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On Decentralized Adaptive Controller Design

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Abstract

This paper presents a decentralized model reference adaptive control scheme for an interconnected linear system composed of a number of single-input single-output subsystems in which outgoing interactions pass through the measurement channel and are subjected to bounded external disturbances. The scheme can treat the unknown strength of interactions as well as uncertainties in subsystem dynamics, and allows for the case when the relative degree of each decoupled subsystem does not exceed two.

1. Introduction

In order to handle an uncertain interconnected system composed of a number of subsystems, decentralized adaptive mathods whereby each subsystem is independently controlled on the basis of its own adjustment mechanism and locally available information have been proposed [1-3].

A brief summary of the research results on decentralized adaptive control so far is as follows. Robust model reference adaptive controllers have been first applied to the control of unknown subsystems as if they were decoupled each other. Then, the sufficient conditions so called the M - matrix conditions have been given to establish the bounds on interconnections, which guarantee that stability of the closed-loop isolated subsystems is not destroyed when the subsystems are interconnected [1, 2]. Recently, Gavel and Siljak [3] presented a stsble adaptive decentralized design in the face of unknown interconnections strengths and under certain structural condition placed on the interconnections. To be more concrete, for an uncertain interconnected system composed of single-input single-output subsystems in which either incoming interactions enter through the mesurement channels or outgoing interactions pass through the measurement channels, they suggested a decentralized high-gain feedback scheme which automatically adjusts the local adaptation gains to levels that assure stability of the overall system. But, the design technique for the class that incoming interactions enter through the control channels has the disadvantage of disconnecting the interconnection signal from the input of the plant. And moreover, the main result is restricted to the case when the relative order of each subsystem is equal to 1.

In this paper, a decentralized adaptive scheme is developed based on the design techniques of [3, 4] for an uncertain interconnected continuous linear system composed of single-input single-output subsystems in which outgoing interactions pass through the mesurement channels and the relative degree of which does not exceed two. It is noted that when the relative degree is equal to 1, the present controller is similar in form to that of [3], while in case that the relative order is equal to 2, a new type of controller is deviced. The stability of the overall closed-loop system is investigated. Also, a numerical example is illustrated via computer simulations.

2. Problem Statement

Consider an interconnected continuous linear time-invariant system composed of N subsystems

$$\dot{x}_i = A_i x_i + b_i u_i + d_i + \sum_{j=1}^{N} A_{ij} x_j \qquad (1)$$

$$y_i = c_i^T x_i , \qquad i = 1, 2, \cdots, N$$

where $x_i \in R^{n_i}$, $u_i \in R$, $y_i \in R$, and $d_i \in R^{n_i}$ are the state vector of the i-th subsystem, its control input, its output, and bounded external disturbance vector affecting to the i-th subsystem, respectively. A_i, A_{ij}, b_i, and c_i are constant matrices or vectors of appropriate dimensions. A_{ij} represents the interconnection pattern between the i-th and j-th subsystems. The transfer function of esch isolated disturbance-free subsystem is given by

$$G_i(s) = \frac{y_i(s)}{u_i(s)} = \frac{\kappa_i\beta_i(s)}{\alpha_i(s)} \tag{2}$$

where $\alpha_i(s)$ and $\beta_i(s)$ are polynomials of order n_i and m_i, respectively, and κ_i is the high-frequency constant gain.

It is assumed here that

i) u_i and y_i are available only at the i-th subsystem.

ii) n_i and m_i are exactly known, and the relative order defined by $n_i^* = n_i - m_i$ does not exceed two.

iii) The elements of A_i, b_i, c_i, A_{ij} are not known.

iv) The pair (A_i, b_i) is controllable, and (c_i^T, A_i) is observable.

v) Outgoing interactions pass through the measurement channel, i.e, $A_{ij} = a_{ij}c_j^T$.

vi) $\beta_i(s)$ is a Hurwitz polynomial, that is, each decoupled subsystem is of minimum phase.

vii) The pair of κ_i is known. Without loss of generality, let its sign be positive.

