'92 韓國自動制御學術會議論文集 1992. 10. 19~21

최적유도법칙의 Closed-Form 해와 근사식

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The Closed-Form Solution and Its Approximation of the Optimal Guidance Law

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Abstract

In this paper, the optimal homing guidance problem is investigated for the general missile/target models described in the state-space. The closed-form solution of the optimal guidance law is derived, and its asymptotic properties are studied as the time-to-go goes to infinity or zero. Futhermore, several approximate solutions of the optimal guidance law are suggested for real-time applications.

Keywords : Optimal Guidance Law
Proportional Navigation
Missile Homing Guidance

Introduction

Homing guidance is based on the seeker measuremetns such as target range and LOS angle or rate. The purpose of homing guidance is to minimize miss distance. The most common guidance law for terminal homing is proportional navigation which has been adopted for most anti-aircraft missiles since 1950's. Some inital studies on proportional navigation can be found in Refs. [Adle] and [Murt].

On the other hand, optimal control theory evolved in 1960's has been widely applied to homing guidance problem. It was found that if the missile is treated as a particle mass a proportional navigation with guidance gain (or navigation constant) of 3 is the optimal guidance law [Brys]. Other applications of optimal control theory on the homing guidance problem can be found in [Garb] , [Cott], [Ashe], [Stoc], [Dick], [Kim] and [York], where various missile/target models and terminal constraints are investigated.

In this paper, the optimal guidance law is derived for the general missile/target models expressed in the state space. The closed-form solution of the optimal guidance law is found, and its asymptotic properties are analyzed for the cases where the time-to-go goes to infinity or zero. Futhermore, several approximate solutions of the optimal guidance law are suggested for real-time applications.

Problem Formulation

Suppose that missile dymamics are represneted by

$$\begin{aligned} \dot{x}_m &= F_m x_m + G_m u \\ a_m &= H_m x_m \end{aligned} \tag{1}$$

and target dynamics are given as

$$\begin{aligned} \dot{x}_t &= F_t x_t + w_t \\ a_t &= H_t x_t \end{aligned} \tag{2}$$

where the target manuever input, u_t, is a white noise as assumed in a Singer model [Sing]. This assumption is justified since u_t is neither known nor predictable in real homing engagements.

The overall system dynamics for homing guidance is then represneted as

$$\dot{x} = Fx + Gu \tag{3}$$

where $u = u_m$ and the state variable x is defined as

$$x = \begin{bmatrix} x_k^T & x_t^T & x_m^T \end{bmatrix}^T . \tag{4}$$

The matrices F, G, K are given as

$$F = \begin{bmatrix} F_k & F_{c_t} & F_{c_m} \\ 0 & F_t & 0 \\ 0 & 0 & F_m \end{bmatrix}$$

$$F_k = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ F_{c_t} = \begin{bmatrix} 0 \cdots 0 \\ H_t \end{bmatrix},\ F_{c_m} = \begin{bmatrix} 0 \cdots 0 \\ -H_m \end{bmatrix} \tag{5}$$

$$G = \begin{bmatrix} 0 \\ 0 \\ G_m \end{bmatrix}$$

General homing guidance problems can be reformualted as an optimal control problem for which the cost function is defined as

$$J = \frac{1}{2}\int_{t_o}^{t_f} (x^T A x + u^T B u) dt \tag{6}$$

and the terminal constraint is specified as

$$R_f^T x_f = 0$$
$$R_f = [\,1 \quad 0 \quad 0 \quad \cdots \quad 0\,]^T \tag{7}$$

The optimal control u(t) is given as

$$u = -B^{-1}G^T(S - RQ^{-1}R^T)x \tag{8}$$

where S(t), R(t), Q(t) is computed from the following matrix differential equations [Brys].

$$\dot{S} = -SF - F^T S - A + SGB^{-1}G^T S, \; S(t_f) = 0 \tag{9}$$
$$\dot{R} = -(F^T - SGB^{-1}G^T)R, \quad R(t_f) = R_f \tag{10}$$
$$\dot{Q} = R^T GB^{-1}G^T R, \quad Q(t_f) = 0\,. \tag{11}$$

