(\tau) + u^T(\tau)R_u u(\tau)]d\tau\} \qquad (2.3)$$

where

$$F \geq 0,\ Q_y \geq 0 \text{ and } R_u > 0$$

are weighting matrices. It is also assumed that available informations at present time t are Q_y and R_u on the interval $[t,\ t+T_c]$ and the measurement data y on the past $[t-T_f,\ t]$.

The solution to the RHLQG/FIRF problem states that the optimal result is achieved by the following procedure. The first subproblem is to find the control law which will minimize the cost function. The control law is determined by RHC(Receding Horizon Control) method as following algorithm:

$$u(t) = -R_u^{-1}B^TK(t,t+T_c)\hat{x}(t|T) := -P(t)\hat{x}(t|T) \qquad (2.4)$$

where $K(t,t+T_c)$ satisfies the matrix Riccati equation given by

$$-\frac{\partial}{\partial s}K(s,t+T_c) = A^TK(s,t+T_c)+K(s,t+T_c)A+C^TQ_yC-K(s,t+T_c)BR_u^{-1}B^TK(s,t+T_c)$$
$$t \leq s < t + T_c \qquad (2.5)$$

$$K(t+T_c,t+T_c) = F\ (= \infty I) \qquad (2.6)$$

$$P(t) := R_u^{-1}B^TK(t,t+T_c)$$

and $\hat{x}(t|T)$ is the FIR filter.

The second subproblem is to determine the optimal FIR filter, which is shown to have the structure of a state observer as follows:

$$\hat{x}(t|T) = \int_{t-T}^{t} H(t,\tau;T)z(\tau)d\tau + \int_{t-T}^{t} H_u(t,\tau;T)u(\tau)d\tau \qquad (2.7)$$

where the impulse response $H(t,\cdot;T)$ and $H_u(t,\cdot;T)$ are derived as follows:

$$H(t,s;T) = S^{-1}(t,T)L(t,s;T),\ t-T \leq s \leq t,\ T \geq \lambda_o \qquad (2.8)$$

$$\frac{\partial}{\partial\sigma}L(t,s;\sigma) = -[A^T+S(t,\sigma)BQB^T]L(t,s;\sigma),\ 0 \leq T-t+s < \sigma \leq T \qquad (2.9)$$

$$L(t,s;T-t+s) = C^T$$

$$\frac{\partial}{\partial\sigma}S(t,\sigma) = -S(t,\sigma)A - A^TS(t,\sigma) + C^TC - S(t,\sigma)BQB^TS(t,\sigma) \qquad (2.10)$$

$$S(t,0) = 0,\ 0 < \sigma \leq T$$

$$H_u(t,s;T) = \int_{t-T}^{s} H(t,\tau;T)C\Phi(\tau,s)Gd\tau,\ t-T \leq s \leq t \qquad (2.11)$$

where λ_o in Eq. (2.8) is the observability index and $\Phi(\cdot,\cdot)$ in (2.11) is the transition matrix of A.

To judge the closed-loop stability of the candidate design method, we show the closed-loop stability condition, which is given by the following theorem:

<u>[Theorem 2.1]</u> : (Stability of RHLQG/FIRF)

If the system is completely controllable and completely observable then the RHLQG/FIRF stabilizes the system with $T \geq l_o$ and $T_c \geq l_c$, where l_o and l_c are the controllability and the observability index, respectively.

[Proof] The closed loop system without noise is given as follows:

$$\dot{x} = Ax(t) + Bu(t)$$
$$= Ax(t) - BP(t)\hat{x}(t|T)$$
$$= [A-BP(t)]x(t) + BP(t)\tilde{x}(t)$$

where $\tilde{x}(t) := x(t) - \hat{x}(t|T)$. Since the FIRF becomes a deadbeat observer for noise free systems under the completely observability [3] the asymptotic stability of $\tilde{x}(t)$ is guaranteed automatically. The complete controllability then guarantees the asymptotic stability of $x(t)$ using the RHC method [4], which completes the proof. □□□

In the case of linear time-invariant systems, the RHLQG/FIRF has very simple forms with the constant gain feedback as $u(t) = -P\hat{x}(t|T_f)$ and the time-invariant state estimator

$$\hat{x}(t|T) = \int_0^T H(\tau;T)y(t-\tau)d\tau + \int_0^T H_u(\tau;T)u(t-\tau)d\tau.$$

Note that in LQG/LTR the parameter Q_y, R_u, Q and R are design parameters[8] but, on the other hand, they are given as *a priori* information available in the case of RHLQG/FIRF, which guarantees the optimality in the sense of receding horizon. Moreover, RHLQG/FIRF is robuster to parameter uncertainty than LQG/LTR and LQG as shown later in this paper.

