Leak Detection in a Pipeline Based on Estimation Theory

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ABSTRACT

A leak detection method for diagnosis of the leak position in a pipeline was developed using an estimation theory with the assumption that the measured flow rates and pressures are stochastic processes. A notch filter was designed using power spectral density analysis of measurements to reduce the effects of disturbances. The noise model dimension was determined by hypothesis testing and then recursive extended least square method was applied to estimate the leak position in real time. The proposed method was applied to an experimental system for evaluation of its performance.

INTRODUCTION

Undesirable leaks in pipelines for transportation of liquids or gases can cause problems in safety and environmental pollution as well as loss of money. Therefore it is important to early detect and localize the leaks when they occur.

Usually the leak detection system has only a limit number of sensors for the measurement of flow rate and pressure. In addition, as pipelines are spread over a wide place, they are susceptible to various disturbances. The leak detection system should be designed to be reliable enough while overcoming these unfavorable situations.

Leak detection methods for pipelines using nonlinear adaptive state observers and correlation techniques were introduced by Billmann and Isermann (1984). Lee et al. developed a leak detection system based on physical modeling (1991).

In this study a leak detection method for localization of leaks in a pipeline was developed using an estimation theory with the assumption that the measurements of flow rates and pressures at the beginning and the end of a pipeline are stochastic processes. In order to reduce the effects of disturbances on measurements a filter was designed using an analysis of the power spectral density of the process. The parameters of the model with this filter was estimated using recursive extended least square method, i.e, the leak position in a pipeline was evaluated in real time. For the determination of the noise model dimension hypothesis testing was used. The obtained results are shown for leak experiments with a water pipeline.

MATHEMATICAL PIPELINE MODELS

A mathematical model of the pipeline is generally obtained by the mass, momentum and heat balances. For simplifying the modeling of the gas and liquid flow dynamics in a pipeline a constant diameter, a turbulent flow and isothermic condition are assumed.

For the pipe element of length dz the mass and momentum balances are

$$A\frac{\partial \rho}{\partial t} + A\frac{\partial(\rho v)}{\partial z} = 0 \quad (1)$$

$$A\frac{\partial(\rho v)}{\partial t} + A\frac{\partial P}{\partial z} + A\frac{\partial(\rho v^2)}{\partial z} = -\rho A g \sin\theta - A\frac{2f}{D}\rho v^2 \quad (2)$$

where A, D, ρ, v, P, t and z are constant sectional area, constant diameter, density, velocity of flow, pressure, time coordinate and length coordinate, respectively. The influence of the height is represented by the first term of the right side of Eq. (2) and the friction influence by the second term.

Introducing the flow rate

$$q = A\rho v \tag{3}$$

Equations (1) and (2) become

$$A\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial z} = 0 \tag{4}$$

$$\frac{1}{A}\frac{\partial q}{\partial t} + \frac{\partial P}{\partial z} + \frac{1}{2\rho A^2}\frac{\partial q^2}{\partial z} = -\rho g \sin\theta - \frac{2fq^2}{D\rho A^2} \tag{5}$$

The friction factor, f, in Eq. (5) can be obtained from the relation between N_{Re} and f for turbulent flow in smooth round pipes, known as the von Kármán equation

$$\frac{1}{\sqrt{f}} = 4.06\log\left(N_{Re}\sqrt{f}\right) - 0.60 \tag{6}$$

Reynolds number, N_{Re}, can be calculated by the equation

$$N_{Re} = \frac{4q}{\mu D\pi} \tag{7}$$

where μ is viscosity of fluid; and then the friction factor, f, is calculated from the flow rate.

MODELING FOR LIQUID PIPELINES

As the dynamics of a liquid pipeline are very fast in comparison to gas pipelines and the incompressibility of liquid is very strong, it is possible to localize the leak position in a liquid pipeline with static model.

For the liquid pipeline shown in Fig. 1 with a positive flow rate and a constant density Eqs. (4) and (5) are reduced to

$$\frac{\partial q}{\partial z} = 0 \tag{8}$$

$$\frac{\partial P}{\partial z} = -\frac{2\alpha f q^2}{D\rho A^2} - \rho g \sin\theta \tag{9}$$

where the correction factor, α, is introduced to correct the error by model specification, calculation of friction factor, experimental condition, etc. Equations (8) and (9) are the static model for liquid pipelines.

