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Receding Horizon Predictive Controls and Generalized Predictive Controls with their Equivalence and Stability

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ABSTRACT

In this paper, we developed a Receding Horizon Predictive Control for Stochastic state space models(RHPCS). RHPCS was designed to minimize a quadratic cost function. RHPCS consists of Receding Horizon Tracking Control(RHTC) and a state observer. It was shown that RHPCS is equivalent to Generalized Predictive Control(GPC) when the underlying state space model is equivalent to the I/O model used in the design of GPC. The equivalence between GPC and RHPCS was shown through the comparison of the transfer functions of the two controllers. RHPCS provides a time-invarient optimal control law for systems for which GPC can not be used. The stability properties of RHPCS was derived. From the GPC's equivalence to RHPCS, the stability properties of GPC were shown to be the same as those for RHTC.

1. Introduction

There are several control methods[1-6] which can be classified as predictive controls. Many successful applications of these methods to industrial processes have been reported. The GPC suggested by Clarke et. al.[6] is regarded as a generalization of the Minimum-Variance(MV)[1] and Generalized Minimum-Variance(GMV) control[2]. GPC has been believed to stabilize nonminimum-phase and open-loop unstable plants. However, this ability has not yet completely proven.

There have been many attempts to examine the stability properties of GPC due to its popularity. However the fact that GPC is based on I/O models and minimizes a finite receding horizon cost function has been an obstacle in searching for the stability properties of GPC. There have been several approaches[7-10] to search for the properties of GPC by relating it with control methods for state space models such as RHTC or LQ(Linear Quadratic) control. Clarke et. al.[7,9] discussed the stability properties of GPC by relating GPC with LQ control for state space models. However they only mentioned the limit cases in which the cost horizon grows infinitely or the control weighting decreases to zero. Clarke and Scattolini[11] suggested a constrained receding horizon predictive control which guarantees the stability of the closed loop system. Kwon et. al.[10] showed that the GPC algorithm for I/O models has the same solution as RHTC. In effect they showed that GPC has the same stability properties with RHTC when the system has no disturbances.

Since the works[7,9,10] discussed the system without disturbances, they could compare GPC with state feedback controllers such as LQ control or RHTC under the assumption that the exact state was available. However, for systems with disturbances, a form of a state estimator must be employed, which makes the comparison more difficult.

Based on the fact that GPC is designed to mininized the same cost function as LQG, Bitmead *et. al*[12] claimed that GPC is eqivalent to a receding horizon LQG. However, they did not show the equivalence between GPC and LQG directly.

In this paper, we derive Receding Horizon Predictive Control for Stochastic models(RHPCS) minimizing a quadratic cost function which is somewhat different from that of LQG. The cost function containes the discrepancy between reference sequence and predicted future output sequence. RHPCS has some variations according to how to make the prediction of future output sequence. It will be shown that RHPCS has the same transfer function as GPC when the predicted future output sequence is made from a steady state kalman filter. RHPCS provides a stable predictive control method for a stochastic system for which GPC can not be used.

In order to derive RHPCS, an one-shot solution for stochastic state space models will be derived first. The derived one-shot solution for stochastic state space models will be denoted by GPC/SM. RHPCS can be obtained from GPC/SM using the results of Kwon *et al.*(to appear). RHPCS consists of RHTC and a state observer. In this paper, we will call the GPC solution for stochastic I/O models(Clarke *et al.* 1987) as GPC/IM to clearly distinguish it from GPC/SM.

In Section 2, we state the problem and in Section 3, we develop GPC/SM and RHPCS. The equivalence between GPC/IM and RHPCS is proved in Section 4. Stability properties of GPC/IM are discussed through the equivalence between GPC/IM and RHPCS in Section 5.

2. Problem statement

We consider a linear system given in the following stochastic state space model:

$$x(t+1) = Ax(t) + B\Delta u(t) + D\xi_1(t)$$
$$y(t) = Hx(t) + \xi_2(t) \qquad (2.1)$$

where $\xi_1(t)$ and $\xi_2(t)$ are zero-mean, white noise sequences with variances given by:

$$E\left\{\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}[\xi_1(t)^T \ \xi_2(t)^T]\right\} = \begin{bmatrix}\Gamma_1 & \Gamma_{12}\\ \Gamma_{12}^T & \Gamma_2\end{bmatrix}$$

and x(t), Δu(t), and y(t) are vectors of order n, m, and l respectively. The matrix A is assumed to be nonsingular.