<u>3. Robustness Analysis of RHLQG/FIRF</u>

In this section we consider robustness property of RHC and optimal FIR filter. It will be shown that since RHC and FIR filter are based on the finite horizon they are robuster than LQ control and Kalman filter which are based on the infinite horizon. We prove this analytically here.

To analyse the robustness in frequency domain, we assume that the plant is time-invariant in this section.

3.1. Robustness of the RHC

The LQ controller has been known to have excellent gain and phase margin in time invariant systems. However, it appears to have very poor performance when applied to the parameter variation, model uncertainty and time varying systems. We here compare the frequency domain robustness of RHC with that of LQ, which shows that RHC has better robustness than LQ controller.

First of all, we introduce the Fake ARE[6,7] for the frequency domain equality of Eq.(2.5)

[Lemma 3.1] Consider the FARE

$$\dot{Q}(s) = K(s,t+T_c)B^TR_u^{-1}BK(s,t+T_c)-A^TK(s,t+T_c)-K(s,t+T_c)A \quad (3.1)$$

and assume that

i) [A,B] is a controllable pair,
ii) $\dot{Q}(s) \geq 0$ and $[A,\dot{Q}^{1/2}(s)]$ is a stabilizable pair.

Then RHC feedback law (2.4) asymptotically stabilizes the plant.

Define $\dot{Q}(s) = Q + \dot{K}(s,t+T_c)$. Then Eq.(3.1) comes from (2.5). The connection between monotonicity of $\{K(s,t+T_c)\}$ and stabilizability of $[A,\dot{Q}^{1/2}(s)]$ then emerges [7]. Note that it is the stabilizability of the pair $[A,\dot{Q}^{1/2}(s)]$(when $\dot{Q}(s) \geq 0$) that determines the asymptotic stability of (3.1), since then $K(s,t+T_c)$ satisfies a legitimate ARE.

□□□

Let us investigate the frequency domain characteristics of RHC as shown in the following theorem:

[Theorem 3.1] : The return difference matrix $T_{RHC}(s)$ of the system with RHC satisfies the following relation in the frequency domain:

$$T_{RHC}^T(-jw)R_uT_{RHC}(jw) = R_u + B^T\Phi^T(-jw)\dot{Q}\Phi(jw)B \quad (3.2)$$

where

$$\Phi(s) := (sI - A)^{-1} \text{ and } \dot{Q} = C^TQ_yC + \frac{\partial}{\partial\tau}K(\tau,t+T_c)\Big|_{\tau=t.} \quad (3.3)$$

□□□

The above result comes from the same procedure as that of LQ controller [8] using the definition (3.3). From Theorem 3.1 we intuitively recognize that it is possible to show the robustness of RHC comparing to that of LQ controller.

Note that in RHC problem $K(s,t+T_c)$ is a monotone increasing function and that $\frac{\partial}{\partial\tau}K(\tau,t+T_c)\Big|_{\tau=t} \geq 0$ for all t.

Thus we have $\dot{Q} \geq C^TQ_yC$ and $\bar{\sigma}\{T_{RHC}^{-1}(jw)\} \leq \bar{\sigma}\{T_{LQ}^{-1}(jw)\}$ where $T_{LQ}(s)$ is the return difference matrix. We also have

$$\bar{\sigma}\{I - T_{RHC}^{-1}(jw)\} \leq \bar{\sigma}\{I - T_{LQ}^{-1}(jw)\}, \quad (3.4)$$

which implies that RHC is robuster than LQ controller since the smaller the upper bound of $\bar{\sigma}\{I - T(jw)\}$ is, the robuster is the feedback control system.

3.2. Robustness of the FIR Filter

Robustness of the FIR filter can be analysed by a dual form of RHC.

The return difference matrix $T_{FIR}(s)$ of FIR filter satisfies the following relation in the frequency domain:

$$T_{FIR}(jw)T_{FIR}^T(-jw) = I + C\Phi(jw)\bar{R}\Phi^T(-jw)C^T \quad (3.5)$$

where

$$\bar{R} := BQB^T - \frac{\partial}{\partial\sigma}R(t,\sigma)\Big|_{\sigma=T} \quad (3.6)$$

and $R(t,\sigma) := S^{-1}(t,\sigma)$

Since $R(t,\sigma)$ is a monotone decreasing function, we have $\frac{\partial}{\partial\sigma}R(t,\sigma) \leq 0$ for all t and $\bar{R} \geq BQB^T$ from Eq.(3.6). Hence the following inequalities are derived from (3.5):

$$\bar{\sigma}\{T_{FIR}^{-1}(jw)\} \leq \bar{\sigma}\{T_{KF}^{-1}(jw)\}$$

$$\bar{\sigma}\{I - T_{FIR}^{-1}(jw)\} \leq \bar{\sigma}\{I - T_{KF}^{-1}(jw)\}, \quad (3.7)$$

where $T_{KF}(s)$ is the return difference matrix of the Kalman filter. Eq.(3.7) implies that the FIR filter is robuster than the Kalman filter.

