'92 KACC 1992. 10. 19～21

Order Identification of Transfer Function-Noise Model

Seongju Park, Hankyung Bae, Kyungmoo Huh

DAEWOO HEAVY INDUSTRIES LTD., Central R & D Institute

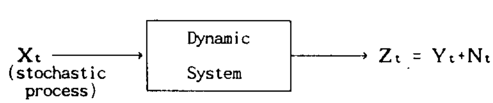
Abstract

Classical methods for estimating transfer function models have not always been successful. A statistic approach to the identification of transfer function models which is corrupted by disturbances or noise is presented. The estimated impulse response is obtained from the autocorrelation function and cross correlation function between the measured input and output. Several data analysis tools such as R- , S- and GPAC array for the estimated impulse response give us pretty clear information on the order of transfer function models.

I. Introduction

The reason control is necessary is that there are inherent disturbances or noise in the process. Noise contains some information on the system in question. There have been many studies on the identification of stochastic models to forecast future values and control process. H.L.Gray, G.D.Kelly and D.D.McIntire proposed R-array and S-array in 1978 to identify order(p,q) of the Autoregressive-Moving Average(ARMA) process driven by white noise[1]. With their proposal the order of ARMA process can be determined uniquely. In 1979 Woodward and Gray proposed GPAC array based on the generalized partial autocorrelation function, which tells us p and q more effectively.

In this paper, transfer function models with noise as shown in Fig.1 is considered.



(Z_t : measured output
N_t : noise contained in measurement)

Fig.1 Time series in relation to a dynamic system

Classical methods for estimating transfer function models based on deterministic perturbations of the input such as step, pulse, and sinusoidal changes have not always been successful because the response of the system may be masked by uncontrollable disturbances collectively referred to as a noise. In this paper statistical method for estimating transfer function models with noise is presented.

II. Autoregressive-Moving Average(ARMA) Process Modeling

Let's consider linear filter system (Fig.2) with white noise input a_t and measurable filtered output Z_t.

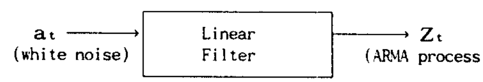


Fig.2 Linear filter system driven by white noise

Filtered Z_t may be described in general form of ARMA(p,q) process.

$$Z_t = \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + \cdots + \Phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} \cdots - \theta_q a_{t-q} \qquad (2\text{-}1)$$

2.1 Definitions and Theorems

2.1.1 R-array and S-array

Let m be an integer, h>0, and f be a real-valued function. Also let $f_m = f(mh)$,

$$H_n(f_m) = \begin{vmatrix} f_m & f_{m+1} & \cdots & f_{m+n-1} \\ f_{m+1} & f_{m+2} & \cdots & f_{m+n} \\ \vdots & \vdots & & \\ f_{m+n-1} & f_{m+n} & \cdots & f_{m+2n-2} \end{vmatrix} \qquad (2\text{-}2)$$

$$H_0(f_m) = 1 \qquad (2\text{-}3)$$

and

$$H_{n+1}(1;f_m) = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ f_m & f_{m+1} & \cdots & f_{m+n} \\ \vdots & \vdots & & \\ f_{m+n-1} & f_{m+n} & \cdots & f_{m+2n-1} \end{vmatrix} \qquad (2\text{-}4)$$

Then we define

$$R_n(f_m) = \frac{H_n(f_m)}{H_n(1:f_m)} \qquad (2\text{-}5)$$

$$S_n(f_m) = \frac{H_{n+1}(1:f_m)}{H_n(f_m)} \qquad (2\text{-}6)$$

Pye and Atchison have shown that $R_n(f_m)$ and $S_n(f_m)$ can be calculated recursively by the following relations. Define

$$S_0(f_m) = 1 \ , \quad m = 0,\ \pm 1,\ \pm 2, \cdots \qquad (2\text{-}7)$$

$$R_1(f_m) = f_m \ , \quad m = 0,\ \pm 1,\ \pm 2, \cdots \qquad (2\text{-}8)$$

Then

$$S_n(f_m) = S_{n-1}(f_{m+1}) \left[\frac{R_n(f_{m+1})}{R_n(f_m)} - 1 \right] \qquad (2\text{-}9)$$

and

$$R_{n+1}(f_m) = R_n(f_{m+1}) \left[\frac{S_n(f_{m+1})}{S_n(f_m)} - 1 \right] \qquad (2\text{-}10)$$

for $n=1,2,\cdots$ and $m=0,\ \pm 1,\ \pm 2,\cdots$.