When the system (2.1) is a single input single output system, $\xi_1(t)=\xi_2(t)$ and the matrices A, B, D and H are given as:

$$A = \begin{bmatrix} -a_1 & 1 & 0 & \cdot & \cdot & \cdot & 0\\ -a_2 & 0 & 1 & 0 & \cdot & \cdot & 0\\ \cdot & & & \cdot & \cdot & & \cdot\\ \cdot & & & & \cdot & \cdot & \cdot\\ \cdot & & & & & \cdot & 0\\ \cdot & & & & & & 1\\ -a_n & 0 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix} \quad B = \begin{bmatrix} b_1\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ b_n \end{bmatrix} \quad D = \begin{bmatrix} c_1-a_1\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ c_n-a_n \end{bmatrix}$$

$$H = [1 \ 0 \cdot \cdot \cdot 0]. \qquad (2.2)$$

System (2.1) is equivalent to the following CARIMA(Controlled AutoRegressive Integrated Moving Average) model:

$$a_e(q^{-1})\Delta\, y(t) = b(q^{-1})\Delta u(t) + c(q^{-1})\xi_1(t)$$
$$a(q^{-1}) \triangleq a_e(q^{-1})\Delta = 1+a_1q^{-1}+\cdots+a_nq^{-n}, \quad a_n \neq 0$$
$$b(q^{-1}) = b_1q^{-1}+b_2q^{-2}+\cdots+b_nq^{-n}$$
$$c(q^{-1}) = 1+c_1q^{-1}+\cdots+c_nq^{-n} \qquad (2.3)$$

where q^{-1} is the unit delay operator, Δ is the differencing operator $1-q^{-1}$.

GPC/IM was designed for the CARIMA model of Equation (2.3)[6]. The cost function to be minimized is:

$$J = \sum_{j=1}^{N}\{[\hat{y}(t+j/t)-y_r(t+j)]^T[\hat{y}(t+j/t) - y_r(t+j)]+\lambda\Delta u^T(t+j-1)\Delta u(t+j-1)\} \qquad (2.4)$$

where the reference sequence $[y_r(t+j), j=1,2,\ldots,N]$ are supposed to be available at time t and $\hat{y}(t+j/t)$ is the expected value of y(t+j), conditioned on the measured output sequence up to time t. The future control sequences u(t), u(t+1), ..., u(t+N-1) are determined from the data available up to time t at each sample instance t. After obtaining the optimal control sequences u(t), u(t+1),...,u(t+N-1),only the first element u(t) is applied at time t, and at the next time t+1, the overall procedure is repeated.

Our aim is to derive RHPCS which takes the same strategy as GPC for the state space model (2.1), and to show the equivalence between the steady state RHPCS and GPC/IM when system (2.1) is equivalent to the CARIMA model (2.3). Utilizing the equivalence between GPC/IM and RHPCS, we will investigate the stability properties and internal structures of GPC/IM.

3. Development of GPC/SM and RHPCS

In this section, we will derive an optimal output feedback control minimizing the cost function (2.4) for system (2.1), where the matrices A, B, D and H may be different from Equation (2.2).

Using $\hat{x}(t+1/t)$, which is the expected value of x(t+1) conditioned on the measured output sequence up to time t, the expected values of the future outputs $\hat{y}(t+i/t)$, for i=1,2, .., N, can be obtained as follows:

$$\hat{y}(t+1/t) = H\hat{x}(t+1/t)$$
$$\hat{y}(t+2/t) = HA\hat{x}(t+1/t) + HB\Delta u(t+1)$$
$$\vdots \qquad (3.1)$$
$$\hat{y}(t+N/t) = HA^{N-1}\hat{x}(t+1/t) + \sum_{j=2}^{N} HA^{N-j}B\Delta u(t+j-1)$$

In the following development, we will use $\hat{x}(t+1)$ instead of $\hat{x}(t+1/t)$.

