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Estimation of Solid Friction in Mechanical Systems

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ABSTRACT

This paper describes the estimation of the solid friction in mechanical systems by using the extended Kalman filtering techniques. We proposed two stochastic model for the estimation. The one is the 'parametric model' which represents the friction characteristics by an exponential function with unknown parameters. The other is the 'blind model' which does not assume an explicit model but regard the effect of the friction as an unknown input to a known dynamic system. For both models, we give estimation algorithms to generate the filtered estimate and the smoothed estimate with a fixed lag. The filtered estimate can be generated on-line for compensating the solid friction in mechanical systems. Although on-line applications are impossible, the smoothed estimate is more accurate and can be used to grasp precise friction characteristics. Simulation and experimental results are presented to show the effectiveness of the proposed techniques.

1. INTRODUCTION

Solid friction in mechanical control systems causes undesirable effects such as a jerking motion. Since, for example, the occurrence of the jerking motion depends on the negative gradient in the velocity characteristics of the solid friction[1], it is important to identify the velocity characteristic of the solid friction. Bell et al.[2] and Dahl [3] have measured the solid friction of slideways and that of ball bearing, respectively. Walrath[4] has discussed effective application of the measured data to compensate the solid friction in mechanical control systems. However, these methods require several sensors or a special apparatus for the measurement. To compensate the effect of the solid friction without special hardwares, Canudas et al.[5] have proposed to use an observer which estimates the friction.

In this paper we consider the estimation of the solid friction by using extended Kalman filtering techniques. We propose two stochastic models for the estimation. The one is the parametric model which represents the friction characteristics by an exponential function with three unknown parameters. The unknown parameters contained in the model are estimated by an extended Kalman filter together with the state variables. The other is the blind model which does not assume an explicit model but estimates the friction as an unknown input to a known dynamic system. For both models, we derive estimation algorithms to generate the filtered estimate and smoothed estimate with a fixed-lag. The filtered estimate can be generated on-line for compensating the solid friction in mechanical systems. The smoothed estimate, which can not be generated on-line, is more accurate than the filtered estimate. Although the smoothed estimated can not directly be used for the on-line compensation, it is extremely useful to grasp friction characteristics.

To confirm the effectiveness of the proposed algorithms, we give simulation results for a simple mechanical system.

Then we give experimental results where the proposed techniques are applied for estimation of the solid friction acting on the axis of an experimental pendulum system. Using the change of pendulum angle, we can estimate the relationship between the solid friction and the velocity by the proposed techniques.

2. ESTIMATION METHOD OF SOLID FRICTION

2.1 Mechanical systems with solid friction

Consider a mechanical system such as Fig.1. Let the continuous-time state equation of the mechanical system be

$$\dot{x}=\Psi(x)+Bu+Gf+w \tag{1}$$

where x is a state variable vector, $\Psi(x)$ is a vector function representing the system dynamics, u is a system input vector, f is a solid friction, w is a process noise vector. Since the measurement is performed by a microcomputer system, we use a discrete-time model of (1) described by

$$x_{t+1}=\psi(x_t)+Bu_t+Gf_t+w_t \tag{2}$$

$$y_t=Cx_t+v_t \tag{3}$$

where suffix t denote the t th sampling time, y_t is a observation vector, v_t is a observation noise vector.

2.2 Friction models

The solid friction is regarded as a function of the velocity. We mainly consider here the friction model which shows the negative gradient for the velocity. This model is represented in Fig.2(a). The characteristics of the solid friction with a hysteresis loop[2] is shown in Fig.2(b).