For homing guidance problems the magnitudes of the state variables are not important except for the terminal time so that we can set

$$A = 0, \tag{12}$$

and the cost function becomes

$$J = \frac{1}{2}\int u^T B u\, dt \tag{13}$$

Then, from Eq. (9) we see that S(t) = 0 and optimal solution is simplified as

$$u = B^{-1}G^T R Q^{-1} R^T x \tag{14}$$

The differential equations of R(t) and Q(t) are also simplified as

$$\dot{R} = -F^T R, \qquad R(t_f) = R_f \tag{15}$$
$$\dot{Q} = R^T GB^{-1}G^T R, \qquad Q(t_f) = 0 \tag{16}$$

We can show that Eq.(14) is equal to Eq.(17).

$$u = B^{-1}G^T R Q^{-1} \cdot (ZEM) \tag{17}$$

where ZEM denotes the zero-effort miss, which is defined as the miss obtained by setting the guidance command to zero from the current time to the terminal time[Brys].

The differentail equation of R(t) is decomposed as

$$\dot{R}_k = -F_K^T R_k, \qquad R_k(t_f) = [\,1 \quad 0\,]^T \tag{18}$$
$$\dot{R}_t = -F_{c_t}^T R_k - F_t^T R_t, \quad R_t(t_f) = [\,0 \cdots 0\,]^T \tag{19}$$
$$\dot{R}_m = -F_{c_m}^T R_k - F_m^T R_m, \quad R_m(t_f) = [\,0 \cdots 0\,]^T \tag{20}$$

Define a scalar p as

$$p = G^T R = G_m^T R_m \tag{21}$$

and denote Q(t) as q(t). Then the equation for q(t) is rewritten as

$$\dot{q} = R^T G G^T R = R_m^T G_m G_m^T R_m, \; q(t_f) = 0 \tag{22}$$

or

$$\dot{q} = p^2, \qquad q(t_f) = 0 \tag{23}$$

$$u = \frac{p}{q}(R_k^T x_k + R_t^T x_t + R_m^T x_m) \tag{24}$$

Define guidance gain Λ as

$$\frac{\Lambda}{t_{go}^2} = \frac{p}{q} \tag{25}$$

Then optimal guidance command becomes

$$u = \frac{p}{q} \cdot (ZEM) = \frac{\Lambda}{t_{go}^2} \cdot (ZEM) \tag{26}$$

where ZEM is given as

$$ZEM = R_k^T x_k + R_m^T x_m \tag{27}$$

Denote time-to-go as $t_{go} = t_f - t$, then ZEM is rewritten as

$$ZEM = z + v\,t_{go} + R_t^T x_t + R_m^T x_m \tag{28}$$

and optimal guidance command is given as

$$u = \frac{\Lambda}{t_{go}^2}(z + v\,t_{go} + R_t^T x_t + R_m^T x_m)\,. \tag{29}$$

The Closed-Form Solution of the Optimal Guidance Law

The closed-form solution of optimal guidance law is obtained by integrating Eqs. (18), (19), (20), and (23). Since Eq. (18) gives

$$R_k = \left[1 \quad (t_f - t)\right]^T \tag{30}$$

Eqs. (19) and (20) can be rewritten as

$$\dot{R}_m = -F_{c_m}^T R_k - F_m^T R_m = H_m^T(t_f - t) - F_m^T R_m, \quad R_m(t_f) = 0$$
$$\dot{R}_t = F_{c_t}^T R_k - F_m^T R_t = -H_m^T(t_f - t) - F_m^T R_m, \quad R_t(t_f) = 0 \tag{31}$$

which are integrated to result in

$$R_m(t_{go}) = \left[I - e^{F_m^T t_{go}}\right]\left(F_m^{-2}\right)^T H_m^T + t_{go}\left(F_m^{-1}\right)^T H_m^T \tag{32}$$
$$R_t(t_{go}) = -\left[I - e^{F_m^T t_{go}}\right]\left(F_m^{-2}\right)^T H_m^T - t_{go}\left(F_m^{-1}\right)^T H_m^T \tag{33}$$

