

# 두 로봇의 위치 및 힘 제어의 부하분배에 관한 응용

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## Position and Force Control for Two Robots with Application to Load Distribution

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### 요약문

두말 robot의 dynamic model 에 위치 및 힘의 제어를 적용했을때의 부하분배의 관한 연구가 논문의 주요내용이다. 두말 robot의 완전한 control 을 위해서는 위치 뿐만아니라 힘의 제어를 하여야하며 그럴때의 전체 동력학 식의 타당성 여부를 알기 위해서는 안정도 연구를 하여야 한다. 그리고 위치 및 힘의 제어를 했을때의 Load의 distribution 문제에 대하여 언급했다. 이에 관한 연구도 현재 많이 진행되고 있으며 여기서 소개하는 방법은 Load 및 end-effector의 힘의 분포에 주안점을 두어서 해석을 하였다. 이경우에 Load의 분포에 관한 최적해를 얻기 위해서는 두말에서 소모되는 에너지의 최소치를 찾는 방법과 또 하나는 Load에서 형성되는 힘의 기하학적 구조에 대한 2가지 해석을 하였다. Load의 힘의 분포를 해석하기 위해서는 Load의 동작에 무관하게 적용되는 internal force를 가정하였다.

### Abstract

Stability analysis and load distribution problem of two coordinating robots using full dynamic model is studied in this paper. Dynamic models of two robots are combined with the position force control strategy and the Liapunov 2nd method is used for the proof of the stability. This analysis shows that the position and force control of two coordinating robot is always stable. Also, load distribution problem is mentioned with respect to the end-effector forces minimizing joint torques.

### 1. Introduction

Recently, coordinating robots have received much attention in automated manufacturing. In case of handling long or heavy object and assembling complex parts, coordinating robots are superior to the single robots. But, the coordinating robots are rarely used in factory automation, because they have to use not only position but force control simultaneously [1]. Two coordinating robots can be expanded to multi coordinating robots easily by using master slave concept. So the coordinating robots problem is limited to two coordinating robots problem in this paper.

When two robots grasp a single object, a set of kinematic constraints are imposed on two end-effectors. From this constraints, joint position, velocity and acceleration of two robots are derived. So the coordinating motion can be planned and executed [2,3].

The dynamics of two coordinating robots are combined to form one unified dynamic equation through the control mechanisms. Thus the applied joint torques of two robots can be calculated in accordance with the motion trajectory of the object [2,4].

Motion control of two coordinating robots in handling a single object was studied by a number of works [5-7]. Control mechanisms such as nonlinear [5], adaptive [7] and variable structures [6] etc. have been proposed to control the coordinated motion.

Force control in coordination of two arms was once studied by Ishida [9]. A two robots system was operated in master-slave mode to handle a

single object. The master was position controlled, and the slave was force controlled. The purpose of this force control was to counteract the interactive force caused by the master through the object rather than to maintain a desired interaction force. Koivo proposed to use adaptive control strategy to realize position-velocity-force control of two robots [7]. This force control was to reduce the error between the actual and desired values of the generalized forces, not the interaction forces between the object and end-effectors.

As for the load distribution, [8] addressed the optimal load distribution with respect to the joint torques. But the internal force factors of the end-effectors are not considered. [11] use the linear programming technics, but they have still too much computational burden for the real time operation. [11-13] use the closed kinematic chain for the control of the multiple robots and load distribution problem.

In this paper, a new problem associated with the two coordinating robots will be studied using the simultaneous position and force control strategy. After the proposal of the robot dynamics and control strategy, the stability analysis and the load distribution problem will be mentioned.

### 2. Stability Analysis

For the proof of control strategy, the stability analysis has to be investigated. Some papers are published for the stability of the robot control system [14-15].

The stability analysis is conducted with robot dynamics included. First, the closed-form dynamic equation of the robot is derived, then it is transformed into the operational space (cartesian space) for further analysis. Finally, the Liapunov method is applied to the dynamic equation in the operational space.