$\hat{x}(t+1)$ is obtained from the Kalman filter of the following form:

$$\hat{x}(t+1)=A\hat{x}(t)+B\Delta u(t)+K^*(t)(y(t)-H\hat{x}(t)) \quad (3.2)$$
$$\hat{x}(t_0)=E\{x(t_0)\}.$$

The optimal gain matrix $K^*(t)$ is given by[11]:

$$K^*(t)=(AP^*(t)H^T+D\Gamma_{12})(HP^*(t)H^T+\Gamma_2)^{-1} \quad (3.3)$$

$P^*(t)$ satisfies the following recursion:

$$P^*(t+1)=AP^*(t)A^T+D\Gamma_1D^T-K^*(t)\cdot[HP^*(t)H^T+\Gamma_2]K^*(t)^T \quad (3.4)$$

with

$$P^*(t_0)=P_0 \quad for \ some \ t_0<t. \quad (3.5)$$

$P^*(t)$ is the state error covarience, that is:

$$P^*(t) = E\{[x(t)-\hat{x}(t)][x(t)-\hat{x}(t)]^T/t\}$$

From Equation (3.2), $\hat{x}(t+1)$ is given by:

$$\hat{x}(t+1)=A\hat{x}'(t)+B\Delta u(t) \quad (3.6)$$

where

$$\hat{x}'(t)=\hat{x}(t)+A^{-1}K^*(t)(y(t)-H\hat{x}(t)). \quad (3.7)$$

Combining Equation (3.6) with (3.1), we get:

$$\begin{aligned}\hat{y}(t+1/t) &= HA\hat{x}'(t)+HB\Delta u(t)\\ \hat{y}(t+2/t) &= HA^2\hat{x}'(t)+HAB\Delta u(t)+HB\Delta u(t+1)\\ &\vdots \\ \hat{y}(t+N/t) &= HA^N\hat{x}'(t)+\sum_{j=1}^{N} HA^{N-j}B\Delta u(t+j-1).\end{aligned} \quad (3.8)$$

From the above relations, the cost function (2.4) can be rewritten as:

$$J=(WU+V\hat{x}(t)-Y_r(t))^T(WU+V\hat{x}(t)-Y_r(t))+\lambda U(t)^TU(t) \quad (3.9)$$

where

$$W \triangleq \begin{bmatrix} HB & 0 & \cdot & \cdot & \cdot & 0 \\ HAB & HB & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ HA^{N-1}B & HA^{N-2}B & \cdot & \cdot & \cdot & HB \end{bmatrix} \quad V \triangleq \begin{bmatrix} HA \\ HA^2 \\ \cdot \\ \cdot \\ HA^N \end{bmatrix}$$

$$U(t) \triangleq \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \cdot \\ \cdot \\ \Delta u(t+N-1) \end{bmatrix}$$

$$Y_r(t) \triangleq [y_r(t+1)\ y_r(t+2) \cdots y_r(t+N)]^T.$$

The control increment vector U(t) which minimizes J can be obtained from (3.9) as:

$$U(t) = (W^TW+\lambda I)^{-1}W^T(Y_r(t) - V\hat{x}'(t)).$$

The first m elements of U(t) constitute the current control input vector as follows:

$$\Delta u_S(t) = Z_S(Y_r(t) - V\hat{x}'(t)) \quad (3.10)$$

where

$$Z_S= [I_m 0 \cdots 0](W^TW+\lambda I)^{-1}W^T.$$

The current control u(t) is given by:

$$u(t) = u(t-1)+\Delta u_S(t). \quad (3.11)$$

The control (3.10) will be called as 'GPC solution for stochastic space state models' and will be denoted by GPC/SM.