Substituing Eq. (32) into Eq. (21) results in

$$p = G_m^T\left[I - e^{F_m^T t_{go}}\right]\left(F^{-2}\right)^T H_m^T + t_{go} G_m^T \left(F_m^{-1}\right)^T H_m^T \tag{34}$$

Then

$$\begin{aligned}\dot{q} &= p^T p \\ &= \left\{G_m^T\left[I - e^{F_m^T t_{go}}\right]\left(F_m^{-2}\right)^T H_m^T + t_{go} G_m^T\left(F_m^{-1}\right)^T H_m^T\right\}^T \\ &\quad \times\left\{G_m^T\left[I - e^{F_m^T t_{go}}\right]\left(F_m^{-2}\right)^T H_m^T + t_{go} G_m^T\left(F_m^{-1}\right)^T H_m^T\right\}\end{aligned} \tag{35}$$

Eq. (35) may be rewritten as

$$\begin{aligned}\dot{q} &= k_2^2 + 2k_1k_2\,t_{go} + k_1^2\,t_{go}^2 + \\ &\quad - H_m F_m^{-2}\left(G_m G_m^T e^{F_m^T t_{go}} + e^{F_m t_{go}} G_m G_m^T\right)\left(F_m^{-2}\right)^T H_m^T \\ &\quad - k_1\left(H_m F_m^{-2} e^{F_m t_{go}} G_m + G_m^T e^{F_m^T t_{go}}\left(F_m^{-2}\right)^T H_m^T\right) t_{go} \\ &\quad + H_m F_m^{-2} e^{F_m t_{go}} G_m G_m^T e^{F_m^T t_{go}}\left(F_m^{-2}\right)^T H_m^T\end{aligned} \tag{36}$$

where $k_1 = H_m F_m^{-1} G_m$ and $k_2 = H_m F_m^{-2} G_m$ is used for simplicity: Eq. (36) is integrated to result in $q(t_{go})$ as

$$\begin{aligned}q(t_{go}) &= -k_2^2 t_{go} - k_1 k_2 t_{go}^2 - \frac{1}{3}k_1^2 t_{go}^3 - 2k_2 H_m F_m^{-3}\left(I - e^{F_m t_{go}}\right)G_m \\ &\quad + 2k_1\left\{H_m F_m^{-3} e^{F_m t_{go}} t_{go} + H_m F_m^{-4}\left(I - e^{F_m t_{go}}\right)\right\}G_m \\ &\quad + \left(H_m F_m^{-2}\right)X\left(H_m F_m^{-2}\right)^T\end{aligned} \tag{37}$$

where $k_1 = H_m F_m^{-1} G_m$ and $k_2 = H_m F_m^{-2} G_m$ and the matrix X satisfies

$$F_m X + X F_m^T = G_m G_m^T - e^{F_m t_{go}} G_m G_m^T e^{F_m t_{go}} \tag{38}$$

The optimal guidance gain Λ is directly computed form the solutions of p and q given by Eqs. (34) and (38). Also, the optimal guidance command u is computed by substituting Eqs. (32) and (33) into Eq. (29). The closed-form solution of the optimal guidance gain does not require numerical integration or look-up table searches. However, it requires the evaluation of exponential functions and a Lyapunov equation solver. For this reason, it is not adequate for real-tim applications.

Asymptotic Properties of the Optimal Guidance Law

It is not easy to analyze the relationship between the structure of the optimal guidance law and missile model parameters. In this paper, we analyze the asymptotic properties of the optimal guidance gain as t_{go} goes to zero or infinity. As shown in Eq.(21), (29), the guidance gain is independent of target model. For this reason, we neglect the target model from now on.

(a) In case that $t_{go} \to \infty$

From Eq.(21) and (32) we see that

$$p_\infty = G_m^T R_{m_\infty} \approx (H_m F_m^{-1} G_m)^T t_{go} \tag{39}$$

where the subscript ∞ denotes the limit of the variable as $t_{go} \to \infty$.