LEAK DETECTION BASED ON STATIC MODEL

Solving the static mass and momentum balances for q and P to the boundary condition $q = q_0$ and $P = P_0$ at $z = 0$, respectively, we obtain

$$q = q_0 = constant \tag{10}$$

$$P = P_0 - \left(\frac{2\alpha f_0 q_0^2}{D\rho A^2} + \rho g \sin\theta\right) z \tag{11}$$

where f_0 is the friction factor based on inlet flow rate q_0 and is calculated from Eqs. (6) and (7). Equations (10) and (11) represent the flow rate and pressure distributions in a liquid pipeline which has no leaks. The pressure-distribution curve for a liquid pipeline in the case of no leaks is shown in Fig. 2.

When a leak occurs at location z_L, however, it is convenient for solving the mass and momentum balances that the pipeline is considered as a system composed a part from inlet to z_L and the next part from z_L to outlet. The flow rate and pressure distributions of the part from inlet to leak position is represented by Eqs. (10) and (11). The static mass and momentum balances of the part from leak position to outlet are solved for q and P using the boundary conditions that $q = q_N$ and $P = P_N$ at $z = z_N$, respectively. The resulting equations to be solved are

$$q = q_N = constant \tag{12}$$

$$P = P_N + \left(\frac{2\alpha f_N q_N^2}{D\rho A^2} + \rho g \sin\theta\right)(z_N - z) \tag{13}$$

The correction factor, α, is evaluated from Eq. (11) using the condition $P = P_N$ at $z = z_N$ when no leaks occur. The correction factor thus becomes

$$\alpha = \frac{D\rho A^2}{2f_0'q_0'^2}\left(\frac{P_0'-P_N'}{z_N} - \rho g\sin\theta\right) \quad (14)$$

where q_0', f_0', P_0' and P_N' are inlet flow rate, friction factor based on q_0', inlet pressure and outlet pressure in the case of no leaks, respectively. The pressure-distribution curve for a liquid pipeline in the case of leak occurrence is shown in Fig. 3.

If the pipeline correction factor, α, is evaluated using the measurements before leaks occur, then the leak position, z_L, can be obtained from Eqs. (11) and (13) using the condition that the pressure-distributions of the part from inlet to z_L and the part from z_L to outlet have the same value at $z = z_L$. Thus

$$z_L = \frac{D\rho A^2(P_0-P_N) - \left(2\alpha f_N q_N^2 + D\rho^2 A^2 g\sin\theta\right) z_N}{2\alpha\left(f_0 q_0^2 - f_N q_N^2\right)} \quad (15)$$

LEAK DETECTION BASED ON ESTIMATION THEORY

As pipelines are spread over a wide place, they are susceptible to various disturbances. The leak detection system should be designed to be reliable enough while overcoming these unfavorable situations. The main noise may be the environmental elements like the vibration by pumps. If an appropriate model for these colored noises is obtained, the more accurate estimation of the leak position in a pipeline can be performed. Thus the pipeline model with the noise elements have to be used, and then the diagnosis of the leak position in a pipeline can be performed using an estimation theory with the assumption that the measured flow rates and pressures are stochastic processes.

Consider a model of the form

$$y(t) = u(t)\,\theta + v(t) \quad (16)$$

where $y(t)$ is a measurable quantity, $u(t)$ is a known quantity, θ is an unknown parameter and $v(t)$ is a colored noise. The colored noise, $v(t)$, can be represented by filtered white noise and then Eq. (16) becomes

$$y(t) = u(t)\,\theta + \frac{C(q^{-1})}{A(q^{-1})}e(t) \quad (17)$$

where $e(t)$ is a white noise, q^{-1} is the backward shift operator ($q^{-1}e(t) = e(t-1)$, etc.) and

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{na}q^{-na} \quad (18)$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_{nc}q^{-nc} \quad (19)$$

Equation (17) is rewritten as

$$A(q^{-1})y(t) = A(q^{-1})u(t) + C(q^{-1})e(t) \quad (20)$$

or

$$y^F(t) = u^F(t)\,\theta + C(q^{-1})e(t) \quad (21)$$

where superscript F denotes the filtered signal and

$$y^F(t) \equiv A(q^{-1})y(t) \quad (22)$$

$$u^F(t) \equiv A(q^{-1})u(t) \quad (23)$$

If $y(t)$ and $u(t)$ are essentially zero-frequency signals and their power spectral density are profiled as the curve shown in Fig. 4, then the frequency ω may be the domain frequency of the noise. Thus for well-modeling of the noise elements $1/A(q^{-1})$ has to be designed to have a resonance peak at the same frequency.
The filter $A(q^{-1})$ can be designed as the simple form of

$$A(q^{-1}) = (1 - a e^{j\omega T})(1 - a e^{-j\omega T}) \quad (24)$$

where T is a sampling time and a is the value in range of 0.9 to 1.0. The Bode diagram of $1/A(q^{-1})$ from Eq. (24) is shown in Fig. 5.