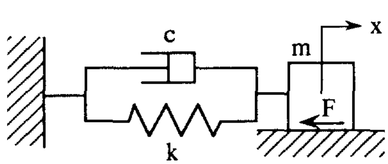
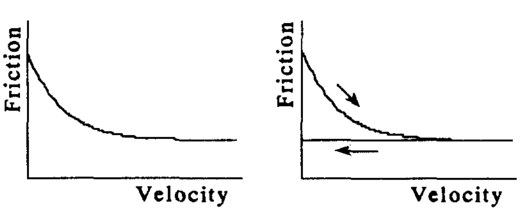


Fig.1 Mechanical system with solid friction



(a) (b)

Fig.2 Friction model

2.3 Estimation method

Parametric model:

Assume that the friction characteristics can be approximated by the exponential function

$$f(v,\theta)=\theta_1\exp(-\theta_2|v|)+\theta_3 \tag{4}$$

where v is the sliding velocity, $\theta=[\theta_1\ \theta_2\ \theta_3]$ is a parameter vector which determines the friction characteristics. Substituting $f_t(n,\theta)$ for f_t in eq.(2), we obtain

$$x_{t+1}=\psi(x_t)+Bu_t+Gf(v_t,\theta_t)_t+w_t \tag{5}$$

Assume that the parameter in eq.(4) is the stochastic process defined by

$$\theta_{t+1}=\theta_t+\tilde{w}_t \tag{6}$$

Then, we introduce an extended state vector

$$z_t=\begin{bmatrix} x_t \\ \theta_t \end{bmatrix} \tag{7}$$

The extended system can be written as

$$z_{t+1}=\phi(z_t)+Du_t+W_t \tag{8}$$

$$y_t=Hz_t+v_t \tag{9}$$

$$H=[C\ \ 0]$$

where

$$\phi(z_t)=\begin{bmatrix} \psi(x_t)+Gf(z_t) \\ \theta_t \end{bmatrix},\ D=\begin{bmatrix} B \\ 0 \end{bmatrix},\ W_t=\begin{bmatrix} w_t \\ \tilde{w}_t \end{bmatrix}$$

The state vector z_t of the eq.(8) can be estimated by extended Kalman filter or fixed-lag smoother. Putting the estimated θ_t into the friction model eq.(4), we obtain the estimate of the solid friction.

Blind model:

Let the solid friction f_t in eq.(2) be

$$f_{t+1}=f_t+\tilde{w}_t \tag{10}$$

where $\tilde{w}_t$ is a white noise process. Then, we introduce an extended state vector

$$z_t=\begin{bmatrix} x_t \\ f_t \end{bmatrix} \tag{11}$$

From eq.(3) and eq.(11), eq.(2) can be rewritten as

$$z_{t+1}=\phi(z_t)+Du_t+W_t \tag{12}$$

$$y_t=Hz_t+v_t \tag{13}$$

where

$$\phi(z_t)=\begin{bmatrix} \psi(x_t)+Gf_t \\ f_t \end{bmatrix}$$

$$D=\begin{bmatrix} B \\ 0 \end{bmatrix}, W_t=\begin{bmatrix} w_t \\ \tilde{w}_t \end{bmatrix}$$

$$H=[C \quad 0]$$

If we can estimate the state variable vector z_t, we have an estimate of the solid friction f_t.

2.4 Estimation algorithms

For the nonlinear system eq.(8) or eq.(12), we define

$$\Phi_t=\left.\frac{\partial\phi(x)}{\partial x}\right|_{x=\hat{x}_{t/t}} \tag{14}$$

Extended Kalman filter algorithm for the generation of the filtered estimate is given as follows:

(i) initial condition

$$\hat{z}_{0/-1}=\bar{z}_0, \; P_{0/-1}=\Sigma_0 \tag{15}$$

(ii) Kalman gain

$$K_t=P_{t/t-1}H^T[HP_{t/t-1}H^T+R]^{-1} \tag{16}$$

(iii) measurement update

$$\hat{z}_{t/t}=\hat{z}_{t/t-1}+K_t[y_t-H\hat{z}_{t/t-1}] \tag{17}$$

(iv) time up date

$$\hat{z}_{t+1/t}=\Phi_t\hat{z}_{t/t}+D_tu_t \tag{18}$$

(v) covariance of estimation error

$$P_{t+1/t}=\Phi_tP_{t/t}\Phi_t^T+GQG^T$$

$$P_{t/t}=P_{t/t-1}-K_tHP_{t/t-1} \tag{19}$$

The algorithm for the generation of the smoothed estimated with fixed-lag is given as follows:

(i) initial condition

$$\hat{z}_{0/-1}=\bar{z}_0, \; \hat{z}_{-j/-1}=0, \quad j=0,1,\cdots,L$$

$$P_{0/-1}(0,0)=\Sigma_0, \; P_{0/-1}(j,l)=0$$

$$j,l=0,1,\cdots,L \;\; (j^2+l^2\neq0) \tag{20}$$

(ii) smoothing gain

$$K_t(j)=P_{t/t-1}(j,0)H^T[HP_{t/t-1}(0,0)H^T+R]^{-1} \tag{21}$$

(iii) measurement update

$$\hat{z}_{t-j/t}=\hat{z}_{t-j/t-1}+K_t(j)[y_t-H\hat{z}_{t/t-1}] \quad j=0,1,\cdots,L \tag{22}$$

(iv) time update

$$\hat{z}_{t+1/t}=\phi(\hat{z}_{t/t})+Du_t \tag{23}$$

(v) covariances of estimation error

$$P_{t+1/t}(0,0)=\Phi_tP_{t/t}(0,0)\Phi_t^T+GQG^T$$

$$P_{t+1/t}(j,0)=\Phi_tP_{t/t}(j,0)$$

$$P_{t+1/t}(0,j)=(P_{t+1/t}(j,0))^T$$

$$P_{t/t}(j,l)=P_{t/t-1}(j,l)-K_t(j)HP_{t/t-1}(0,l) \tag{24}$$

Figure 3 illustrates measurement data used by Kalman filter and fixed-lag smoother. Note that the fixed-lag smoother provides more accurate estimate than Kalman filter by making full use of measured data.

3. SIMULATION

The simulation study given here considers the mechanical system consisting of a spring, a damper and a mass showing in Fig.1. The solid friction acts on the mass of the system. The equation of motion is

$$m\frac{d^2X}{dt^2}=-kX-c\frac{dX}{dt}\pm F \tag{25}$$

where X is a position, m is a mass, k is a spring constant, and c is a damping coefficient, F is a solid friction. The eq.(25) can be rewritten in state space form as

$$\begin{cases} \dot{x}_1=x_2 \\ \dot{x}_2=-x_1-2\zeta x_2+f \end{cases} \tag{26}$$

where

$$\tau=pt, \; x_1=\frac{X}{X_0}, \; x_2=\frac{dx_1}{d\tau}, \; p^2=\frac{k}{m}, \; 2\zeta x_2=\frac{c}{m},$$

$$f=sgn(x_2)\frac{F}{kX_0}, \; u=\frac{u_m}{X_0}, \; sgn(x_2)=\begin{cases} -:x_2>0 \\ +:x_2<0 \end{cases}$$

Now, consider the two parametric models of the friction with negative gradient for the velocity :

Model A:

$$f=0.1exp(-6.2|x_2|)+0.02 \tag{27}$$

Model B:

$$f=0.1/(1.0+10|x_2|)+0.02 \tag{28}$$

Using the friction model A or B, we solve eq.(26) by the 4th order Runge-Kutta method. Adding pseudo random numbers to the solutions, we generate 'observed data' contaminated by the observation noise. And the obtained results are added to observation noise. Applying the estimation technique described in the previous section, we can obtain the estimation of the solid friction. Here we present simulation results corresponding the

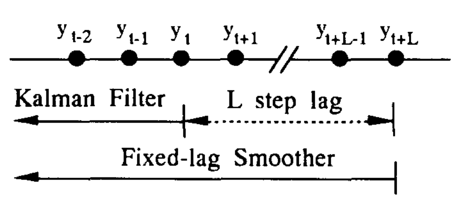


Fig.3 Kalman filter and fixed-lag smoother

following five conditions.

Case 1 : The extended Kalman filter based on the blind model (KFBM) for model A.