According to the result of Kwon et. al.[10], the feedback and feedforward gains of Equation (3.10) can be obtained from the following recursive equations:

$$\begin{aligned} Z_SV &= [\lambda I+B^TF(N)B]^{-1}B^TF(N)A \\ Z_SY_r(t) &= -[\lambda I+B^TF(N)B]^{-1}B^Tg_N(t+1)\end{aligned} \quad (3.12)$$

where F(N) is obtained from the discrete time Riccati equation:

$$\begin{aligned} F(i+1) &= A^TF(i)A-A^TF(i)B[\lambda I+B^TF(i)B]^{-1}\cdot B^TF(i)A+H^TH \quad for \ i>1 \\ F(1) &= H^TH \end{aligned} \quad (3.13)$$

and $g_N(t+1)$ is obtained from the following recursion:

$$g_N(t+j)=A^T\{I-F(N-j)B[\lambda I+B^TF(N-j)B]^{-1}B^T\}\cdot g(t+j+1)-H^Ty_r(t+j)$$

$$g_N(t+N) = -H^Ty_r(t+N). \quad (3.14)$$

Thus, Equation (3.10) can be rewritten as:

$$\Delta u_S(t) = -[\lambda I+B^TF(N)B]^{-1}B^T\cdot[F(N)A\hat{x}'(t)+g_N(t+1)] \quad (3.15)$$

where $\hat{x}'(t)$ is obtained from the state observer

(3.2) and Equation (3.7). Since A is nonsingular, $\hat{x}'(t)$ is obtained from $\hat{x}(t)$ as in Equation (3.7). We must take note of the fact that the state feedback gain in Equation (3.15) is equal to that of RHTC[8] when the cost functions are the same. The control (3.15) will be called RHPCS since it consists of RHTC and a Kalman filter of a special form(i.e. Equation (3.2) and (3.7)).

In order to see the equivalence between RHPCS and GPC/IM in the next section, now we achieve the steady state RHPCS under the condition that system (2.1) is equivalent to the CARIMA model (2.3). We will call GPC/SM and RHPCS with a steady state value of $K^*(t)$ as 'steady state GPC/SM' and 'steady state RHPCS' respectively. The steady state value of $K^*(t)$ and $P^*(t)$ are given as the following lemma.

Lemma 3.1 If system (2.1) is equivalent to the CARIMA model (2.3) and the polynomial $c(q^{-1})$ is exponentially stable, then the steady state solutions of Equation (3.3) and (3.4) are:

$$\lim_{t_0\to-\infty} K^*(t) \triangleq \underline{K} = D, \qquad \lim_{t_0\to-\infty} P^*(t) \triangleq \underline{P} = 0$$

for any P_0.
Proof: Omitted

In the next lemma, the convergence properties of the state observer (3.2) with $K^*(t) = \underline{K} = D$ are summarized.

Lemma 3.2: If system (2.1) is equivalent to the CARIMA model (2.3) and the gain matrix $K^*(t)$ of the state observer (3.2) is chosen to be equal to D and $t_0=0$, then:

(i) If $c(q^{-1})$ is exponentially stable, then $\lim_{t\to\infty} e(t) = 0$ for all e(0).

(ii) If e(0)=0, then $\hat{x}(t) = x(t)$ for all $t \geq 0$.

(iii) If $c(q^{-1})=1$, then $\hat{x}(t) = x(t)$ for all $t \geq n$.

where $e(t) \triangleq x(t) - \hat{x}(t)$
Proof: Omitted

From Lemma 3.1 and 3.2, we get the steady state RHPCS for system (2.1) when system (2.1) is equivalent to the CARIMA model (2.3) as follows:

$$\Delta u_s(t) = -[\lambda + B^T F(N) B]^{-1} \cdot B^T [F(N) A \hat{x}'(t) + g_N(t+1)] \quad (3.16)$$

where

$$\hat{x}'(t) = \hat{x}(t) + A^{-1}K^*(t)(y(t) - H\hat{x}(t)) \quad (3.17)$$

$$\hat{x}(t+1) = A\hat{x}(t) + B\Delta u(t) + D(y(t) - H\hat{x}(t)) \quad (3.18)$$
$$\hat{x}(t_0) = E\{x(t_0)\}.$$

The steady state GPC/SM is also given as follows:

$$\Delta u_S(t) = Z_S(Y_r(t) - V\hat{x}'(t)) \quad (3.19)$$

where $\hat{x}'(t)$ is obtained from Equation (3.17) and (3.18). Previous studies[9][10] did not mention the state observer when the relation between GPC/IM and RHTC or LQ control was discussed.