2.1.2 GPAC array

The generalized partial autocorrelation function (GPAC) is defined as

$$\Phi_{kk}{}^{j} = \frac{\begin{vmatrix} R_{zz}(j) & R_{zz}(j-1) & & R_{zz}(j-k+2) & R_{zz}(j+1) \\ R_{zz}(j+1) & R_{zz}(j) & & R_{zz}(j-k+3) & R_{zz}(j+2) \\ \vdots & \vdots & & \vdots & \vdots \\ R_{zz}(j+k-1) & R_{zz}(j+k-2) & \cdots & R_{zz}(j+1) & R_{zz}(j+k) \end{vmatrix}}{\begin{vmatrix} R_{zz}(j) & R_{zz}(j-1) & \cdots & R_{zz}(j-k+2) & R_{zz}(j-k+1) \\ R_{zz}(j+1) & R_{zz}(j) & \cdots & R_{zz}(j-k+3) & R_{zz}(j-k+2) \\ \vdots & \vdots & & \vdots & \vdots \\ R_{zz}(j+k-1) & R_{zz}(j+k-2) & \cdots & R_{zz}(j+1) & R_{zz}(j) \end{vmatrix}} \qquad (2\text{-}11)$$

(where $R_{zz}(j)$ is the autocorrelation funtion of Z_t.)

Woodward and Gray proposed powerful method generating the generalized partial autocorrelation function.

$$\Phi_{kk}{}^{j} = -\frac{S_k(f_{-k+j+1})}{S_k(f_{-k-j})} \quad : \text{if } f_m = R_{zz}(m) \qquad (2\text{-}12)$$

$$\Phi_{kk}{}^{j} = (-1)^{k+1}\frac{S_k(f_{-k+j+1})}{S_k(f_{-k-j})} \quad : \text{if } f_m = (-1)^m R_{zz}(m) \qquad (2\text{-}13)$$

2.2 R-, S- and GPAC array of ARMA(p,q)

Autocorrelation function of Z_t can be derived from eq.(2-1) by multiplying Z_{t-m} on both sides and taking the expectation.

$$R_{zz}(m) = \Phi_1 R_{zz}(m-1) + \Phi_2 R_{zz}(m-2) + \cdots + \Phi_p R_{zz}(m-p) + R_{za}(m) - \theta_1 R_{za}(m-1) - \cdots - \theta_q R_{za}(m-q) \qquad (2\text{-}14)$$

where $R_{za}(\cdot)$ is the cross correlation function between Z_t and a_t. Noting that a_t and Z_t are uncorrelated, eq.(2-14) can be written for $m>q$ as follows.

$$R_{zz}(m) = \Phi_1 R_{zz}(m-1) + \Phi_2 R_{zz}(m-2) + \cdots + \Phi_p R_{zz}(m-p) \qquad (2\text{-}15)$$

If we calculate R-array, S-array and GPAC array for the autocorrelation function instead of f_m in eq.(2-9) and eq.(2-10), we get particular patterns in R-array, S-array and GPAC array as shown in Table 1,2 and 3.

m \ n	1	p+1
		0
.		.
-q-p-1		0
-q-p		non zero
.		.
.		.
q-p		non zero
q-p+1		0
.		.

Table 1. R array for autocorrelation function of ARMA(p,q)

m \ n	1	p	p+1	p+2
.		.		*
.		c2	*	
.		c2		
-q-p		c2		
-q-p+1				
.				c1
q-p			-c1	
q-p+1		c1		
.		c1		
.		c1		
		.		

* : infinite number
$c_1 = (-1)^p (1-\Phi_1-\Phi_2-\cdots-\Phi_p)$
$c_2 = -(c_1/\Phi_p)$

Table 2. S-array for autocorrelation function of ARMA(p,q)

	1	...	p	p+1	...	p+i	...
0							
.							
.							
q			Φ_p	0	...	0	...
q+1			Φ_p				
.			.				
.			.				