$P_{0/-1}$=diag[10^{-4} 10^{-4} 10^{-2}],

Q=diag[0.0 10^{-7} 1.0]

Case 2 : The extended Kalman filter based on the parametric model A (KFPMA) for model A.

$P_{0/-1}$=diag[10^{-4} 10^{-4} 10^{-2} 1.0 10^{-2}],

Q=diag[0.0 10^{-7} 10^{-3} 1.0 10^{-3}]

Case 3 : KFPMA for model B.

$P_{0/-1}$=diag[10^{-4} 10^{-4} 10^{-2} 1.0 10^{-2}],

Q=diag[0.0 10^{-7} 10^{-2} 10 10^{-2}]

Case 4 : The fixed-lag smoother based on the blind model (FSBM) for model B.

$P_{0/-1}$=diag[10^{-4} 10^{-4} 10^{-2}],

Q=diag[0.0 10^{-7} 1.0], lag=800

Case 5 : The fixed-lag smoother based on the parametric method A (FSPMA) for model B.

$P_{0/-1}$=diag[10^{-4} 10^{-4} 10^{-2} 1.0 10^{-2}],

Q=diag[0.0 10^{-7} 10^{-2} 10 10^{-2}], lag=100

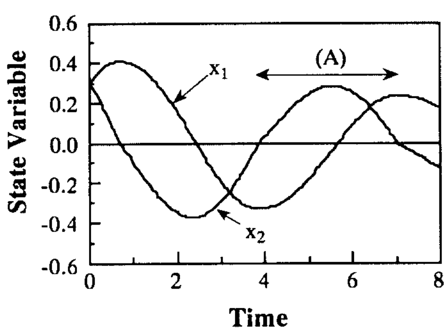


Fig.4 Estimated state variables for case 1

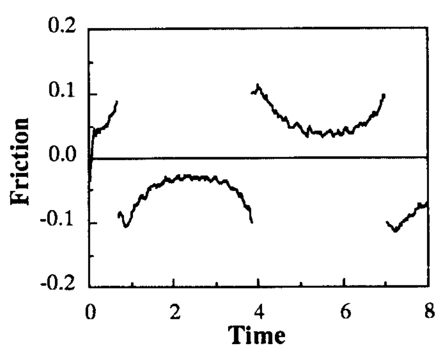


Fig.5 Estimated solid friciton for case 1

The simulation results are shown in Fig.4-10. Figure 4 and 5 show the estimated state variables and solid friction, respectively, for case 1. Figure 6 shows the velocity characteristics of the solid friction in the (A) period in Fig.4. Figure 7 is the results for case 2. The estimated parameters of the friction model approach to the real values quickly. The result for case 3 is shown in Fig.8. Small estimation error exists due to the modelling error of the friction characteristics. Figure 9 and 10 show the results for the smoothers corresponding to case 4 and case 5, respectively. Note that the result for FSBM shown in Fig.9 is inferior to that for FSPMA although the lag for FSBM is larger than that for FSPMA. Table 1 summarizes the mean square of the estimation errors.

For on-line applications , KFPMA provides better performance than KFBM. However, the computation time required for KFPMA is over ten time as long as KFBM. In addition, the performance of KFPMA deteriorates if the friction characteristic can not be approximated by the assumed model. For economical on-line applications, DMKF is more appropriate for economical on-line applications.

