유한요소법과 SUMT를 이용한 편측식 선형유도전동기의 설계

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DESIGN OF SINGLE-SIDED LINEAR INDUCTION MOTOR USING FINITE ELEMENT METHOD AND SUMT

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ABSTRACT

This paper describes the optimization of design variables of SLIM using finite element method and SUMT(Sequential Unconstrained Minimization Technique). Thrust is taken as an objective function in order to maximize thrust under constant current drive, and seven independent design variables and nine constraints are chosen.

As a result, τ/g(pole pitch/airgap) and τ/d_{AL}(pole pitch/aluminum depth) of good criteria in SLIM design are determined.

INTRODUCTION

Single-sided Linear Induction Motor(SLIM) is very useful at the places requiring linear motion since it produces thrust directly. So the need for industry application is recently increasing. It is possible to divide linear motors into the following categories from application viewpoint[1]: (1)force machines (2)power machines, and (3)energy machines. Force machines are short-duty machines operating at very low speeds, and efficiency is not a major consideration with regard to overall performance. Power machines are often operated at medium or high speeds and are continuous-duty machines; they must have high efficiencies. Energy machines are short-duty machines and have found applications as an accelerator. At present most of the linear machines find applications at low speeds and at standstill. In this paper, SLIM as a force machine is designed.

Also optimum design of SLIM is necessary because the design is very different according to its application. In this study the design variables of SLIM for conveyance are optimized by using finite element analysis and SUMT(Sequential Unconstrained Minimization Technique). Starting thrust is adopted as an objective function to find conditions that the thrust is maximum under constant current drive. Seven independent design variables - slot width/slot pitch, slot depth, aluminum depth, back iron depth, pole pitch, air gap, yoke depth - and nine constraints are chosen.

FINITE ELEMENT ANALYSIS OF SLIM

For an electromagnetic field where displacement current can be neglected in low frequencies, Maxwell's equations can be written as follows.

$$\mathrm{rot}\, H = J \qquad (1)$$

$$\mathrm{rot}\, E = -\frac{\partial B}{\partial t} \qquad (2)$$

$$\mathrm{div}\, B = 0 \qquad (3)$$

$$B = \mu H \qquad (4)$$

The magnetic vector potential A and scalar potential Φ is respectively related to B and E by

$$B = \mathrm{rot}\, A \qquad (5)$$

$$E = -\frac{\partial A}{\partial t} - \mathrm{grad}\, \Phi \qquad (6)$$

Also because J consists of forcing current and secondary induced current, it can be written as

$$J = J_o + J_e \qquad (7)$$

where J_o : forcing current density[A/m^2],
J_e : induced current density[A/m^2]

If the conductivity of secondary reaction plate is denoted by σ, then

$$J_e = \sigma E = -\sigma \left(\frac{\partial A}{\partial t} + \mathrm{grad}\, \Phi \right) \qquad (8)$$

Eq.(1) ~ eq.(8) can be combined to yield

$$\mathrm{rot}\, \frac{1}{\mu} (\mathrm{rot}\, A) = J_o - \sigma \left(\frac{\partial A}{\partial t} + \mathrm{grad}\, \Phi \right) \qquad (9)$$

If the secondary induced current is symmetric, eq.(9) can be rewritten as

$$\frac{1}{\mu} \left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \right) = -J_z + \sigma \frac{\partial A_z}{\partial t} \qquad (10)$$

where A_z and J_z are z-component of A and J_o , respectively.

And complex number approximation is used to compute the time differentiation term in eq.(10). This method is to assume that all quantities in the magnetic field change sinusoidally with the time. We may account for the relative motion by making use of the concept of slip frequency by putting $\partial/\partial t = js\omega$, where ω is the primary angular frequency and s is a slip.

$$\frac{1}{\mu}\left(\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}\right) = -J_z + js\omega A_z \quad (11)$$

Using variational principle to find energy functional,

$$\chi = \int\left[\frac{1}{2\mu}\left\{\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}\right\} - J_z A_z + \frac{js\omega}{2} A_z^2\right]dS \quad (12)$$

Therefore eddy current term in the right hand side of eq.(12) with regard to an element is given by

$$\frac{\partial \chi^{(e)}}{\partial A_{zi}^{(e)}} = \frac{js\omega\Delta^{(e)}}{12}\sum_{l=1}^{3}(1 + \delta_{il})\, A_{zi}(e) \quad (13)$$

$$(i = 1, 2, 3)$$

where, $\Delta^{(e)}$: Area of an element,

δ: Kronecker's delta function

Thrust and normal force calculation

Force acting on volume V equals surface integral of Maxwell stress tensor P to the surface. Denoting x, y component of Maxwell stress tensor producing in an element by p_x, p_y, respectively

$$p_x = \frac{1}{2\mu_0}\{(B_x^2 - B_y^2)n_x + 2n_y B_x B_y\}$$

$$p_y = \frac{1}{2\mu_0}\{(B_y^2 - B_x^2)n_y + 2n_x B_x B_y\} \quad (14)$$

where n_x, n_y are unit normal direction vectors.