The closed form dynamic model of a robot can be derived using Newton-Euler or Lagrangian formulation.

$$D(q)\ddot{q} + H(q, \dot{q})\dot{q} + G(q) = T + J^T F \quad (1)$$

where

- $D(q)$  : moment of inertia matrix,
- $H(q, \dot{q})$  : coriolis and centrifugal matrix,
- $G(q)$  : gravity term,
- $T$  : joint torque vector,
- $F$  : external force at the end-effector,
- $J$  : jacobian.

$$q = [q_1 \dots q_n]^T \quad \text{: joint variables.}$$

In order to change equation (1) to the operational space model, let the end-effector trajectory vector be  $x$  ( $\dot{x} = J \cdot \dot{q}$ ), then one has

$$D(x)\ddot{x} + H(x, \dot{x})\dot{x} + G(x) = (J^T)^{-1} \cdot T + F \quad (2)$$

where

$$D(x) = (J^T)^{-1} \cdot D(q) \cdot J^{-1}$$

$$|T_2|^2 = |B_2 + J_2^T \cdot F - J_2^T \cdot F_0 + \theta \cdot J_2^T \cdot F_0|^2 \quad (35)$$

From these, the total absolute value becomes

$$|T_1|^2 + |T_2|^2 = a^2 \cdot \{|J_1^T \cdot F_0|^2 + |J_2^T \cdot F_0|^2\} - 2ab + g \quad (36)$$

where

$$b = (J_1^T \cdot F_0)^T (B_1 - J_1^T \cdot F_1) + |J_2^T \cdot F_0|^2 - (J_2^T \cdot F_0)^T (B_2 + J_2^T \cdot F_1),$$

$$g = |B_1 - J_1^T \cdot F_1|^2 + |B_2 + J_2^T \cdot F_1 - J_2^T \cdot F_0|^2.$$

Differentiate (36) with respect to  $\theta$ , and let it be zero, we get

$$\theta = \frac{b}{|J_1^T \cdot F_0|^2 + |J_2^T \cdot F_0|^2} \quad (37)$$

Second derivative of (36) with respect to  $\theta$  is

$$\frac{d^2(|T_1|^2 + |T_2|^2)}{d\theta^2} = 2[|J_1^T \cdot F_0|^2 + |J_2^T \cdot F_0|^2] \geq 0, \quad (38)$$

which is always positive. Therefore,  $\theta$  minimizes the energy consumption. If we neglect the internal force  $F_i$ ,  $\theta$  becomes same as the result of Zheng and Luh's ([9]) which does not consider the internal force. So it extends the result of [9] to the general case which includes the internal force.

Meanwhile,  $\theta$  has a boundary of  $0 < \theta < 1$ . If  $\theta$  is greater than 1,  $\theta$  should be selected to be 1. In this case, load falls totally on the leader, and follower does nothing on the load. On the other hand, if  $\theta$  is less than 0, then  $\theta$  becomes 0. In this case, load falls totally on the follower, and the leader do nothing on the load. From (37),

the condition when  $\theta$  becomes  $\frac{1}{2}$  is

$$J_1 \cdot T_1 = J_2 \cdot T_2 \quad (39)$$

In this case, the load is evenly distributed with minimum energy consumption. (39) represents that the infinitesimal linear momentum at the leader and follower end-effectors are the same. This means that load on two end-effectors are same.

#### 4. Conclusions

Position and force control of two coordinating robots in handling a single object is proposed, analyzed and applied to the load distribution. Theoretical result shows that given system is stable and good for the practical use. As for the optimal load distribution problem, minimum energy consumption technic at the joints is used to determine the load partition parameter.

For the continuing work, the unknown load distribution problem and the multi-limbed robots, i.e., coordination of arms and legs will be the next topics of my study.

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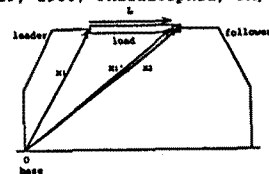