Corresponding to the interconnected system (1), let us choose a reference model with desired properties.

$$G_{iM}(s) = \frac{y_{iM}(s)}{u_{iM}(s)} = \frac{\kappa_{iM}\beta_{iM}(s)}{\alpha_{iM}(s)}. \tag{3}$$

$$i = 1, 2, \cdots, N$$

where $\alpha_{iM}(s)$ and $\beta_{iM}(s)$ are Hurwitz polynomial of order n_i and m_i, respectively, and κ_{iM} is a positive constant. Now the problem is to determine a local control for each subsystem (1) to track the reference model (3) as close as possible. For this, we first present a method to design a decentralized adaptive controller, and then show that the resultant closed-loop system is assured to be globally stable.

3. Controller Design

Since the state variables are not accessible, we use, instead, auxilary signals generated by the State Variable Filter (SVF).

$$\begin{aligned} \dot{v}_i &= F_i v_i + h_i y_i \\ \dot{w}_i &= F_i w_i + h_i u_i \end{aligned} \tag{4}$$

where v_i and w_i are auxilary signal vectors of order (n_i-1), and, (F_i, h_i) is a controllable pair with stable matrix F_i. Also, F_i is chosen such that determinant of $(sI_i - F_i)$ has $\beta_{iM}(s)$ in (3) as a factor.

Augumenting SVF (4) into (1), we get the following nonminimal representation.

$$\begin{bmatrix}\dot{x}_i\\ \dot{v}_i\\ \dot{w}_i\end{bmatrix} = \begin{bmatrix} A_i & 0 & 0\\ h_i c_i^T & F_i & 0\\ 0 & 0 & F_i\end{bmatrix}\begin{bmatrix}x_i\\ v_i\\ w_i\end{bmatrix} + \begin{bmatrix}b_i\\ 0\\ h_i\end{bmatrix} u_i + \sum_{j=1}^{N}\begin{bmatrix}a_{ij}\\ 0\\ 0\end{bmatrix} y_j + \begin{bmatrix}d_i\\ 0\\ 0\end{bmatrix}$$

$$y_i = [\, c_i^T \;\; 0^T \;\; 0^T \,]\begin{bmatrix}x_i\\ v_i\\ w_i\end{bmatrix}, \quad i = 1, 2, \cdots, N \tag{5}$$

or

$$\begin{aligned} \dot{\bar{x}}_i &= \bar{A}_i\bar{x}_i + \bar{b}_i u_i + \sum_{j=1}^{N}\bar{a}_{ij} y_j + \bar{d}_i \\ y_i &= \bar{c}_i^T\bar{x}_i, \quad i = 1, 2, \cdots, N \end{aligned} \tag{5.a}$$

Introducing a stable matrix $\bar{A}_{iM}$ given by.

$$\bar{A}_{iM} = \begin{bmatrix} A_i + b_i\theta_{i1}^{*}c_i^T & b_i\theta_{i2}^{*T} & b_i\theta_{i3}^{*T} \\ h_i c_i^T & F_i & 0 \\ h_i\theta_{i1}^{*}c_i^T & h_i\theta_{i2}^{*T} & F_i + h_i\theta_{i3}^{*T} \end{bmatrix} \tag{6}$$

(5.a) can be rewritten as follows :

$$\begin{aligned} \dot{\bar{x}}_i &= \bar{A}_{iM}\bar{x}_i + \bar{b}_i u_i - \bar{b}_i\bar{\theta}_i^{*T}\bar{\Psi}_i + \bar{d}_i + \sum_{j=1}^{N}\bar{a}_{ij} y_j \\ y_i &= \bar{c}_i^T\bar{x}_i. \quad i = 1, 2, \cdots, N \end{aligned} \tag{7}$$

where $\bar{\theta}_i^* = [\theta_{i1}^* \;\; \theta_{i2}^{*T} \;\; \theta_{i3}^{*T}]^T$, $\bar{\Psi}_i = [\, y_i \;\; v_i^T \;\; w_i^T \,]^T$.