In the next section, we will show the equivalence between the steady state RHPCS given by Equation (3.16) and GPC/IM. This will make the internal structure of GPC/IM clear in terms of the state space framework.

4. The equivalence between RHPCS and GPC/IM

In this section we will show the equivalence between the steady state RHPCS and GPC/IM. Before showing the equivalence between the steady state RHPCS and GPC/IM, we will review the GPC/IM algorithm. In the development of the GPC/IM algorithm, a set of optimal output predictors over the cost horizon N is used. The optimal i-step ahead prediction, $\hat{y}^*(t+i)$, which is equal to $\hat{y}(t+i/t)$, satisfies:

$$c(q^{-1})\hat{y}^*(t+i) = P_i(q^{-1})y(t) + F_i(q^{-1})b(q^{-1})\Delta u(t+i). \quad (4.1)$$

$P_i(q^{-1})$ and $F_i(q^{-1})$ are the unique polynomials satisfying:

$$c(q^{-1}) = F_i(q^{-1})a(q^{-1}) + q^{-i}P_i(q^{-1})$$
$$F_i(q^{-1}) = 1 + f_1 q^{-1} + \cdots + f_{i-1}q^{-i+1} \quad (4.2)$$
$$P_i(q^{-1}) = p_0^i + p_1^i q^{-1} + \cdots + p_{n-1}^i q^{-n+1}.$$

Let

$$S_i(q^{-1}) \triangleq F_i(q^{-1})b(q^{-1}).$$

Then $S_i(q^{-1})$ can be divided into two polynomials as follows[7]:

$$S_i(q^{-1}) = S_i'(q^{-1})c(q^{-1}) + q^{-i}\Gamma_i(q^{-1}) \quad (4.3)$$

where $S_i'(q^{-1}) = F_i'(q^{-1})b(q^{-1})$. $F_i'(q^{-1})$ satisfies the following identity:

$$1 = F_i'(q^{-1})a(q^{-1}) + q^{-i}P_i'(q^{-1})$$

where $F_i'(q^{-1})$ and $P_i'(q^{-1})$ are polynomials of the same order with $F_i(q^{-1})$ and $P_i(q^{-1})$ respectively. Combining Equation (4.1) with (4.2), we get:

$$\hat{y}^*(t+i) = S_i'(q^{-1})\Delta u(t+i) + \Gamma_i(q^{-1})\frac{\Delta u(t)}{c(q^{-1})} + P_i(q^{-1})\frac{y(t)}{c(q^{-1})} \quad (4.4)$$

where s_j' is the j'th coefficient of the the polynomial $s_i'(q^{-1})$ associated with q^{-j}.
q^{-j}. From Equation (4.3), we can see that $c(q^{-1})[\sum_{j=1}^{n-1} s_{j+i}'\cdot q^{-j}] + \Gamma_i(q^{-1})$ is a polynomial of order n-1.

If we use the following notation,:

$$\hat{y}_p^*(t+i) \triangleq \frac{c(q^{-1})[\sum_{j=1}^{n-1} s_{j+i}'\cdot q^{-j}] + \Gamma_i(q^{-1})}{c(q^{-1})}\Delta u(t) + \frac{P_i(q^{-1})}{c(q^{-1})}y(t) \quad (4.5)$$

then $\hat{y}^*(t+i)$ can be written as:

$$\hat{y}^*(t+i) = [\sum_{j=1}^{i} s_i'\cdot q^{-j}]\Delta u(t+i) + \hat{y}_p^*(t+i) \quad (4.6)$$