Table 3. GPAC array for autocorrelation function of ARMA(p,q)

A modification of S-array called "shifted S-array" defined in eq.(2-16) enables us to see the pattern more easily as shown in Table 4.

$$S_k{}^{*}(f_j) = S_k(f_{j-k+1}) \qquad (2\text{-}16)$$

Table 1 represents the behaviour of the R-array when Z_t is ARMA(p,q) and $f_m = R_{zz}(m)$. Elements in a column p+1 are zero for rows k, k<-q-p and k>q-p. The behaviours of the S-array and the S^*-array are shown in Table 2 and Table 4 respectively. S^*-array represents

m	p	p+1	p+i
n			
.	.		
.	c_2		
-q-2	c_2		
-q-1	c_2	*	* . .
.			
.			
.			
q	c_l	$-c_l$	c_l . .
q+1	c_l		
.	.		
.	.		

Table 4. Shifted S-array for auto. function of ARMA(p, q)

that constant behaviour occurs in a column p for rows k, k<-q and k>q-1. And in columns p+i, (i=1, 2, · ·), the value $\pm\infty$ occur at a row -q-1 and $(-1)^i c_l$ occur at a row q.

More powerful pattern appers in GPAC array in which constant behaviour occurs in a column p for rows k, k>q-1 and zero values appear in a row q for columns k, k>p. From these patterns orders p and q of ARMA process can be determined uniquely.

Ⅲ. Transfer Function Models

3.1 Nature of transfer function

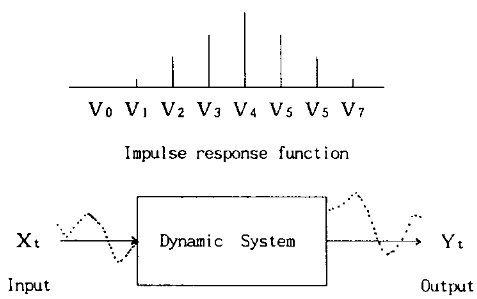


Fig. 3 Input to, and output from, a dynamic system

We suppose that pairs of observations (X_t, Y_t) are available at equispaced intervals of time from some dynamic system. It frequently happens that, to an adequate approximation, the inertia of the system can be represented by a linear filter of the form

$$\begin{aligned} Y_t &= V_0X_t + V_1X_{t-1} + V_2X_{t-2} + \cdots \\ &= (V_0 + V_1B + V_2B^2 + \cdots)X_t \\ &= V(B)\ X_t \end{aligned} \quad (3\text{-}1)$$

The weights $V_0, V_1, V_2, \cdots$ in eq.(3-1) are called the impulse response function of the system. The operator $V(B)$ is called the transfer function where B is backward shift operator.

On the other hand, discrete dynamic systems are also represented by the general linear difference equation.

$$(1-\delta_1B-\delta_2B^2-\cdots-\delta_rB^r)Y_t = (W_0-W_1B-W_2B^2-\cdots-W_sB^s)X_{t-b} \quad (3\text{-}2)$$

or

$$\delta(B)Y_t = W(B)B^bX_t \quad (3\text{-}3)$$

Substituting eq.(3-1) into eq.(3-3) yields

$$(1-\delta_1B-\delta_2B^2-\cdots-\delta_rB^r)(V_0+V_1B+\cdots) = (W_0-W_1B-\cdots-W_sB^s)B^b \quad (3\text{-}4)$$

On equating coefficients of B, we find

$$V_j = 0 \qquad (j=0, 1, \cdots, b-1) \quad (3\text{-}5)$$
$$V_j = \delta_1V_{j-1}+\delta_2V_{j-2}+\cdots+\delta_rV_{j-r} + W_0 \quad (j=b) \quad (3\text{-}6)$$
$$V_j = \delta_1V_{j-1}+\delta_2V_{j-2}+\cdots+\delta_rV_{j-r} - W_{j-b} \quad (j=b+1, \cdots, b+s) \quad (3\text{-}7)$$
$$V_j = \delta_1V_{j-1}+\delta_2V_{j-2}+\cdots+\delta_rV_{j-r} \quad (j > b+s) \quad (3\text{-}8)$$

In practice, the output could not be expected to follow exactly pattern determined by the transfer function model, even if that model were entirely adequate. Disturbances of various kinds other than input normally corrupt the system. A disturbance might originate at any point in the system, but it is often convenient to consider it in terms of its net effect on the output Z, as indicated in Fig. 4.