Note that the steady-state gain, or static gain, denoted as k_s, of the missile dynamics is given as

$$k_s = H_m (sI - F_m)^{-1} G_m \big|_{s=0} = -H_m F_m^{-1} G_m \ . \tag{40}$$

Then, we see that

$$p_\infty \approx -k_s t_{go} \tag{41}$$

$$q_\infty \approx -\frac{1}{3} k_s^2 t_{go}^3 \ . \tag{42}$$

Therefore, as $t_{go} \to \infty$, the optimal guidance command is expressed as

$$\begin{aligned} u_\infty &= \frac{p_\infty}{q_\infty}(z + v t_{go} + R_{m_\infty}^T x_m) \\ &\approx \frac{3}{k_s t_{go}^2}(z + v t_{go} + (H_m F_m^{-1})^T x_m t_{go}) \end{aligned} \tag{43}$$

and then the guidance gain is given as

$$\Lambda_\infty = \frac{3}{k_s} \tag{44}$$

If the input and output to the missile dynamics are acceleration command and achieved acceleration, respectively, then the steady state gain of the missile dynamics k_s becomes unity. Hence, regardless of missile dynamics, the guidance gain becomes 3 as $t_{go} \to \infty$.

(b) In case that $t_{go} \to 0$

Assume R_m can be expressed as a series of t_{go} when the time-to-go approaches to 0.

$$e^{F_m^T t_{go}} = I + F_m^T t_{go} + \frac{1}{2}(F_m^T t_{go})^2 + \frac{1}{6}(F_m^T t_{go})^3 + \cdots\cdots \tag{45}$$

From Eq.(21) we obtain

$$\begin{aligned} p_o &= G_m^T R_{m_o} \\ &= -\frac{1}{2}(H_m G_m)^T t_{go}^2 - \frac{1}{6}(H_m F_m G_m)^T t_{go}^3 + \cdots\cdots \\ &= -\frac{1}{2}\mu_1 t_{go}^2 - \frac{1}{6}\mu_2 t_{go}^3 + \cdots\cdots \\ &= -\sum_{i=1}^{\infty} \frac{1}{(i+1)!}\ \mu_i\, t_{go}^{(i+1)} \end{aligned} \tag{46}$$

where μ_i is defined as $\mu_i = H_m F_m^{i-1} G_m$, which is identified as the i-th Markov parameter of the missile dynamic model represented by (F_m, G_m, H_m) [Kail].

Suppose that μ_k is the first nonzero Markov parameter. Then, we can show that as the time-to-go goes to zero,

$$q_o \approx \frac{1}{[2(k+1)+1]\,(k+1)!\,(k+1)!}\ \mu_k^2\, t_{go}^{[2(k+1)+1]} \tag{47}$$

and

$$\Lambda_o = \frac{p_o}{q_o} t_{go}^2 \approx \frac{[2(k+1)+1]\,(k+1)!}{\mu_k\, t_{go}^k} \tag{48}$$

We previously observed that as the time-to-go goes to infinity the optimal guidance gain becomes independent of the missile dynamics. But as the time-to-go decreases, the optimal guidance gain becomes dependent upon the missile dynamics as seen in Eq. (48).

Suppose that the transfer function of the missile dynamics is expressed as

$$T_m(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n} \tag{49}$$

Then, we can show that the coefficients of the numerator of $T_m(s)$ are related to the Markov parameters as follows [Kail];

$$b_1 = \mu_1, \quad b_2 = \mu_2 + a_1\mu_1, \quad b_3 = \mu_3 + a_1\mu_2 + a_2\mu_1, \quad \cdots\cdots \tag{50}$$

We observe from Eq. (48) that if the pole excess number of $T_m(s)$ is k, then μ_k is the first nonzero Markov parameters. Thus, the larger the pole excess number is, the faster the guidance gain increases, as shown in Eq. (48). Also, the sign of the optimal guidance gain is determined by the sign of the first nonzero Markov parameter μ_k .