The transfer function of reference model (3) can also be realized in nonminimal representation.

$$\begin{aligned} \dot{\bar{x}}_{iM} &= \bar{A}_{iM}\bar{x}_{iM} + \bar{b}_i\theta_{i4}^{*}r_i \\ y_{iM} &= \bar{c}_i^T\bar{x}_{iM}. \quad i = 1, 2, \cdots, N \end{aligned} \tag{8}$$

Note that there exist such θ_i^* and θ_{i4}^* that (8) is reduced to (3) (refer to [4]).

Defining the state error and the output error as

$$e_i = \bar{x}_i - \bar{x}_{iM}, \quad \varepsilon_i = y_i - y_{iM} \tag{9}$$

the state error equations are obtained from (7) and (8).

$$\begin{aligned} \dot{e}_i &= \bar{A}_{iM} e_i + \bar{b}_i u_i - \bar{b}_i\theta_i^{*T}\bar{\Psi}_i + \bar{d}_i + \sum_{j=1}^{N}\bar{a}_{ij} y_j \\ \varepsilon_i &= \bar{c}_i^T e_i, \quad i = 1, 2, \cdots, N \end{aligned} \tag{10}$$

where $\theta_i^* = [\bar{\theta}_i^{*T} \ \theta_{i4}^*]^T$, $\Psi_i = [\bar{\Psi}_i^T \ r_i]^T$. (10.a)

3.1 The Case When $n^* = 1$

In this case, the transfer function of the reference model (3) is strictly positive real. To stabilize the error system (10), the following decentralized adaptive controller is derived based on the design concept of [3].

$$u_i = \theta_i(t)^T \Psi_i - \rho_i(t)\,\varepsilon_i, \qquad i = 1, 2, \cdots, N \quad (11)$$

where $\theta_i(t)$ and $\rho_i(t)$ are estimates of θ_i^* in (10.a) and ρ_i^* to be defined later. The estimates are updated via the adaption laws given by

$$\begin{aligned} \dot{\theta}_i &= \Gamma_i(-\sigma_{i1}\theta_i - \varepsilon_i \Psi_i) \\ \dot{\rho}_i &= \gamma_i(-\sigma_{i2}\rho_i + \varepsilon_i^2) \end{aligned} \quad (11.a)$$

where σ_{i1} and σ_{i2} are positive decaying constants, Γ_i is a symmetric positive definite weighting matrix, and γ_i is a positive weighting constant. Note that the second term of the control law (11) is included to override some destabilizing effects by interactions from order subsystems to the i-th subsystem

Applying (11) to (10), the closed loop error system becomes

$$\begin{aligned} \dot{e}_i &= \bar{A}_{iM} e_i + \bar{b}_i \phi_{i1}^T(t)\Psi_i - \bar{b}_i \rho_i(t)\,\varepsilon_i + \bar{d}_i + \sum_{j=1}^{N} \bar{a}_{ij} y_j \\ \varepsilon_i &= \bar{c}_i^T e_i, \quad i = 1, 2, \cdots, N \end{aligned} \quad (12)$$

where

$$\begin{aligned} \phi_{i1}(t) &= \theta_i(t) - \theta_i^*, \quad \dot{\phi}_{i1} = \dot{\theta}_i \\ \phi_{i2}(t) &= \rho_i(t) - \rho_i^*, \quad \dot{\phi}_{i2}(t) = \dot{\rho}_i \end{aligned} \quad (12.a)$$

3.2 The Case When $n^* = 2$

In this case, it s not possible to choose the transfer function (3) strictly positive real, and hence this case should be treated in a different way from the afore-mentioned case. To reduce the model following error e_i of (10), the following decentralized adaptive controller is deviced based on the design techniques of [3, 4].

$$u_i = \theta_i^T \Psi_i + \dot{\theta}_i^T \eta_i - \rho_i \varepsilon_i - \mathrm{sgn}(\varepsilon_i)\dot{\rho}_i \xi_i \qquad i = 1, 2, \cdots, N \quad (13)$$

where $\theta_i(t)$ and $\rho_i(t)$ are adjustable parameters, and calculated through the adaptation laws given by

$$\begin{aligned} \dot{\theta}_i &= \Gamma_i(-\sigma_{i1}\theta_i - \varepsilon_i \eta_i) \\ \dot{\rho}_i &= \gamma_i(-\sigma_{i2}\rho_i + |\varepsilon_i|\xi_i) \end{aligned}$$

$$\begin{aligned} \dot{\eta}_i &= -g_i \eta_i + \Psi_i \\ \dot{\xi}_i &= -g_i \xi_i + |\varepsilon_i| \end{aligned} \quad (13.a)$$

where g_i is a positive constant, $|\cdot|$ means the absolute value, and $\mathrm{sgn}(\cdot)$ in (13) indicates the sign function.