So thrust T_x and normal force T_y acting on integration path ℓ are

$$T_x = \int_\ell w\, p_x\, d\ell \quad (15)$$

$$T_y = \int_\ell w\, p_y\, d\ell \quad (16)$$

w : stack height of primary

If SLIM end effects are to be included, periodicity condition do not exist and whole machine structure must be discretized. But one pole analysis is very effective in optimizing the design variables of SLIM.

Fig. 1 represents the data normalized for the second pole thrust and normal force. This figure explains end effect which is specific phenomenon in SLIM. Thrust at each end is much smaller than that at center. In the normal force distribution, (+) is repulsive, (-) is attractive force. The resultant normal force of this model is near to zero. If the absolute value of normal force is much greater than zero, each value has the same sign. So total thrust and normal force can be approximated by the following equations, respectively.

$$T_x \fallingdotseq np \times F_x$$

$$T_y \fallingdotseq np \times F_y \quad (17)$$

where np is the number of poles, F_x and F_y is thrust and normal force respectively by one pole analysis. The approximation of normal force has somewhat errors as the absolute value approaches zero.

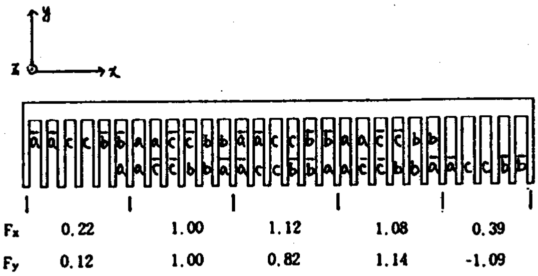

Fig. 1 Thrust and normal force distribution (slip = 1)

FORMULATION OF OPTIMIZATION PROBLEM

In general optimization problem can be stated as follows.

Find X which minimize f(X) subject to

$$g_j(X) \le 0 \quad (j = 1,2,\ldots,m) \quad (18)$$

where $X = (X_1, X_2, \ldots, X_n)$, X are independent design variables, n is the number of design variables and j is the number of constraints.

In this study SUMT(Sequential Unconstrained Minimization Technique) is used to optimize design variables. This method transforms the basic optimization problem into alternative formulation (19) such that numerical solutions are sought by solving a sequence of unconstrained minimization problems[2].

$$\phi_k = \phi(X, r_k) = f(X) + r_k \sum_{j=1}^{m} G_j[g_j(X)] \quad (19)$$

where G_j is some function of the constraint g_j, and r_k is a positive constant as the penalty parameter. The penalty function formulations for inequality constrained problems can be divided into two categories, i.e, the interior and exterior methods. In the interior formulations, the popularly used forms of G_j is given by

$$G_j = -1/g_j(X) \quad (20)$$

Therefore eq.(19) is written by

$$\phi(X, r_k) = f(X) - r_k \sum_{j=1}^{m} 1/g_j(X) \quad (21)$$

In the interior methods, all the unconstrained minima of ϕ_k lie in the feasible region and converge to the solution of eq.(18) as r_k is varied in a particular manner.

Variable metric method[2] is used for unconstrained minimization in penalty function method, and the algorithm is as follows.

step 1 : Start with an initial point X_1 and a $n \times n$ positive definite symmetric matrix H_1. Usually H_1 is taken as an identity matrix I.

step 2 : Compute the gradient of the function, ∇f_i, at the point X_i, and set search direction S_i.

$$S_i = - H_i \nabla f_i$$

step 3: Find the optimal step length λ_i^* in the direction S_i and set

$$X_{i+1} = X_i + \lambda_i^* S_i$$

Here cubic interpolation method is used to find λ_i^*.

step 4 : Test the new point X_{i+1} for optimality. If X_{i+1} is optimal, terminate the iterative process. Otherwise, go to step 5.

step 5 : Update the H matrix.

$$H_{i+1} = H_i + M_i + N_i$$

where

$$M_i = \lambda_i^* \frac{S_i S_i^T}{S_i^T Q_i} \qquad N_i = - \frac{(H_i Q_i)(H_i Q_i)^T}{Q_i^T H_i Q_i}$$

and

$$Q_i = \nabla f(X_{i+1}) - \nabla f(X_i) = \nabla f_{i+1} - \nabla f_i$$

step 6 : Set the new iteration number and go to step 2.

Fig. 2 shows the flow chart of an optimum design program.