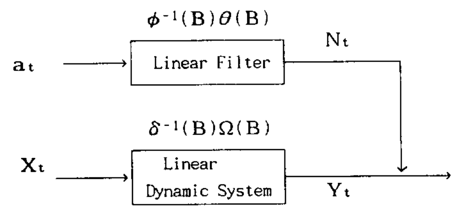


Fig. 4 Transfer function model with added noise

If we assume that the disturbance, or noise N, is independent of X, and is additive with respect to the influence of X, then we can write

$$Z_t = Y_t + N_t \quad (3\text{-}9)$$

If the noise model can be represented by an ARMA process

$$N_t = \phi^{-1}(B)\theta(B)a_t \quad (3\text{-}10)$$

where a_t is white noise, the model (3-9) can be written as

$$Z_t = \delta^{-1}(B)\omega(B)X_{t-b} + \phi^{-1}(B)\theta(B)a_t \quad (3\text{-}11)$$

or

$$Z_t = V(B)X_t + N_t \quad (3\text{-}12)$$

3.2 The estimated impulse response, $\widehat{V}_j$

In the same way that the autocorrelation function was used to identify stochastic models, the data analysis tool employed for the identification of transfer function models is the cross correlation function between the input and the output. We can get cross correlation function by multiplying X_{t-m} on both sides of eq.(3-12) and taking expectation.

$$R_{xz}(m) = V_0R_{xx}(m) + V_1R_{xx}(m-1) + V_2R_{xx}(m-2) + \cdots \quad (3\text{-}13)$$

$(E[X_{t-m}N_t]=0$ because X_t and N_t are uncorrelated.)

When the process are nonstationary it is assumed that stationarity can be induced by suitable differencing. Nonstationary behaviour is suspected if the estimated auto-and cross-correlation functions of the (X_t, Z_t) series fail to damp out quickly.

Suppose that the weights V_j are effectively zero for m>k. Then the first k+1 of the equations (3-12) can be written

$$\begin{bmatrix} R_{xz}(0) \\ R_{xz}(1) \\ \vdots \\ R_{xz}(k) \end{bmatrix} = \begin{bmatrix} R_{xx}(0) & R_{xx}(1) & \cdots & R_{xx}(k) \\ R_{xx}(1) & R_{xx}(0) & \cdots & R_{xx}(k-1) \\ \vdots & \vdots & & \vdots \\ R_{xx}(k) & R_{xx}(k-1) & \cdots & R_{xx}(0) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ \vdots \\ V_k \end{bmatrix} \quad (3\text{-}14)$$

Noting that above matrix is symmetric, we can get estimates $\hat{V}_j$ using matrix algebra algorithm. Delay factor, b, in eq.(3-11) can be determined from eq.(3-5) and estimated $\hat{V}_j$.

3.3 Identification of transfer function models

If we substitute $\hat{V}_j$ calculated in section 3.2 into fm in eq.(2-9) and (2-10), we get R-, S*- and GPAC array for the estimated impulse response. Table 5, Table 6 and Table 7 represent behaviour of R-, S*- and GPAC array for the estimated impulse response. In R-array elements at a column r+1 for rows k, k>b+s-r and k<-b-s-r are zero. S*-array represents constant behaviour in a column r for rows k, k>b+s-1 and k<-b-s. In columns r+i, (i=1,2,··), the value $(-1)^i c_i$ occur at a row b+s and infinite number at a row -b-s-1.

GPAC array represent constant behaviour in a column r for rows k, k>b+s-1 and zero values appear in a row b+s. From these behaviours, order of transfer function models, r and s, and delay factor b can be determined.

m \ n	1	r+1
		0
		0
-b-s-r-1		0
.		
.		
b+s-r+1		0
.		0
.		0
.		.

Table 5. R-array for the estimated impulse response

m \ n	r	
.	.	
	c_2	
-b-s-1	c_2	* * . . .
.		
b+s	c_1	$-c_1$ $-c_1$ . . .
.	c_1	
.	c_1	
.	.	
.	.	

* : infinite number

$c_1 = (-1)^r(1-\delta_1-\delta_2-\cdots-\delta_r)$

$c_2 = -(c_1/\delta_r)$

Table 6. S*-array for the estimated impulse response

	1	r . . .	
0			
.			
b+s		δ_r	0 0 0 · ·
.		δ_r	
.		δ_r	
.		.	
.		.	