Tail-controlled missiles and the canard-controlled missiles are most common for the air-to-air missile application. For tail-controlled systems, the transfer function is nonminimum phase so that the first Markov parameter is negative. Hence, the sign of optimal guidance gain changes to be negative as the time-to-go decreases, as shown in Fig.1. For canard-controlled systems, the optimal control gain is always positive due to the minimum phase dynamics, as shown in Fig.2. In Fig.1 and Fig.2, $k_s\Lambda$ instead of Λ is shown to exclude the effects of the static gain of missile dynamics.

Approximations of the Optimal Guidance Law

It is not desirable to store the optimal solution into guidance computer since $\Lambda(t_{go})$ and $R_m(t_{go})$ are dependent on t_{go}. It is rather convenient to derive simple approximate equations for $\Lambda(t_{go})$ and $R_m(t_{go})$, as described in this section.

(a) Approximate guidance gain for large t_{go}'s.

Suppose that missile dynamics is stable and the eigenvalue of state matrix F_m are represented as follows;

$$\lambda_k = -\sigma_k \quad \text{(real root)} \tag{51}$$

$$\lambda_k = -\sigma_k \pm jw_k \quad \text{(complex roots)}$$

Let σ_{min} be the minimum σ_k. For t_{go} much larger than $\frac{1}{\sigma_{min}}$

$$\left\| e^{F_m T} \right\| << 1, \qquad t_{go} >> \frac{1}{\sigma_{min}} \tag{52}$$

and we see that

$$R_m(t_{go}) \approx \left(F_m^{-2}\right)^T H_m^T + t_{go}\left(F_m^{-1}\right)^T H_m^T, \qquad t_{go} \ge t_s \tag{53}$$

where t_s may be chosen to be $\frac{3}{\sigma_{min}}$ or $\frac{4}{\sigma_{min}}$.

Eq.(52) is substituted into Eq.(34) to result in

$$p \approx k_1 t_{go} + k_2 \tag{54}$$

where

$$k_1 = H_m F_m^{-1} G_m, \qquad k_2 = H_m F_m^{-2} G_m \tag{55}$$

Also q is obtained by substituting Eq.(52) to Eq.(37)

$$q \approx -\frac{1}{3} k_1^2 t_{go}^3 - k_1 k_2 t_{go}^2 - k_2^2 t_{go} \tag{56}$$

Then, when $t_{go} > t_s$, the approximate optimal guidance law becomes

$$u = \frac{\tilde{\Lambda}}{t_{go}^2}\left[z + vt_{go} + \left(H_m F_m^{-2} + t_{go} H_m F_m^{-1}\right)x_m\right] \tag{57}$$

where the approximate guidance gain is given as

$$\tilde{\Lambda} = -\frac{\left(k_1 t_{go}^2 + k_2 t_{go}\right)}{\left(\frac{1}{3} k_1^2 t_{go}^2 + k_1 k_2 t_{go} + k_0^2\right)} \tag{58}$$

As $t_{go} \to \infty$, Eq. (58) becomes

$$\tilde{\Lambda}_\infty = -\frac{3}{k_1} \, . \tag{59}$$

Also, from Eq.(40) and (55), $k_1 = -k_s$. Hence, Eq. (59) is identical to the result of Eq. (44).

(b) Correction of the approximate equation

Eq.(54) is a good approximate equation for p when $t_{go} \ge t_s$, but Eq.(56) does not approximate q precisely even for $t_{go} < t_s$. hence, we correct p and q as follows:

Let the exact p and q be p_s, q_s at $t_{go} = t_s$. To match the approximation of p with the exact value at $t_{go} = t_s$, Eq.(54) is modified as

$$p = p_s + k_1(t_{go} - t_s) \tag{60}$$

Then, q is computed by using Eq. (23) with a new boundary condition $q(t_{go} = t_s) = q_s$.

$$q = q_s - \int_{t_s}^{t_{go}} \left[p_1 + k_1(\tau - t_s)\right]^2 d\tau \tag{61}$$

$$q = q_s - \frac{1}{3} k_1^2 \left(t_{go} - t_s\right)^3 - k_1 p_s \left(t_{go} - t_s\right)^2 - p_s^2 \left(t_{go} - t_s\right) \tag{62}$$

Note that if Eq.(60) is sufficiently close to the exact solution of p for $t_{go} \ge t_s$, then Eq.(62) is also an appropriate approximate equation of q.