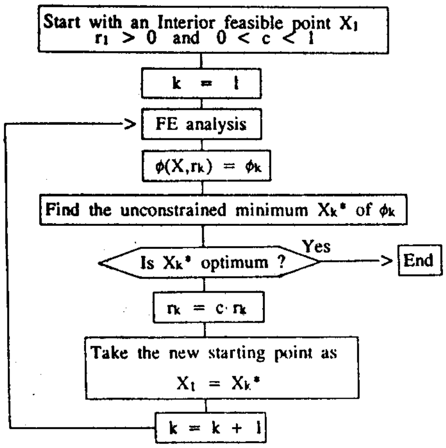

Fig. 2 Flow chart of optimization program

OPTIMIZATION OF THE DESIGN VARIABLES

To obtain maximum starting thrust under constant current drive(I = 12[A]), objective function is chosen as

$$f(X) = \int_{\ell} w\, p_x\, d\ell \ [N] \tag{22}$$

where p_x is x component of Maxwell stress tensor, w is stack height of the primary and ℓ is the path of integration.

Also constraints and independent design variables are shown in Table 1. Seven independent design variables and nine constraints are chosen. The goodness factor[1] introduced by Laithwaite is a convenient measure for assessing the quality of an Linear electric machine and given by

$$G = \frac{2f\mu_0 \sigma \tau^2}{\pi g} \tag{23}$$

Goodness factor G of SLIM must be less than one so that it may have drooping characteristic. As the results of optimization, G is 0.65, maximum flux density in teeth and back iron is 0.9[T], 1.07[T], respectively.

Fig 4. illustrates magnetic flux lines of the optimized model by one pole analysis.

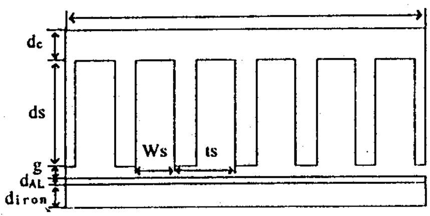

Fig. 3 Independent design variables

Table 1. Constraints and independent design variables

Constraints	Design variables
g_1 : maximum flux density in teeth $B_{tm} \le 1.3$[T]	X_1 : $k_t = w_s/t_s$ (slot width/slot pitch)
g_2 : maximum flux density in back iron $B_{bim} \le 1.1$[T]	X_2 : d_s (slot depth)
g_3 : 2 [mm] $\le d_{AL} \le$ 4 [mm]	X_3 : d_{AL}(aluminum depth)
g_4 : 7 [mm] $\le d_{iron} \le$ 8.5 [mm]	X_4 : d_{iron} (back iron depth)
g_5 : 2 [mm] $\le g \le$ 5 [mm]	X_5 : τ (pole pitch)
g_6 : motor length $\le$ 40 [cm]	X_6 : g (air gap)
g_7 : 0.5 $\le$ slot-filled factor $\le$ 0.6	X_7 : d_c(yoke depth)
g_8 : Goodness factor $\le$ 1.0	
g_9 : |normal force/thrust| $\le$ 0.15	

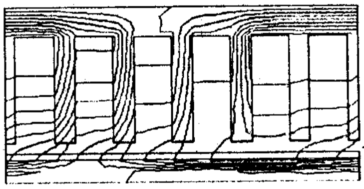

Fig. 4 Flux lines of the optimized model (slip=1, f=60Hz)

Table 2. History of optimization process

w_s/t_s	d_s[m]	d_{AL}[m]	d_{iron}[m]	τ[m]	g[m]	d_c[m]	F_x[N]
0.6500000	3.5200000E-02	3.0000000E-03	7.0000002E-03	8.1600003E-02	4.0000002E-03	9.9999998E-03	74.03014
0.6500037	3.5310972E-02	2.6137233E-03	7.4233967E-03	8.0022767E-02	3.8713049E-03	1.0117318E-02	84.54140
0.6500039	3.5319168E-02	2.2573329E-03	7.4474015E-03	7.9997197E-02	3.8007398E-03	1.0122076E-02	94.42560
0.6500039	3.5319645E-02	2.1984440E-03	7.4971081E-03	7.9989515E-02	3.7955602E-03	1.0122271E-02	96.20768
0.6500039	3.5319638E-02	2.1977059E-03	7.4986834E-03	7.9989381E-02	3.7934901E-03	1.0122232E-02	96.24012
0.6500039	3.5319638E-02	2.1971751E-03	7.4986760E-03	7.9989359E-02	3.7933930E-03	1.0122232E-02	96.25752

Final results of optimization are shown in Table 2. From Table 2 it can be understood that the determination of values of τ/g(pole pitch/airgap) and τ/d_{AL}(pole pitch/aluminum depth) is very important because objective function is very sensitive to air gap(g), aluminum depth(d_{AL}) and pole pitch(τ). Also it can be found that τ/g is 21.088, τ/d_{AL} is 36.403 and $(g+d_{AL})/\tau$ is 13.351.

CONCLUSION

In this paper, low speed SLIM as a force machine has been designed by finite element analysis and SUMT. The values of τ/g, τ/d_{AL} which are very important factors in designing SLIM have been found, and input current has been able to be reduced about 12.5% from this study.

REFERENCES

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[2] S.S.RAO, "Optimization-Theory and Applications", WIELY, 1984