Equations (14) and (15) can be rewritten for α and z_l in the form of Eq. (17), respectively.

$$P_0' - P_N' - \rho g z_N \sin\theta = \frac{2f_0'q_0'^2 z_N}{D\rho A^2}\alpha + \frac{C(q^{-1})}{A(q^{-1})}e_k \quad (25)$$

$$P_0 - P_N - \left(\frac{2\alpha f_N q_N^2}{D\rho A^2} + \rho g \sin\theta\right) z_N = \frac{2\alpha\left(f_0 q_0^2 - f_N q_N^2\right)}{D\rho A^2} z_L + \frac{C(q^{-1})}{A(q^{-1})} e_k \qquad (26)$$

The correction factor, α, is evaluated from Eq. (25) using the measurements before leak occurrence, and after a leak occur the leak position, z_L, can be evaluated from Eq. (26) using the calculated α and the measured signals. $A(q^{-1})$ is designed by the analysis of the power spectral density for the terms in Eqs. (25) and (26) corresponding to $y(t)$ and $u(t)$ in Eq. (17). Then Eqs. (25) and (26) can be rewritten in the form of Eq. (21), and the recursive extended least squares (RELS) method is applied to estimate α and z_L in real time.

An appropriate noise model dimension, *nc*, in Eq. (19) can be obtained by hypothesis testing method.

INTERVAL ESTIMATES OF PARAMETERS

Suppose that data are generated according to Eq. (16). Assume further that $v(t)$ is a stochastic variable with zero mean and variance σ^2, $N(0,\sigma^2)$. In matrix form Eq. (16) is written as

$$Y = \Phi\theta + E \qquad (27)$$

Under these conditions $\hat{\theta}$ has normal distribution with mean θ and variance $(\Phi^T\Phi)^{-1}\sigma^2$,

$$\hat{\theta} \sim N\left(\theta, (\Phi^T\Phi)^{-1}\sigma^2\right) \qquad (28)$$

Let a_{ii} be ith diagonal element of $(\Phi^T\Phi)^{-1}$, then

$$\hat{\theta}_i \sim N\left(\theta_i, a_{ii}\sigma^2\right) \qquad (29)$$

Introducing

$$z = \frac{\hat{\theta}_i - \theta_i}{\sigma\sqrt{a_{ii}}} \sim N(0,1) \qquad (30)$$

with confidence level γ = 95%,

$$-1.96 \le z \le 1.96 \qquad (31)$$

Then the interval of parameters is represented by

$$\hat{\theta}_i - 1.96\sigma\sqrt{a_{ii}} \le z \le \hat{\theta}_i + 1.96\sigma\sqrt{a_{ii}} \qquad (32)$$

The interval of the correction factor, α, and the leak position, z_L, can be estimated by the above method.

EXPERIMENTS AND RESULTS

The proposed method was applied to an experimental system for evaluation of its performance. The experimental system consists a straight long pipe of 3/8" inside diameter and 97.6m length, a water storage tank, a centrifugal pump, two absolute pressure transducer (AP), two differential pressure transducer (DP), a 12bit analog-to-digital converter (ADC), PC (80286), etc., as shown in Fig. 6. The leak can occur at 45.0m and/or 69.3m from inlet pressure gage.

Before diagnosis of the leak position in a pipeline the leak has to be detected precedently. Whether leak occur or not was decided using mean of accumulation of the leak ratio to inlet flow rate at that time as

$$\frac{\Delta q(k)}{q_0(k)} = \frac{1}{N}\sum_{n=0}^{N-1}\frac{\Delta q(k-n)}{q_0(k-n)} \qquad (33)$$

where $\Delta q(k) = q_0(k) - q_N(k)$ and N is the number of data to be accumulated. As this value is ranged over 0.061895 to 0.241456 in the case of no leak occurrence, relatively safe reference value for leak detection can be choosed.

The noise model dimension is determined precedently using the hypothesis testing with the measurements of some experiments. The power spectral density of $y(t)$ and $u(t)$ before leak occurrence is shown in Fig. 7. The notch filter, $A(q^{-1})$, is designed from this analysis and correction factor, α, is estimated using RELS with Eq. (25). After leak occurrence the leak position, z_L, is also estimated using RELS with Eq. (26) and the evaluated α. In addition the interval estimation is performed for α and z_L.

The experimental conditions and results are tabled in Table 1 with comparison to the results by Lee et al.