Applying (13) to (10), the closed loop error system becomes [4].

$$\begin{aligned} \dot{e}_i &= \bar{A}_{iM} e_i + \bar{b}_{iPR}\, \phi_{i1}^T \eta_i - \bar{b}_{iPR}\, \mathrm{sgn}(\varepsilon_i)\, \rho_i\, \xi_i + \bar{d}_i + \sum_{j=1}^{N} \bar{a}_{ij} y_j \\ \varepsilon_i &= \bar{c}_i^T e_i, \qquad i = 1, 2, \cdots, N \end{aligned} \quad (14)$$

where $\bar{b}_{iPR} = b_i(s + \delta_i)$, and the transfer function $\bar{c}_i^T(sI - \bar{A}_{iM})^{-1}\bar{b}_{iPR}$ of a decoupled subsystem in (14) is strictly positive real. Also ϕ_{i1} and ϕ_{i2} are defined in (12.a). Note that comparing to (11), (13) have additional terms to make the transfer functions of (14) strictly positive real, which is used as inportant point in stability analysis to follow.

5. Stability Analysis

The error systems (12) to (14) developed according to the relative order can be expressed in an unified form.

$$\begin{aligned} \dot{e}_i &= \bar{A}_{iM} e_i + \bar{b}_{iPR}\, \phi_{i1}^T \eta_i - \bar{b}_{iPR}\, \mathrm{sgn}(\varepsilon_i)\, \rho_i\, \xi_i + \bar{d}_i + \sum_{j=1}^{N} \bar{a}_{ij} y_j \\ \varepsilon_i &= \bar{c}_i^T e_i, \quad i = 1, 2, \cdots, N \end{aligned} \quad (15)$$

$$\begin{aligned} \dot{\phi}_{i1} &= \Gamma_i(-\sigma_{i1}\theta_i - \varepsilon_i \eta_i) \\ \dot{\phi}_{i2} &= \dot{\rho}_i = \gamma_i(-\sigma_{i2}\rho_i + |\varepsilon_i|\,\xi_i) \end{aligned} \quad (15.a)$$

It is noted here that if $\bar{b}_{iPR}$, η_i and ξ_i are replaced by $\bar{b}_i$, Ψ_i and $|\varepsilon_i|$, respectively, then (14) and (14.a) result in (12) and (12.a).

The stability of the overall adaptive system (15) and (15.a) is then established through the following theorem. For use in the theorem, let us first introduce the Kalman-Yakubovich(KY) Lemma [5].

Given an asymptotically stable matrix $\bar{A}_{iM}$, a vector $\bar{c}_i$, a vector $\bar{b}_{iPR}$ such that $(\bar{A}_{iM}, \bar{b}_{iPR})$ is controllable, and a symmetric positive matrix Q_i, there exists a symmetric positive matrix P_i satisfying

$$\begin{aligned} \bar{A}_{iM} P_i + P_i \bar{A}_{iM} &= -Q_i \\ P_i \bar{b}_{iPR} &= \bar{c}_i \end{aligned} \quad (16)$$

if and only if $\bar{c}_i(sI_i - \bar{A}_{iM})^{-1}\bar{b}_{iPR}$ is the strictly positive real. To be specific, let

$$Q_i = NI_i + 2\delta_i I_i , \quad \delta_i > 0 \tag{16.a}$$

Theorem 1 : The overall adaptive system (15) and (15.a) are globally uniformly stable. In other words, $e_i(t)$, $\phi_{i1}(t)$, $\phi_{i2}(t)$, $i = 1, 2, \cdots, N$ in (15) and (15.a) are bounded for any finite $e_i(0)$, $\phi_{i1}(0)$, $\phi_{i2}(0)$, $i = 1, 2, \cdots, N$ and all t.

