여유 자유도 매니퓰레이터를 위한 최적 제한 조건을 기반으로한 Resolved Motion 방법의 특이점에 관한 연구

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On the Singularities of Optimality Constraint-based Resolved Motion Methods for a Redundant Manipulator

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ABSTRACT

Algorithmic or kinematic singularities are inevitably introduced if optimality criteria or augmented kinematic equations are used to resolve the redundancy of almost any manipulator with rotary joints. In this paper, a sufficient condition for a singularity-free optimal solution of the kinematic control of a redundant manipulator is derived and, specifically, algorithmic singularities are analyzed for optimality-based methods. A singularity-free space (SFS) to characterize the performance of a secondary task for a redundant manipulator using the sufficient condition for a redundant manipulator is defined. The SFS is a set of regions classified by the loci of configurations satisfying the inflection condition for manipulability measure in the Configuration space. Using SFS, the topological property of the Configuration space and the invertible workspace without singularities are analyzed.

I. INTRODUCTION

The redundancy resolution of a redundant manipulator for reconfiguration without affecting the end-effector posture has been discussed in the framework of how to optimize some performance measures while carrying out its given task (in general a given task is called a primary task and a task to be optimized is called a secondary task). In redundant manipulators, one of the following performance criteria is used to utilize kinematic redundancy; singularity avoidance, obstacle avoidance, joint-limit avoidance, joint velocity minimization, joint acceleration minimization, joint torque minimization, etc.

In such work of how to resolve the redundancy, many researchers use the null projection operator and Jacobian pseudoinverse. In general, pseudoinverse-based scheme can provide nonconservative joint motions for a task of tracking a closed path in Operational space.[1,3] On the other hand other approach based on optimality constraint can provide the conservative joint motions.[2,5,8] One could choose a function which minimizes a performance criterion mentioned above. And also an adaptive scheme based on an augmented Jacobian matrix is utilized to directly control the configuration variables so as to achieve tracking of some desired reference trajectories throughout the robot motion.

However these approaches have certain drawbacks. First, global optimality cannot be guaranteed because these approaches are formulated by local path information.[4] A manipulator may blunder into unstable regions. Second, algorithmic singularities are inevitably introduced if optimality criteria or augmented kinematic equations are used to resolve the redundancy in almost any manipulator.[2,8,9]

Recently, to solve the above problems in the case of a planar robot manipulator with three joints, the characteristics and topological properties both on the Configuration space and Operational space imposing an optimality constraint function of a dexterity measure are analyzed by the measure constraint locus (MCL) technique.[6] The MCL is the loci of configuration satisfying the necessary condition for optimality of the manipulability measure in the Configuration space. In this case the optimality condition for some performance measure H is described as $Z^T \nabla H = 0$, where ∇H is the gradient of H and Z is the bases of the null space,[2,5] and the superscript T means transpose of the vector. However the MCL is obtained using the necessary condition of optimality only. Therefore, an equilibrium state can be either maximum or minimum. It is necessary to clarify the MCL and to look at the performance of a secondary task using the sufficient condition of optimality for a redundant manipulator. In this context, we propose a sufficient condition for a singularity-free optimal solution for kinematically redundant manipulators.

In this paper, first we derive a sufficient condition for a singularity-free optimal solution of the kinematic control of a redundant manipulator. The derived condition indicates whether the state is maximum or minimum for the resolved motion method. Second, we suggest a singularity-free space (SFS) to characterize the performance of a secondary task for a redundant manipulator using the sufficient condition of optimality for a redundant manipulator. The SFS is a set of regions classified by the loci of configurations satisfying the inflection condition for the optimality of manipulability measure in the Configuration space. Using MCL with SFS, the topological property of the Configuration space and the invertible workspace without singularities are analyzed. And also we make clear that in configurations of algorithmic singularity for the optimality constraint-based method the tangential vectors of MCL exist in null space[10] of Jacobian matrix. Since the MCL with SFS shows a topological property of the manipulability measure and singularity-free joint trajectories in the Configuration space, the SFS gives a clue in the study of the problems associated with inverse kinematic algorithms based on the optimality constraint such as the extended Jacobian method[2] or a closed-form solution.[5] Through numerical examples of cyclic tasks, we show that the invertible workspace is limited according to the initial configuration and why the optimality constraint-based schemes can blunder into a singular configuration during performing the task. Finally we comment that if a maximum configuration of self-motion manifold at initial position in Operational space is selected as an initial configuration, the optimality constraint scheme provides an inverse solution which is a conservative joint trajectory almost entire workspace without kinematic and algorithmic singularities even though it is based on the local optimization.