Figure 1. Schematic Diagram of Robotic System

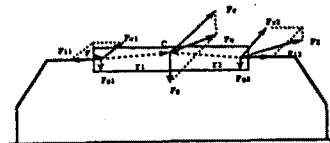


Figure 2. Force Components of Load and End-Effectors due to Gravity and Motion

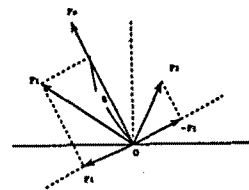


Figure 3. Unevenly Distributed Load

$$H(x, \dot{x}) = (J^T)^{-1} \cdot [D(q) \cdot \ddot{J}^{-1} + H(q, \dot{q}) \cdot J^{-1}],$$

$$G(x) = (J^T)^{-1} \cdot G(q).$$

Equation (2) is the robot dynamic model analyzed in the operational space (Figure 1). Now, the applied torques and external forces are defined. Leader uses position and follower uses force control strategy to the applied torques as follows

$$(J_1^T)^{-1} \cdot T_1 = k_p(x_{1d} - x_1) + k_v(-\dot{x}_1) + G_1 \quad (3)$$

$$(J_2^T)^{-1} \cdot T_2 = k_f(F_d - F_c) + G_2 \quad (4)$$

where

- $k_p$  : position feedback gain,
- $k_v$  : velocity feedback gain,
- $k_f$  : force feedback gain of follower,
- $G$ : gravity compensation term,
- 1 means leader and 2 means follower,
- $d$  means desired value,
- $F_c = k_s(x_2 - x_1 - L)$  actual reaction force,
- $k_s$  : stiffness of the force sensor,
- $L$  : length of the load.

The external forces are as follows :

$$F_1 = -(F_d - F_c) \quad (5)$$

$$F_2 = F_d - F_c \quad (6)$$

The dynamic equations with the position-force control strategy become

$$D_1 \ddot{x}_1 + (H_1 + k_v) \dot{x}_1 + k_p(x_1 - x_{1d}) = -(F_d - F_c) \quad (7)$$

$$D_2 \ddot{x}_2 + H_2 \dot{x}_2 - k_f(F_d - F_c) = F_d - F_c \quad (8)$$

which means if the reaction force is the same as the desired force, the external force to the robotic system is zero. Otherwise, the external force should be regulated.

Define the error function vectors of the leader, follower and force respectively as

$$e_1 = x_1 - x_{1d} \quad (9)$$

$$e_2 = x_2 - x_{2d} \quad (10)$$

$$e_f = F_c - F_d \quad (11)$$

Using (9)-(11) to replace the related terms in dynamic equations (7) and (8), we obtain

$$D_1 \ddot{e}_1 + (H_1 + k_v) \dot{e}_1 + k_p \cdot e_1 - e_f = 0 \quad (12)$$

$$(k_f + 1)^{-1} (D_2 \ddot{e}_2 + H_2 \dot{e}_2) + e_f = 0 \quad (13)$$

There are three independent variables in (12) and (13), namely,  $e_1, e_2,$  and  $e_f$ . The Liapunov function must include all these three variables. Select the Liapunov function as

$$V = \frac{1}{2} (\dot{e}_1^T \cdot D_1 \cdot \dot{e}_1 + e_1^T \cdot k_p \cdot e_1) + \frac{1}{2} \dot{e}_2^T \cdot (k_f + 1)^{-1} \cdot D_2 \cdot \dot{e}_2 + \frac{1}{2} e_f^T \cdot k_s^{-1} \cdot e_f \quad (14)$$

which is clearly positive [1]. Take the time derivative of (14), and rearrange the result using (12) and (13), one obtains

$$\dot{V} = -\dot{e}_1^T \cdot k_v \cdot \dot{e}_1 + (\dot{e}_1^T - \dot{e}_2^T) e_f + \dot{e}_f^T \cdot k_s^{-1} \cdot e_f \quad (15)$$

By using the error equations ((9)-(11)) and reaction force relations

$$F_d = k_s(x_{2d} - x_{1d}) \quad (16)$$

$$F_c = k_s(x_2 - x_1) \quad (17)$$

the relation between the position and force errors can be calculated as

$$e_f = F_c - F_d = -k_s(e_1 - e_2) \quad (18)$$

$$\dot{e}_f^T = -k_s(\dot{e}_1^T - \dot{e}_2^T) \quad (19)$$

Using (19), equation (15) becomes

$$\dot{V} = -\dot{e}_1^T \cdot k_v \cdot \dot{e}_1 \leq 0 \quad (20)$$

Consequently, the given dynamic equation of position-force control is always stable. Equality holds when velocity error of the leader end-effector becomes zero ( $\dot{e}_1=0$ ), which means that the position error becomes constant. In this case, the system is always asymptotically stable.