(Proof) Let us choose a Lyapunov function candidate as

$$V(e_i, \phi_{i1}, \phi_{i2} ; i = 1, 2, \cdots, N)$$
$$= \sum_{i=1}^{N} \{ e_i^T P_i e_i + \phi_{i1}^T \Gamma_i^{-1} \phi_{i1} + \gamma_i^{-1}(\phi_{i2})^2 \} \tag{17}$$

Then, the time derivative of V and eqns. (15), (15.a), (16), (16.a) lead to

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N} [& - N e_i^T e_i - 2\delta_i e_i^T e_i - 2\rho_i|\varepsilon_i|\xi_i \\ & + 2\phi_{i2}|\varepsilon_i|\xi_i - 2\sigma_{i1}\phi_{i1}^T(\phi_{i1} + \theta_i^*) \\ & - 2\sigma_{i2}\phi_{i2}(\phi_{i2} + \rho_i^*) + 2 e_i^T P_i \bar{d}_i \\ & + 2 e_i^T P_i \sum_{j=1}^{N} \bar{a}_{ij}(\varepsilon_j + y_{jM})]\end{aligned} \tag{18}$$

Taking the norm operation and completing squares, (18) is modified as

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N} [& - \{ \sum_{j=1}^{N} (\| e_i \| - \| P_i \bar{a}_{ij} \| |\varepsilon_j|)^2 \} \\ & + 2\rho_i^* |\varepsilon_i| \xi_i - 2\rho_i|\varepsilon_i|\xi_i + 2\phi_{i2}|\varepsilon_i|\xi_i \\ & - \delta_i \|e_i\|^2 - \sigma_{i1}\|\phi_{i1}\|^2 - \sigma_{i2}|\phi_{i2}|^2 \\ & - \delta_i (\|e_i\| - \frac{\mu_i}{\delta_i})^2 + \frac{\mu_i^2}{\delta_i} \\ & - \sigma_{i1} (\|\phi_{i1}\| - \|\theta_i^*\|)^2 + \sigma_{i1}\|\theta_i^*\|^2 \\ & - \sigma_{i2} (|\phi_{i2}| - |\rho_i^*|)^2 + \sigma_{i2}|\rho_i^*|^2]\end{aligned} \tag{19}$$

where

$$\rho_i^* = \frac{1}{2}\sum_{j=1}^{N} \| P_j a_{ji} \|^2$$
$$\mu_i = \sup_t \| P_i \bar{d}_i \| + \sum_{j=1}^{N} \| P_i \bar{a}_{ij} \| \sup_t | y_{jM} | \tag{19.a}$$

Note that during the derivation of (19), the following relations are utilized.

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \| P_i \bar{a}_{ij} \|^2 |\varepsilon_j|^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \| P_j \bar{a}_{ji} \|^2 |\varepsilon_i|^2$$
$$|\varepsilon_i| \le \xi_i \tag{19.b}$$

μ_i in (19.a) is a finite constant, because it involves bounded external disturbance $\bar{d}_i$ and the outputs of the reference model y_{jM}. The notation $\sup_t$ means the supremum with respect to time. Returning to (19), if we neglect the negative square terms and apply the definition $\phi_{i2} = \rho_i - \rho_i^*$, then $\dot{V}$ is simplified as

$$\dot{V} \le \sum_{i=1}^{N} [- \delta_i \| e_i \|^2 - \sigma_{i1} \| \phi_{i1} \|^2 - \sigma_{i2} | \phi_{i2} |^2 + \frac{\mu_i^2}{\delta_i} + \sigma_{i1} \| \theta_i^* \|^2 + \sigma_{i2} | \rho_i^* |^2] \tag{20}$$

Now, let us define a compact region so called 'residual set'.