(c) Extension of the approximate equation for small t_{go}'s

Since the guidance gain diverges as $t_{go} \to 0$, it is customary in practice to fix the guidance gain for the time-to-go smaller than t_h, which is to be determined experimentally;

$$\Lambda(t_{go}) = \Lambda(t_h), \qquad t_{go} \le t_h \, . \tag{63}$$

If t_h is chosen to be larger than t_s, the guidance gain $\Lambda(t_{go})$ given by Eq.(60) and Eq.(62) is acceptable for $t_{go} \in [t_h, \infty]$. However, if $t_h < t_s$, the approximate equation may not be accurate due to the effects of the exponential function of Eqs.(34) and (37).

To solve this problem, assume that

$$p = p_h + k_h\left(t_{go} - t_h\right) \tag{64}$$

$$q = q_h - \frac{1}{3} k_h^2 \left(t_{go} - t_h\right)^3 - k_h p_h \left(t_{go} - t_h\right)^2 - p_h^2 \left(t_{go} - t_h\right) \tag{65}$$

where p_h and q_h are exact at $t_{go} = t_h$. The constant k_h is chosen to make the guidance gain exact at $t_{go} = t_s$;

$$\frac{p_s}{q_s} = \frac{p_h + k_h(t_s - t_h)}{q_h - \frac{1}{3} k_h^2 (t_s - t_h)^3 - k_h p_h (t_s - t_h)^2 - p_h^2 (t_s - t_h)} \tag{66}$$

Let

$$\alpha = \frac{p_s}{q_s}, \qquad \tau = t_s - t_h \tag{67}$$

then we need to solve the following equation for k_h;

$$\frac{1}{3} \alpha \tau^3 k_h^2 + \left(\tau + \alpha p_h \tau^2\right) k_h + \left(p_h + \alpha p_h^2 \tau - \alpha q_h\right) = 0 \tag{68}$$

Since two k_h's are obtained from Eq.(68),we select k_h which is close to k_1 of Eq.(55). The approximate equation of the guidance gain Λ for $[t_h, t_s)$ is then computed from Eqs. (64) and (65). Note that Λ obtained from Eqs. (64) and (65) is exact at $t_{go} = t_h$ and $t_{go} = t_s$.

The approximate optimal guidance gain is compared with the exact one in Fig.3 thru Fig.6. In Fig.3, a first order missile model ($T_m(s) = 10/(s+10)$) is considered while a second order missile model ($T_m(s) = 100/(s^2 + 14.14s + 100)$). Typical tail-controlled and canard-controlled model are also used in Fig.5 and Fig.6. "Approx.A" denotes the approximation obtained by using Eqs.(60) and (62) while "Approx.B" denotes the approximation corrected for small t_{go}'s by solving Eq. (68). For all figures, t_s=0.3 sec and t_h=0.1 sec are used.

Conclusion

In this paper,the application of optimal control theory is investigated for the design of homing guidance law. Terminal controller theory is applied to homing guidance, and the effects of missile/target dynamics on the guidance law is studied.

First, the closed form solution of the optimal guidance law is derived for arbitrary missile and target models. The asymptotic properties of the guidance gain are then analyzed. It is explicitly shown that the guidance gain approaches to 3 regardless of missile dynamics as the time-to-go goes to infinity, whereas the guidance gain depends on the Markov parameters of the missile dynamic model as the time-to-go goes to zero.

The implementation of the exact solution of the optimal guidance law is difficult due to its complexity. To overcome this problem the approximate equation of the optimal guidance gain is derived, and then modified to fit the exact solution more closely. It is believed that these approximate solutions are valuable for real-time applications.