### 3. Optimal Load Distribution

If the force exerted on the load by the two end-effectors is  $F_r$  and the gravitational force is  $F_g$ , the force exerted at the center of the load becomes

$$F_o = F_r + F_g \quad (21)$$

where  $F_o$  is the resultant force of load,  $F_r$  is the sum of the two end-effector forces  $F_{r1}$  and  $F_{r2}$ , and  $F_g$  is also sum of  $F_{g1}$  and  $F_{g2}$ , for the leader and follower, respectively (Figure 2). Namely,

$$F_r = F_{r1} + F_{r2} \quad (22)$$

$$F_g = F_{g1} + F_{g2} \quad (23)$$

where  $F_{r1}$  and  $F_{r2}$  are the forces contributed totally to the load.

In reality, there are also internal forces. The internal force is defined as orthogonal to the force components of  $F_r$  and does not affect the motion of the load. The internal forces at the two end-effectors have the same magnitudes, but are in opposite directions. Let  $F_{i1}$  and  $F_{i2}$  be the internal forces of the leader and follower, respectively, one has

$$F_{i1} = -F_{i2} \quad (24)$$

Consequently, the resultant forces at the end-effectors become

$$F_1 = F_{r1} + F_{g1} + F_{i1} \quad (25)$$

$$F_2 = F_{r2} + F_{g2} + F_{i2} \quad (26)$$

The sum of  $F_1$  and  $F_2$  is

$$F_1 + F_2 = F_o \quad (27)$$

For simplicity, we drop the constant gravity term, i.e., let  $F_g = 0$ . As a result, the resultant force is the same as the motion direction, that is,  $F_o = F_r$ . Equation (27) can be represented by the following matrix-vector form

$$F_o = W \cdot \begin{matrix} F_1 \\ F_2 \end{matrix} \quad (28)$$

where

$W = [I_6, I_6]$ , a  $6 \times 12$  matrix, in which  $I_6$  is a  $6 \times 6$  identity matrix.

In equation (28), we can select  $W$  in many other ways. One alternate form is

$$W = \begin{matrix} I_3 & 0 & I_3 & 0 \\ S_1 & I_3 & S_2 & I_3 \end{matrix} \quad (29)$$

where

$$S_i = r_i \times I_3,$$

$I_3$  is  $3 \times 3$  identity matrix and

$r_i$  is defined in Figure 5,  $i=1,2$ .

But this method does not give any difference in the analysis if we choose  $F_1$  as in equation (31).

In order to distribute the load, we need to specify  $F_1$  and  $F_2$ . In general, the force should not be evenly distributed (Figure 3). For this, we introduce a parameter  $\theta$ ,

$$\begin{matrix} F_1 \\ F_2 \end{matrix} = \begin{matrix} \theta \cdot I_6 \\ (1-\theta) \cdot I_6 \end{matrix} \cdot F_o + (I_{12} - W^T \cdot W) \cdot X \quad (30)$$

where  $\theta$  is the load partition parameter, ( $0 < \theta < 1$ ) and  $X$  is the arbitrary vector.

Using the internal force instead of  $X$ ,

$$\begin{matrix} F_1 \\ F_2 \end{matrix} = \begin{matrix} \theta \cdot I_6 & I_6 \\ (1-\theta) \cdot I_6 & -I_6 \end{matrix} \cdot F_o + \begin{matrix} I_6 \\ -I_6 \end{matrix} \cdot F_i \quad (31)$$

In (31),  $\theta$  can be selected following criteria, i.e., a minimum energy consumption criterion. This method is based on minimizing the consumed energies at the leader and follower joints. In order to do so, the closed form dynamic equation (1) is needed. In (1), the elements of the left side are all known. So, this part be denoted as matrix  $B$ . As a result, the leader and follower dynamic equations become

$$T_1 = B_1 - J_1^T \cdot F_1 \quad \text{and} \quad (32)$$

$$T_2 = B_2 - J_2^T \cdot F_2 \quad (33)$$

The absolute value of equations are

$$|T_1|^2 = |(B_1 - J_1^T \cdot F_1) - \theta \cdot J_1^T \cdot F_o|^2 \quad (34)$$