$$\mathcal{D} = \bigcup_{i=1}^{N} \mathcal{D}_i \tag{21}$$

$$\mathcal{D}_i = \{ (e_i,\phi_{i1},\phi_{i2}) \mid \delta_i\| e_i\|^2 + \sigma_{i1}\| \phi_{i1}\|^2 + \sigma_{i2}| \phi_{i2}|^2 \le \frac{\mu_i^2}{\delta_i} + \sigma_{i1}\| \theta_i^* \|^2 + \sigma_{i2}| \rho_i^* |^2 \} \tag{21.a}$$

Since outside the residual set, $\dot{V} < 0$ and $V > 0$, it follows by Thm. 2.24 of [5] that $e_i(t)$, $\phi_{i1}(t)$, $\phi_{i2}(t)$, $i = 1, 2, \cdots, N$ are bounded for any finite $e_i(0)$, $\phi_{i1}(0)$, $\phi_{i2}(0)$, $i = 1, 2, \cdots, N$ and all t. Moreover, it can be shown via similar arguments as in [2, 4] that there exists a finite time T such that $e_i(t)$, $\phi_{i1}(t)$, $\phi_{i2}(t)$, $i = 1, 2, \cdots, N$ reside within the residual set for $t \ge T$. Note that the size of the residual set depend on the strength of interconnections, the magnitude of external disturbances, local design parameters, and the maximum values of the model outputs.

5. Numerical Example

Consider the unstable linear constant interconnected system with unknown system parameters described by

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ -1 \end{bmatrix} y_2 + \begin{bmatrix} 0.1 \sin 5t \\ 0.1 \cos 5t \end{bmatrix}$$
$$y_1 = [\ 4 \quad 2\ ]\ x_1$$
$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 + \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix} y_2 + \begin{bmatrix} -1 \\ 1 \end{bmatrix} y_1 + \begin{bmatrix} 0.1 \cos 5t \\ 0.1 \sin 5t \end{bmatrix}$$
$$y_2 = [\ 2 \quad 0\ ]\ x_2$$

Note that the transfer functions of decoupled subsystems are given by

$$G_1(s) = \frac{2(s + 2)}{s^2 + 4} , \quad G_2(s) = \frac{2}{s^2 - 2}$$

Let the reference model be

$$G_{1M}(s) = \frac{(s+4)}{(s+2)^2} \quad , \quad G_{2M}(s) = \frac{4}{(s+2)^2}$$

where reference inputs $r_i(t)$, $i = 1, 2$ are square waves of height 5 and period 10 sec.

Now, using the proposed decentralized adaptive scheme, computer simulations are carried out for model reference adaptive control of the example plant. The results with design parameters in (4), (11.a), (13.a)

$$F_i = -4 \quad , \quad h_i = 1$$

$$\Gamma_i = I \quad , \quad \gamma_i = 2 \quad , \quad \sigma_{i1} = \sigma_{i2} = 0.01$$

$$g_i = 4 \quad , \quad i = 1, 2$$

and with the initial values of auxilary signals, adjustable parameters resetting to zeros are presented in Figs. 1-4.

As can be seen in the figures, the output of each subsystem closely tracks that of the reference model after a finite interval, and all the adjustable parameters are bounded for every time.