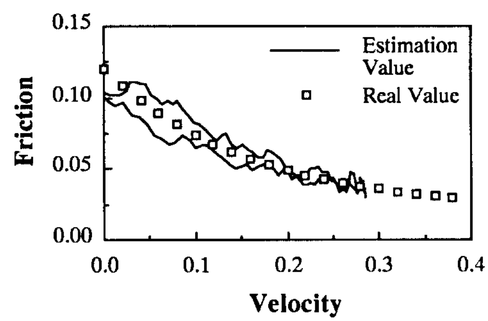


Fig.6 Estimated velocity characteristics of solid friction for case 1

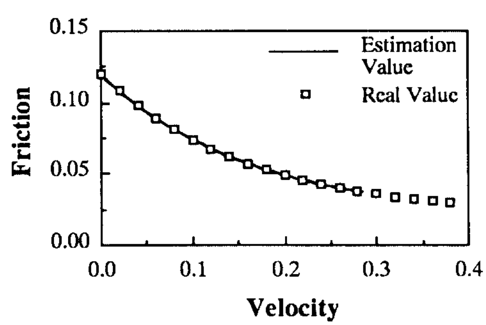


Fig.7 Estimated velocity characteristics of solid friction for case 2

Although FSBM and FSPM cannot be used for on-line estimation, these methods provide better estimation performance than KFBM or KFPMA. Especially, SFBM can estimate any friction characteristic which is difficult to be modeled by simple function such as reported by Dahl[3].

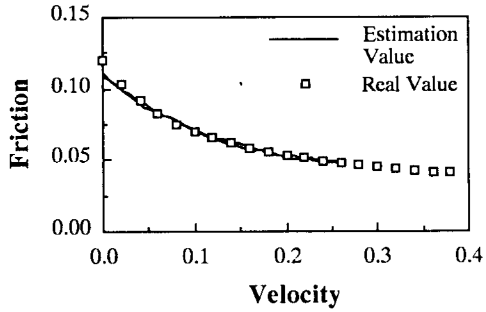


Fig.8 Estimated velocity characteristics of solid friction for case 3

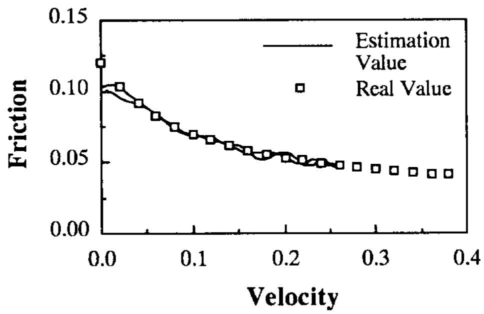


Fig.9 Estimated velocity characteristics of solid friction for case 4

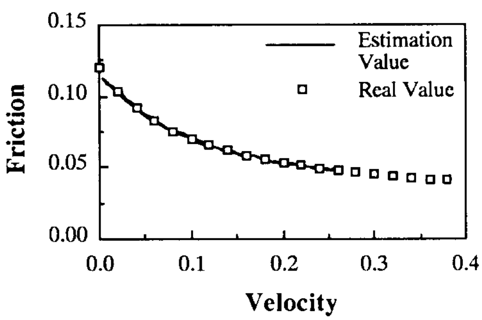


Fig.10 Estimated velocity characteristics of solid friction for case 5

Table1 Estimation error of simulation

simulation condition	mean square of estimation error
case 1	5.8×10^{-5}
case 2	5.2×10^{-8}
case 3	3.2×10^{-6}
case 4	8.0×10^{-6}
case 5	8.3×10^{-7}

4. EXPERIMENTAL RESULTS

In this section, we estimate the solid friction acting on a pendulum axis. Figure 11 shows the schematic diagram of the pendulum. The continuous time state equation of the system is

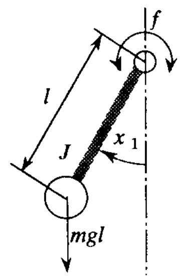


Fig.11 Schematic diagram of pendulum

Table2 Parameter of pendulum

Parameter	Value	Unit
J	2.041×10^{-2}	kgm^2
m	0.1894	kg
l	0.2312	m
c	4.8×10^{-2}	Nm sec.