Table 7. GPAC array for the estimated impulse response

Ⅳ. An Example of Identification of Transfer Function Models

An example to justify proposed method in previous chapter is presented. Let's consider following transfer function model.

$$Z_t = Y_t + N_t \quad (4\text{-}1)$$

The relation between input X_t and uncorrupted output Y_t is written as

$$Y_t = 0.9Y_{t-1} - 0.68Y_{t-2} + X_{t-2} - 0.8X_{t-3} \quad (4\text{-}2)$$

Noise N_t is an ARMA process.

$$N_t = 0.5N_{t-1} + a_t + 0.7a_{t-1} \quad (4\text{-}3)$$

From eq.(4-1) through (4-3), we find

$$r=2,\ s=1,\ b=2,\ p=1,\ q=1. \quad (4\text{-}4)$$

Now, same result can be obtained with proposed method of identification of transfer function model.

The procedure consists of :

a) generating observation pairs (X_t, Z_t) from white noise and arbitrary initial values, X_0, X_1, X_2, X_3.
b) calculating the estimated impulse response $\hat{V}_j$ (Table 8) using eq.(3-14). From this step, delay factor b can be determined.
c) calculating R-, S*- and GPAC array (Table 9,10,11). Order of transfer function model, r and s, can be determined at this point.
d) generating the estimated noise $\hat{N}_t$ using

$$\hat{N}_t = Z_t - \hat{V}_0X_t - \hat{V}_1X_{t-1} - \hat{V}_2X_{t-2} - \cdots \quad (4\text{-}5)$$

e) calculating R-, S*- and GPAC array for autocorrelation function of $\hat{N}_t$. (Table 12,13,14). This step yields p and q.

As we expect, the same result as eq.(4-4) is obtained from Table 8 through Table 14. Table 8 is the estimated impulse response. First two value of $\hat{V}_j$ are quite small so that we can assume that there is delay up to j=3 that is b=2. Table 11 is the GPAC array. Table 11 tells us r=2, b+s=3 and s=1. By the way constant values -0.7080 is close to δ_2 in eq.(4-2). These values can be checked in Table 10, Shifted S-array in which there is a typical pattern of Shifted S-array. From this table we can conclude that the values for r, s and b we have guessed are correct. And constant values in second column of Table 10 can be calculated by hand as follows;

$$c = (-1)^r\,[1 - (\delta_1 + \delta_2 + \cdots + \delta_r)]$$
$$= (-1)^2\,[1 - (0.9 - 0.68)] = 0.78$$

Table 9 is the R-array. Usually R-array itself is difficult to tell the order of model because numbers in R-array are usually small. But it can be used as a supplement table. From the third row(m=2) in the third column, numbers are quite small. It is helpful to keep in mind that numbers in columns greater than r+1 are usually very small. So we can think that first column in which numbers are close to zero is r+1. Table 14 is the GPAC array for noise where p and q in eq.(4-3) can be estimated. 0.445 in first column and second row is the constant value in GPAC array. Numbers in the right side of that value are close to zero. And 0.445 is close to 0.5 that is Φ_1 in eq.(4-3). In the Shifted S-array, Table 13, numbers from second column in m=-2 are very large number. And -0.555 in first column and m=1 is the constant number and is close to -0.5 which can be calculated from $(-1)^1(1-\Phi_1)$.

V(0)	= 0.5035	V(16)	= -0.0059
V(1)	= 0.0844	V(17)	= -0.0494
V(2)	= 1.0035	V(18)	= -0.0339
V(3)	= 0.1302	V(19)	= 0.0152
V(4)	= -0.6019	V(20)	= 0.0303
V(5)	= -0.5936	V(21)	= 0.0242
V(6)	= -0.1405	V(22)	= 0.0018
V(7)	= 0.2790	V(23)	= -0.0153
V(8)	= 0.3470	V(24)	= -0.0119
V(9)	= 0.1138	V(25)	= -0.0171
V(10)	= -0.1232	V(26)	= 0.0104
V(11)	= -0.2023	V(27)	= 0.0079
V(12)	= -0.0958	V(28)	= 0.0052
V(13)	= 0.0371	V(29)	= -0.0002
V(14)	= 0.1023	V(30)	= -0.0036
V(15)	= 0.0586		