So, the output predictors for the horizon [t+1, t+N] can be written in a vector notation as:

$$\hat{Y}^*(t) = Y_P^*(t) + S'U(t)$$

where, the vectors in the above equation are defined as:

$$\hat{Y}^*(t) \triangleq [\hat{y}^*(t+1)\ \ \hat{y}^*(t+2)\ \cdots\ \hat{y}^*(t+N)]^T$$

$$U(t) \triangleq [\Delta u(t)\ \ \Delta u(t+1)\ \cdots\ \Delta u(t+N-1)]^T$$

$$Y_P^*(t) \triangleq [\hat{y}_p^*(t+1)\ \ \hat{y}_p^*(t+2)\ \cdots \hat{y}_p^*(t+N)]^T$$

and the matrix S' are defined as follows:

$$S' \triangleq \begin{bmatrix} s_1' & 0 & \cdot & \cdot & \cdot & 0 \\ s_2' & s_1' & 0 & \cdot & \cdot & 0 \\ \cdot & & \cdot & & & \cdot \\ \cdot & & & \cdot & & \cdot \\ s_N' & s_{N-1}' & \cdot & \cdot & \cdot & s_1' \end{bmatrix}.$$

The optimal control law for I/O models, denoted by Δu_I, is given by[6]:

$$\Delta u_I(t) = Z_I(Y_r(t) - Y_P^*(t)) \quad (4.7)$$

where

$$Y_r(t) = [y_r(t+1)\ \ y_r(t+2)\ \cdots\ y_r(t+N)]$$
$$Z_I = [1\ 0\ \cdots\ 0](S'^T S' + \lambda I)^{-1} S^T.$$

Take note of the fact that the gain matrix Z_I is determined by matrix S', and it does not depend on the polynomial $c(q^{-1})$. The matrix $Y_r(t)$ is composed of the reference sequences on horizon [t+1, t+N].

The control $\Delta u_I(t)$ in (4.7) will be compared with the control $\Delta u_s(t)$ in (3.23) where $\hat{x}'(t)$ is obtained from the state estimator (3.21) and Equation (3.22). Since $\Delta u_S(t)$ in Equation (3.23) can be rewritten as Equation (3.20), the equivalence between steady state RHPCS(i.e. (3.20)) and GPC/IM(i.e. (4.6)) will be proven from the equivalence between Equation (3.23) and (4.6).

Take note of the fact that both of $\Delta u_I(t)$ and $\Delta u_S(t)$ are obtained from the sequences of $\Delta u(\bullet)$, $y(\bullet)$ and $y_r(\bullet)$. So, the sequence $\Delta u(\bullet)$, $y(\bullet)$ and $y_r(\bullet)$ can be regarded as inputs to the two controllers to generate $\Delta u_I(\bullet)$ and $\Delta u_S(\bullet)$. We will show the equivalence between GPC/IM and the steady state RHPCS by proving that they have the same transfer functions. We need the following lemma.

Lemma 4.1. When the matrices A, B, D, H are given by Equation (2.2), the following equalities are satisfied:

$$\sum_{j=1}^{i-1} s_j'\cdot z^{-j} = \sum_{j=1}^{i-1} HA^{j-1}B\cdot z^{-j} \quad (4.8)$$

$$\sum_{j=0}^{n-1}\{s_{i+j}'\cdot z^{-j}\} + \frac{\Gamma_i(z^{-1})}{c(z^{-1})} = HA^{i-1}(I - z^{-1}(A-DH))^{-1}B \quad (4.9)$$

$$\frac{P_i(q^{-1})}{c(q^{-1})} = HA^{i-1}(I - q^{-1}(A-DH))^{-1}D \quad (4.10)$$

where the equalities mean that the left and right hand terms of each equations are the same rational functions of z^{-1}.

Proof: Omitted

Utilizing Lemma 4.1, we can show that GPC/IM has the same transfer function with the steady state RHPCS as the following theorem.

Theorem 4.1 Consider the systems (2.1) and (2.3). When the matrices of system (2.1) are given by Equation (2.2), GPC/IM, Equation (4.7), has the same transfer function with the steady state RHPCS, Equation (3.20).

Proof : Omitted

5. Stability properties of GPC

In section 4, we have shown that steady state RHPCS has the same transfer function as GPC/IM when it is are designed for the same plants. Thus, the stability properties of GPC/IM are equal to those of steady state RHPCS.