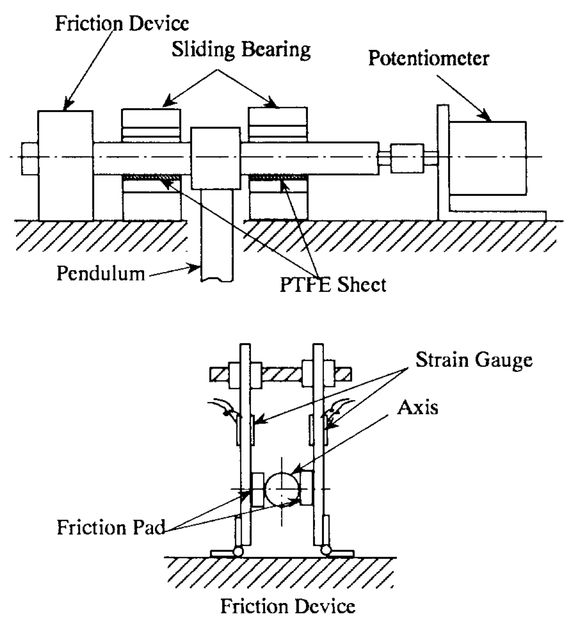


Fig.12 Bearing part of apparatus

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -mgl/J \sin x_1 - c/J x_2 \pm f/J + \omega \end{cases} \quad \begin{pmatrix} +: x_2 < 0 \\ -: x_2 > 0 \end{pmatrix} \tag{29}$$

where x_1 is the pendulum angle, x_2 is the angular velocity, m is the mass of the pendulum, g is the gravity acceleration, l is the length from the center of rotation to the center of gravity, c is a coefficient of a damping, f is a solid friction, J is the inertia of the pendulum, ω is a process noise. The observation equation is

$$y=[1\ 0]x+v \tag{30}$$

Table 2 represents the parameter of the pendulum. The bearing part of the experimental apparatus is shown in Fig.12. The friction device is attached to the axis. The solid friction is almost produced by this device. The material of the axis is mild steel. The friction pads are made by industrial pure aluminum. The surfaces of the friction pads are lubricated by turbine oil. The normal load is measured by the strain gauge and adjusted by the screws of the upper part of the apparatus. The angle of the pendulum is measured by the microcomputer system which has 12bit A/D converter to detect the voltage of the potentiometer.

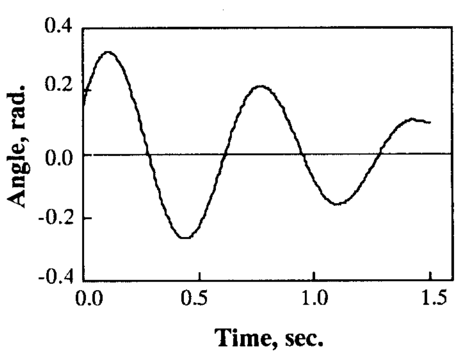


Fig.13 Measured data, pendulum angle vs time

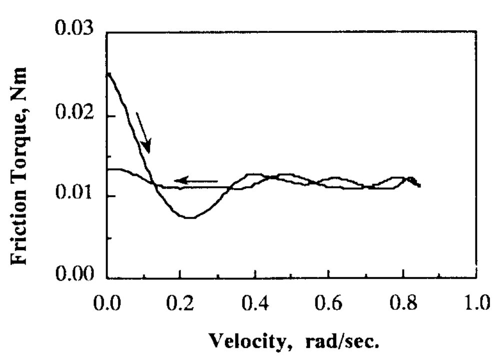


Fig.14 Estimation result for experimental data

The sampling period is 5.0×10^{-4} sec.

We present here an example of the estimation result by use of FSBM. The conditions of FSBM are

$P_{0/-1}$=[0.01 0.01 0.01], Q=[0 10^{-6} 1],

r=10^{-7}, lag=200

The measured data of the pendulum angle are shown in Fig.13. The estimated friction characteristics is shown in Fig.14. Note that the hysteresis characteristic of the friction is captured successfully by the proposed technique.

5. CONCLUSION

Using the extended Kalman filtering techniques, we have discussed methods for estimating the solid friction in mechanical systems. The effectiveness of the proposed methods is illustrated by the experimental result as well as the simulation results.

We recommend two step use of the proposed algorithms. First, the algorithms based on the blind model is applied to obtain a rough model of the friction. Then, the result is used to construct a parametric model possibly with unknown parameters. The improved estimation algorithm can be constructed based on the parametric model.

Applications of the proposed techniques to various practical problems are promising and will be discussed elsewhere.

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