Table 8. The estimated Impulse Response $\hat{V}_j$

m= 0	0.504	-1.189	2.709	0.171	-0.015	-0.052
m= 1	0.084	-1.084	-0.279	-0.079	-0.010	0.003
m= 2	1.003	0.711	-0.082	-0.009	0.000	0.004
m= 3	0.130	0.600	0.050	0.011	-0.003	0.006
m= 4	-0.602	-32.365	-0.016	-0.081	-0.009	0.012
m= 5	-0.594	-0.409	-0.010	-0.005	-0.011	0.001
m= 6	-0.140	-0.302	0.023	0.012	-0.008	-0.043
m= 7	0.279	-1.304	-0.035	0.007	-0.019	0.044
m= 8	0.347	0.239	0.035	0.007	0.035	0.002
m= 9	0.114	0.161	-0.033	0.006	0.002	-0.009
m=10	-0.123	0.368	0.022	0.002	0.000	-0.008
m=11	-0.202	-0.157	-0.034	-0.001	0.009	-0.013
m=12	-0.096	-0.084	0.023	0.000	0.013	0.001

Table 9. R array for the estimated impulse response

m=0	-0.8323	-1.1324	0.7274	4.3589	-4.1854	-8.2939
m=1	10.8836	-0.9609	-13.4936	4.3341	309.383	-8.7334
m=2	-0.8702	1.4412	-1.5899	1.7036	-1.7981	1.8570
m=3	-5.6214	0.8749	-0.6170	0.9019	-0.7310	0.6194
m=4	-0.0137	0.7549	-1.2085	1.0761	-0.3815	3.0881
m=5	-0.7634	0.7537	-0.9972	2.2997	-2.3864	2.5340
m=6	-2.9863	0.7832	-0.2924	2.3752	-22.9600	-4.5053
m=7	0.2436	0.8092	-2.6415	2.4701	4.3613	2.3492
m=8	-0.6720	0.7951	-2.0332	6.6740	1.4740	1.5688
m=9	-2.0825	0.6775	-1.3468	0.5984	-0.1759	0.1556
m=1 0	0.6418	0.8242	-1.6025	-0.0937	-0.1437	4.5589

Table 10. S* array for the estimated impulse response

0.1677	1.9647	-0.4050	-5.0863	0.1735	12.4794	1.3104
11.8836	1.9993	-25.0780	-5.1001	366.095	12.4612	292.236
0.1298	-0.6235	-0.3093	-0.2114	-0.1112	-0.0696	-0.0477
-4.6214	-0.7080	0.1116	-0.0554	0.0212	0.0069	-0.0212
0.9863	-0.6093	-0.3671	-0.0240	0.0427	0.0785	-0.0110
0.2366	-0.6921	-0.1965	-0.3956	-0.0019	0.0165	0.0617
-1.9863	-0.6830	0.4506	-0.4105	-3.5371	0.0222	0.0192
1.2436	-0.7003	-1.5979	-0.2643	1.1807	-1.0348	0.0039
0.3280	-0.6284	-1.0714	-2.8072	0.8717	-0.8237	0.0000
-1.0825	-0.6858	-0.5290	0.3944	0.1777	0.0120	-0.2964
1.6418	-0.7621	-0.7878	0.6662	0.1565	4.4609	-0.3035

Table 11. GPAC array for the estimated impulse response

m=-5	0.0023	0.0174	0.0149	-0.0559	0.1072	-0.2333
m=-4	0.0041	-0.0113	-0.0207	-0.0829	0.2082	0.0053
m=-3	0.0215	-0.0237	0.0771	-0.2390	0.0251	-0.0104
m=-2	0.0640	-0.1000	0.1967	0.0309	0.0104	-2.6447
m=-1	0.1439	-0.3425	0.0424	-0.0057	0.0104	0.0104
m= 0	0.1987	0.1458	-0.0022	-0.0072	0.0044	0.0018
m= 1	0.1439	0.0126	0.0078	0.0038	0.0005	0.0015
m= 2	0.0640	0.0046	0.0068	-0.0042	-0.0018	-0.0010
m= 3	0.0215	-0.0018	0.0073	0.0147	-0.0003	-0.0025
m= 4	0.0041	-0.0144	0.0078	-0.0053	0.0008	-0.0081
m= 5	0.0023	-0.0053	0.0046	-0.0613	0.0077	0.0080
m= 6	0.0077	-0.0104	0.0051	-0.0029	0.0098	0.0057
m= 7	0.0133	-0.0351	-0.0013	-0.0043	0.0059	-0.0009
m= 8	0.0154	0.0208	0.0043	-4.0700	0.0013	-0.0035