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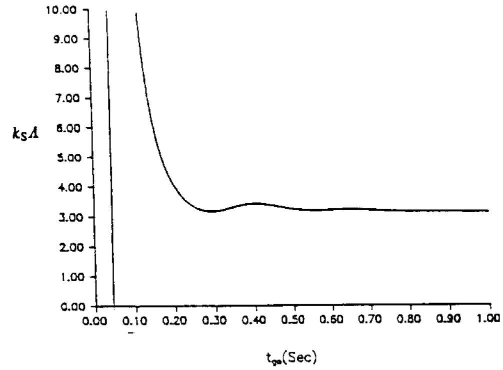

Fig.1 Optimal guidance gain of tail controlled missile

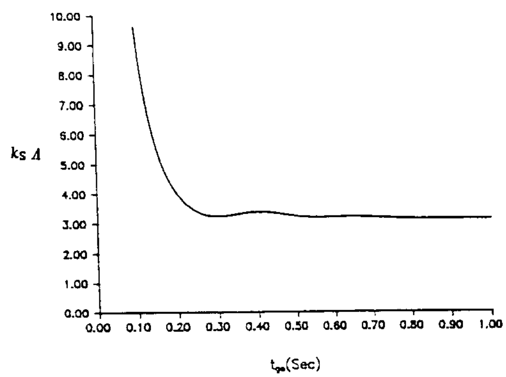

Fig.2 Optimal guidance gain of canard controlled missile

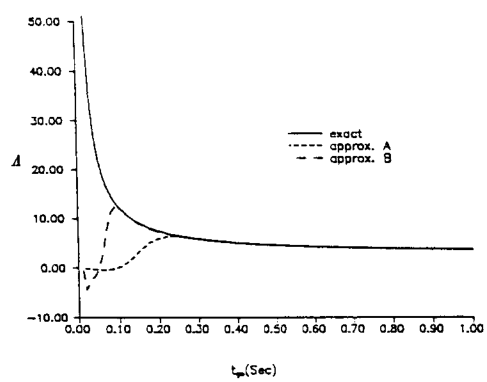

Fig.3 Approximation of optimal guidance gain (first order missile model)

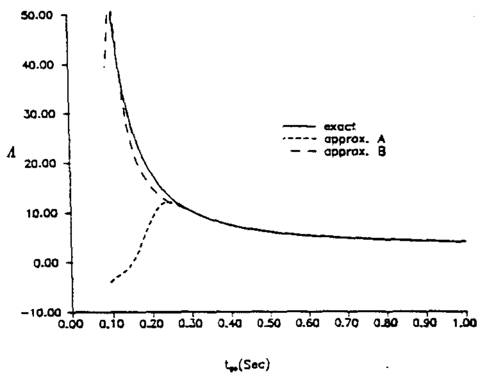

Fig.4 Approximation of optimal guidance gain (second order missile model)

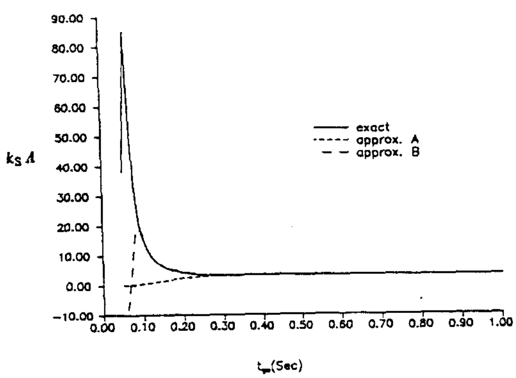

Fig.5 Approximation of optimal guidancd gain (tail controlled missile)

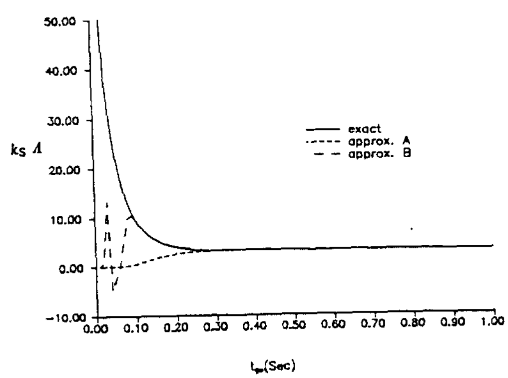

Fig.6 Approximation of optimal guidance gain (canard controlled missile)