Let us consider the stability properties of the steady state RHPCS. The reference sequence $y_r(t+i)$ is assumed to be zero as far as stability is concerned. Substitution of the control law (3.20) into Equation (2.1) and utilizing the relation (3.12) yields:

$$x(t+1) = Ax(t) - BZ_SV\hat{x}'(t) + D\xi(t). \quad (5.1)$$

Substitution of

$$\hat{x}'(t) = x(t) + A^{-1}D\xi(t) - (I - A^{-1}DH)e(t)$$

into (5.1) yields:

$$x(t+1) = (A - BZ_SV)x(t) - (I - A^{-1}DH)e(t) + (D + A^{-1}D)\xi(t). \quad (5.2)$$

Combining (5.2) with (3.2), we obtain

$$\begin{bmatrix} x(t+1) \\ e(t+1) \end{bmatrix} = \begin{bmatrix} A-BZ_SV & BZ_SV(A^{-1}DH-I) \\ 0 & A-DH \end{bmatrix}\begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} D+BZ_SVA^{-1}D \\ 0 \end{bmatrix}\xi(t).$$

Consequently, the closed loop characteristic values comprise the characteristic values of matrix $A-BZ_SV$(the poles of RHTC)[10] and the characteristic values of matrix A-DH(the observer poles). These results are summarized in the following theorem.

Theorem 5.1. When system (2.1) is equivalent to the CARIMA model (2.3), the stability of the steady state RHPCS for model (2.1) is the same as that of the RHTC for model (2.1) without disturbances, provided that $c(q^{-1})$ is exponentially stable.

Proof: The characteristic value of matrix A-DH is $c(q^{-1})$. Since $c(q^{-1})$ is exponentially stable, the stability of the whole system depends on the stability of matrix $A-BZ_SV$. Q.E.D

In general cases, it is assumed that $c(q^{-1})$ is exponentially stable. Thus, we can say that steady state RHPCS has the same stability conditions with those of RHTC. The stability properties of RHTC are described in detail in the work of Kwon and Byun[8], and a part of them is summarized here:

:If the pairs {A, B} and {A, H} of system (2.1) are completely controllable and completely observable respectively, then there exists a finite cost horizon N^* such that for all $N \geq N^*$, the matrix $A-BZ_SV$ is exponentially stable.

The pairs {A, B} and {A, H} of system (2.1) are completely controllable and observable respectively if and only if the polynomials $a(q^{-1})$ and $b(q^{-1})$ have no common modes. The stability properties of GPC/IM is obtained from the stability properties of the steady state RHPCS by the following theorem.

Theorem 5.2. If the polynomial $a(q^{-1})$ and $b(q^{-1})$ have no common modes and $c(q^{-1})$ is an exponentially stable polynomial, then there exists a finite number N^* such that for $N \geq N^*$, the closed loop system with GPC/IM is exponentially stable.

When we design GPC/IM, the dimension of matrices to deal with increases as the cost horizon N becomes larger. However, for RHPCS, the dimension of matrices to deal with does not depend on the cost horizon N. This implies that GPC/IM with a large cost horizon can be replaced by the steady state RHPCS.

6. Conclusions

In this paper, we developed RHPCS for stochastic state space models and showed that GPC is equivalent to the steady state RHPCS which consists of Equation (3.20)-(3.22) when they are applied to the same plants. The RHPCS consists of a state feedback controller and a state observer. The observed state feedback controller is the same as RHTC which minimizes the same cost function as that of GPC. The state observer is a Kalman filter of a special form which consists of Equation (3.1) and (3.7).

The stability of GPC is completely determined by that of RHTC and the state observer. The observer is stable under the assumption that $c(q^{-1})$ is exponentially stable. The stability of $c(q^{-1})$ is a common assumption in the design of GPC. Thus, we can say that GPC has the same stability conditions with RHTC.

We believe that RHPCS will provide a useful tool to design or modify GPC since RHPCS contains GPC as a special case and RHPCS is a state space model based controller. GPC with an infinite cost horizon can be obtained from the steady state RHPCS using the steady state solution of Riccati equation (3.13).

Utilizing the equivalence GPC and the steady state RHPCS, a new type of adaptive GPC can also be derived by applying the well known self-tuning techniques for LQ controller[14] to RHPCS since RHPCS takes a similar form with the controller in [14]. Since there are many studies on the robustness of state space model based control methods, the robustness of GPC can be examined through the study of the robustness of RHPCS.

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