Table 12. R array for the estimated noise

m=-6	-0.703	1.374	0.223	-0.630	0.774	22.317	-31.572
m=-5	0.806	-3.666	0.489	-0.610	122.276	26.198	-193.039
m=-4	4.206	-6.952	0.322	14.896	5.388	2.300	7.281
m=-3	1.982	2.150	-5.146	4.006	-0.009	-3.825	5.949
m=-2	1.249	4.031	-19.047	-9.191	3.840	-3.838	6.178
m=-1	0.381	0.924	1.433	2.699	2.541	13.805	5.074
m= 0	-0.276	0.393	-0.309	0.348	-0.306	0.313	-0.295
m= 1	-0.555	0.507	-0.533	0.632	-0.371	1.098	-0.339
m= 2	-0.665	0.420	-1.941	-0.517	0.001	0.129	-0.362
m= 3	-0.808	1.130	-0.147	0.225	-0.129	0.129	-0.273
m= 4	-0.446	-3.031	-0.229	0.485	-0.432	0.359	-0.363
m= 5	2.365	-1.490	-0.096	0.430	-2.032	0.314	-1.843
m= 6	0.728	0.691	-0.278	0.378	-0.325	0.540	2.120
m= 7	0.159	0.381	0.037	0.396	-1.607	-2.351	-2.438
m= 8	-0.205	0.326	-0.410	0.391	3.509	8.055	-2.329
m= 9	-0.552	0.305	-1.301	-0.657	-0.183	0.365	-0.645

Table 13. S* array for the estimated noise

0.724	-0.426	0.215	-0.129	0.121	-0.023	0.058	-0.008
0.445	-0.126	-0.028	0.069	0.097	0.286	0.055	-0.021
0.335	-0.196	-0.377	0.129	0.056	0.034	0.061	-0.393
0.192	0.163	0.457	-0.015	0.024	-0.056	0.037	-0.068
0.554	-0.827	0.469	0.795	0.004	-0.014	-0.002	0.007
3.365	1.084	0.433	0.682	2.624	-0.014	-0.058	0.025
1.728	-2.018	0.300	0.488	-0.085	0.279	0.119	-0.331
1.159	-1.283	1.222	0.467	1.434	1.033	2.469	0.006
0.795	-0.881	-0.285	0.631	-4.601	10.32	2.517	-27.069
0.448	-0.962	-2.692	1.358	1.355	0.238	0.360	-0.100
-0.492	-0.847	1.412	-1.737	1.046	-1.754	0.416	-0.218

Table 14. GPAC array for the estimated noise

V. Conclusion

In this paper we have demonstrated the use of GPAC, R- and S-array to identify transfer function models in stochastic method. If we get input and output observations, we can plug those pairs into GPAC, R- and S-array ,which tell us pretty clear information on the identification of transfer function model.

REFERENCES

[1] Gray,G.L.,Kelly,G.D. and McIntire,D.D.(1978) "A New Approach to ARMA Modeling," Commun.,Statist.- simula. Computa., B7 (1),1-77

[2] Woodward,Wayne A. and Gray,G.L.(1979),"On the Relationship Between the R and S arrays and the Box-Jenkins Method of ARMA model Identification," Tech Report No.134,Department of Statistics,ONR Contract,SMU

[3] Gene H.Golub and Charles F. Van Loan(1983) , Matrix Computations, The Johns Hopkins University press.

[4] J.Bee Bednar and W.A.Coberly(1981),"Order selection for Lowpass IIR Filters," Proc. of the 1981 Tulsa symposium on Applied Time Series analysis.

[5] Pye,W.C and Atchison,T.A(1973),"An Algorithm for the computation of he higher order G-transformation," SIAM.J. Numerical Analysis 10.

[6] Box,G.I.P and Jenkins,G.M.(1976),"Time Series Anaysis : Forecasting and Control," Holden-Day.