In this section we seperately verify the robustness of RHC and FIR filter and show that RHC and FIR filter are robuster than LQ and Kalman filter, respectively. Thus we can conjecture that RHLQG/FIRF controller is robuster than LQG controller. By the way, even though LQ controller and Kalman filter separately have good robustness each other, however, when two methods are combined to be applied to LQG problem, they may lose robustness. Hence the robustness of RHLQG/FIRF is also in question and requires further research.

4. Simulation

We shall now use the RHLQG/FIRF approach to design a compensator for the nonminimum phase plant given by the equations:

$$\dot{x} = \begin{bmatrix} -4 & -3 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 35 \\ -61 \end{bmatrix} w \quad (4.1)$$

$$y = [\,1 \;\; -2\,]\,x + v \quad (4.2)$$

with $E(w)=E(v)=0$: $E[w(t)w(s)] = E[v(t)v(s)] = \delta(t-s)$. The plant in this simulation is nonminimum phase system with transfer function $G(s)=(s-2)/(s+1)(s+3)$. When we take the sampling period as $T = 0.1$[sec], the discrete time system is represented by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Gw(k) \\ y(k) &= Cx(k) + v(k) \end{aligned} \tag{4.3}$$

where

$$A=\begin{bmatrix} 0.6588 & -0.2460 \\ 0.0820 & 0.9868 \end{bmatrix},\quad B=\begin{bmatrix} 0.0820 \\ 0.0044 \end{bmatrix}$$

$$C=[1\ \ -2],\quad G=\begin{bmatrix} 35 \\ -61 \end{bmatrix}$$

Nyquist diagrams for the LQG/LTR and RHLQG/FIRF are given in Fig.1 and Fig.2. This simulation result shows that the LQG/LTR given in Fig.1 is encircle the loci (-1,0) point and that when LQG/LTR controller is applied to nonminimum phase plant the system become unstable. So LQG/LTR control method is not appropriate to design a plant with modelling error and nonminimum phase zeros. As shown Nyquist diagrams in Fig.2, the RHLQG/FIRF is well suited to stabilize the system even under parameter variation and nonminimum phase zeros.

5. Conclusions

This paper proposed RHLQG/FIRF controller which has better robustness characteristics than conventional LQG or LQG/LTR design methods. We verify that RHC and FIR filter have better robustness than LQ and Kalman filter, respectively. Even though RHC and FIR filter have a good robust characteristics, the robustness of RHLQG/FIRF is also in question and thus requires further research.

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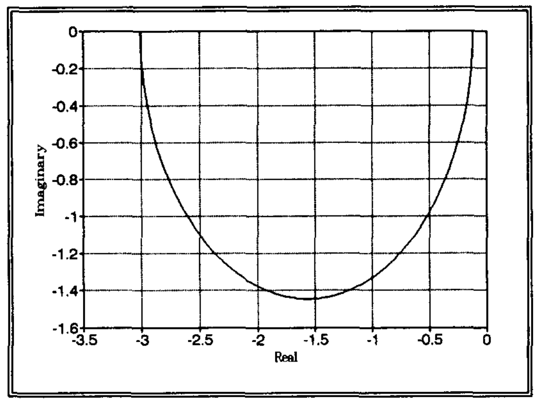

Fig.1 A Nyquist diagram of LQG/LTR controller for system (4.1)

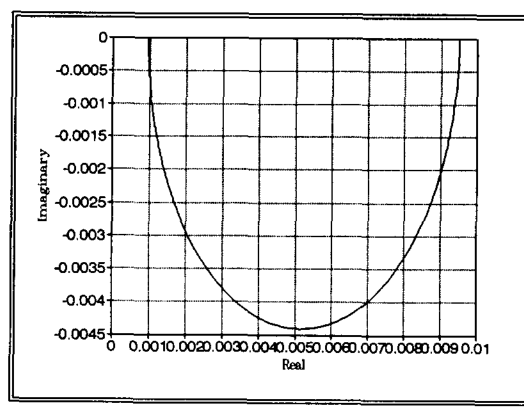

Fig.2 A Nyquist diagram of RHLQG/FIRF for system (4.1)