CONCLUSION

The measurements and experiments show that the early detection and localization of the leaks in a liquid pipeline was improved. A leak detection method for diagnosis of the leak position was developed using an

Leak Detection Method	I	II	I	II	I	II	I	II	I	II
Leak Position [m]	45.00		45.00		69.30		45.00		69.30	
Change of Flow Rate	X		X		X		O		O	
Leak Ratio [%]	3.47		2.45		7.47		14.5		9.62	
Estimated Correction Factor	.2952	.2952	.2940	.2943	.2946	.2939	.2969	.2966	.2977	.2966
Confidence Interval		.0025		.0111		.0002		.0070		.0127
Estimated Leak Position [m]	47.79	45.07	54.81	51.11	68.45	68.77	43.02	42.33	67.24	69.97
Confidence Interval [m]		4.79		25.38		4.64		9.40		2.75
Error of Diagnosis [m]	2.79	0.07	9.81	6.11	0.85	0.53	1.98	1.67	2.06	0.67

estimation theory with the assumption that the measured signals are stochastic processes. The confidence interval for the leak position was evaluated using interval estimation method. Modeling of the noise elements using hypothesis testing and RELS estimation can overcome various disturbances.

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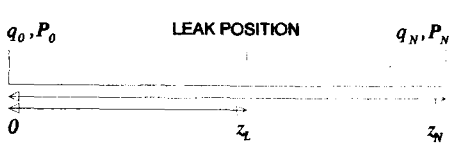

Fig. 1 Representation of the pipeline.

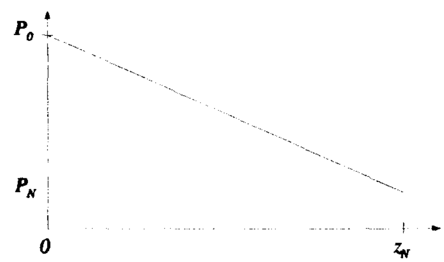

Fig. 2 Pressure distribution in a pipeline without leaks.

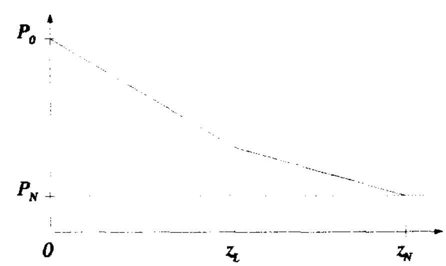

Fig. 3 Pressure distribution in a pipeline with a leak.

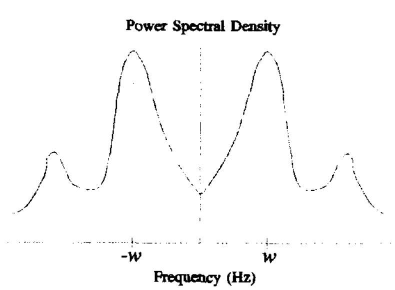

Fig. 4 Power Spectral Density of y(t) and u(t).

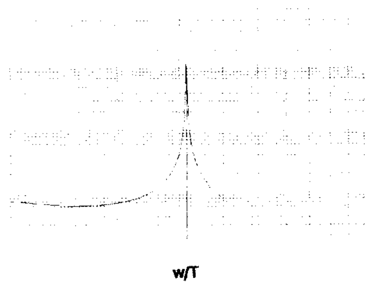

Fig. 5 Bode diagram of $1/A(q^{-1})$.

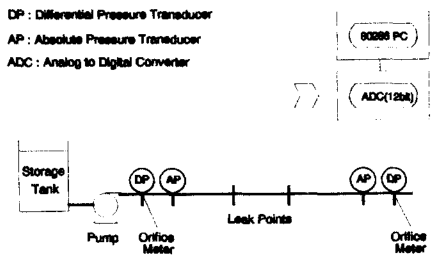

Fig. 6 Schematic diagram of experimental apparatus.

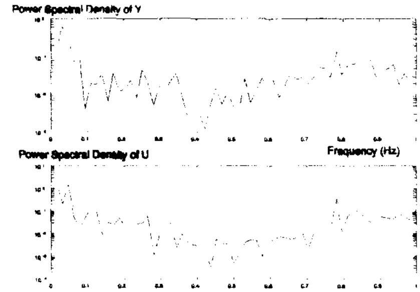

Fig. 7 Power spectral density of y(t) and u(t) for α.

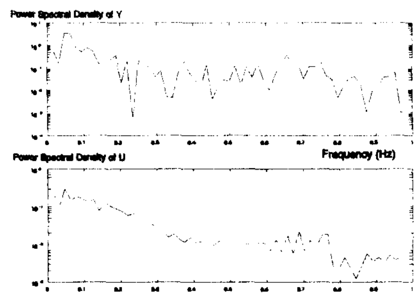

Fig. 8 Power spectral density of y(t) and u(t) for z_1.