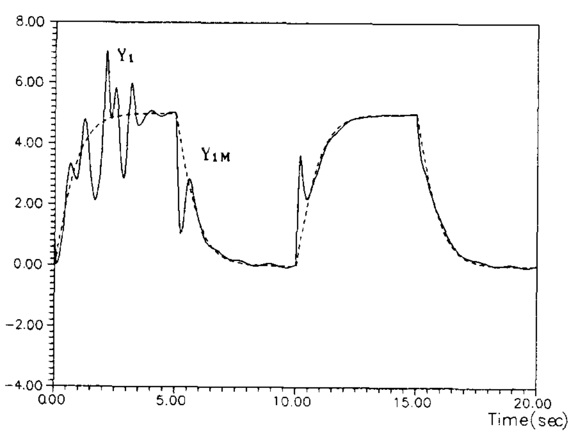

Fig 1. Model following error of subsystem 1

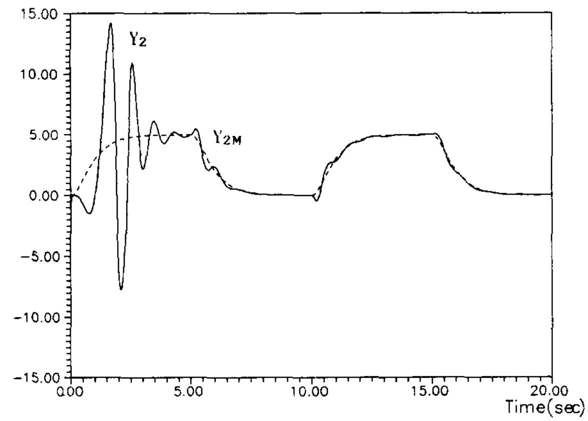

Fig 2. Model following error of subsystem 2

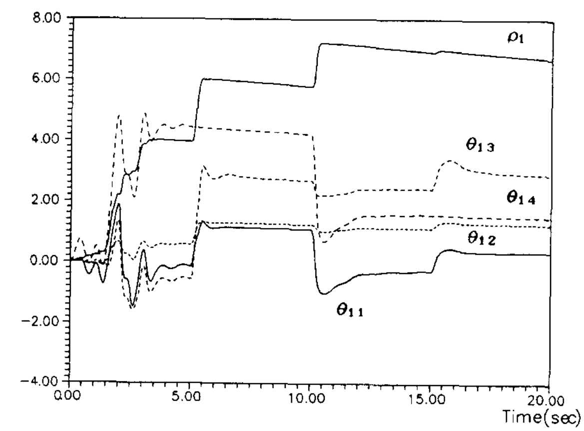

Fig 3. Trajectories of adjustable parameters for subsystem 1

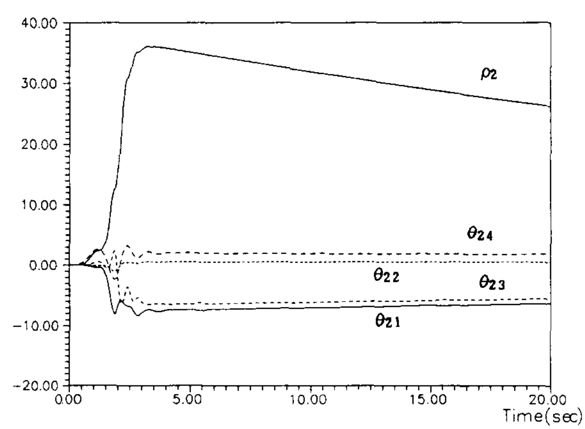

Fig 4. Trajectories of adjustable parameters for subsystem 2

6. Conclusion

A decentralized adaptive controller has been designed for a class of interconnected continuous linear systems in which outgoing interactions pass through the measurement channels and the realtive order of each subsystem does not exceed two. Also, it has been shown that model following errors and adjustable parameters of the overall adaptive system are globally uniformly bounded. The local controllers basically have output error feedback terms to override some destabilizing effects by the interactions. Specifically when the relative order is equal to two, the controllers are reinforced such that the transfer functions of the closed-loop error equations be strictly positive real.

A further research topic of immediate interest is to extend this framework to the case without restriction on the realtive order.

References

[1] A. Hmamed and L. Radouane, "Decentralized Nonlinear Adaptive Feedback Stabilization of Large Scale Interconnected Systems," IEE Proc. Part D, vol. 130, pp. 57-62, 1983.

[2] P. A. Ioannou, "Decentralized Adaptive Control of Interconnected Systems," IEEE Trans. Automat. Contr., vol. 31, pp. 291-298, 1986.

[3] D. T. Gavel and D. D. Siljak, "Decentralized Adaptive Control : Structural Conditions for Stability," IEEE Trans. Automat. Contr., vol. 34, pp.413-426, 1989.

[4] K. S. Narendra and L. S. Valavani, "Stable Adaptive Controller Design-Direct Control," IEEE Trans. Automat. Contr., vol. 23, pp. 570-582, 1978.

[5] K. S. Narendra and A. M. Annaswamy, Stable Adaptive Systems, Prentice-Hall, 1989.