II. A SUFFICIENT CONDITION OF OPTIMAL SOLUTION FOR A SECONDARY TASK

Optimality-based methods may lead to undesirable effects such as algorithmic singularities and the limitations of invertible workspace. These problems have been discussed by using inverse kinematic equations.[3] But if one develops an inverse kinematic algorithm using optimality-based method, one must know whether it is possible to define the inverse kinematic algorithm for a primary task in Operational space. Therefore, it is necessary to look at the performance of a secondary task for the redundant manipulator.

Recently, a method which gives the global aspect of local optimization methods has been suggested.[6] This method is based on the topological properties of the optimality constraint when the local path information is used. A manipulability constraint locus (MCL) which is a set of configurations satisfying the optimality constraint for the manipulability measure is defined. So, the MCL shows a topological property of the manipulability measure and singularity-free joint trajectories in the Configuration space.[6] The MCL and some self-motion manifolds for a three DOF planar manipulator in Fig. 1 are shown in Fig. 2. And, by the forward mapping from the Configuration space to Operational space, the invertible workspace of the inverse kinematic algorithm was discussed. However the MCL is constructed by only the necessary condition of the optimality, the equilibrium state can be either maximum or minimum. Therefore, it is necessary to construct the constraint locus in which the equilibrium state is always one of extrema depending on the measure for a secondary task.

In general, a redundant robot control problem may be considered as follows: maximize (or minimize) a performance measure $H(\theta)$ as a secondary task subject to the kinematic constraints $F_i(\theta) \triangleq f_i(\theta) - x_i = 0,\ \ i = 1,\cdots,m$.

In order to derive the optimality constraint for a secondary task, we employ the following notations: $\nabla H(\cdot) \triangleq \frac{\partial H(\cdot)}{\partial\theta} \in R^n$ and $\nabla^2 H(\cdot) \triangleq \frac{\partial^2 H(\cdot)}{\partial\theta^2} \in R^{n\times n}$ imply the gradient and Hessian of a functional $H(\cdot)$ with respect to $\theta \in R^n$, respectively. Similarly, $\nabla F_i(\cdot) \triangleq \frac{\partial F_i(\cdot)}{\partial\theta} \in R^n$ and $\nabla^2 F_i(\cdot) \triangleq \frac{\partial^2 F_i(\cdot)}{\partial\theta^2} \in R^{n\times n}$ imply the gradient and Hessian of kinematic functions $F_i(\cdot)$ with respect to θ, respectively. All the vectors are represented by row vectors. Let a state θ^* be an optimal configuration for maximization.

To satisfy the kinematic equations $F_i(\cdot)$, it is necessary to move along a self-motion manifold passing through an optimal configuration θ^*. If we define the self-motion manifold as a feasible arc, we can parameterize the arc using a single variable, l. When the redundancy is one, the solution of the kinematic equation is defined as a feasible curve.

Let $\theta(l)$ be an arc, where $\theta(0) = \theta^*$, and let Z be a tangential vector to the arc. That is, $Z \triangleq \frac{d\theta(l)}{dl} \in R^n$ implies the tangential vector on the self-motion manifold which satisfies the kinematic equation, and the variable l is the length from the equilibrium point $\theta(0) = \theta^*$ to the point $\theta(l)$ in the Configuration space. For explanatory convenience, we assume that the redundancy is one from now.

Since the first and second derivatives of the kinematic equation with respect to l are zero, we can obtain the following equations by applying the chain rule.
For all i

$$\frac{d}{dl}F_i(\theta(l)) = \left[\nabla F_i(\theta(l))\right]^T Z = 0, \tag{1}$$

and

$$\frac{d^2}{dl^2}F_i(\theta(l)) = \frac{d}{dl}\left[\left[\nabla F_i(\theta(l))\right]^T Z\right]$$
$$= Z^T \nabla^2 F_i(\theta(l)) Z + \left[\nabla F_i(\theta(l))\right]^T \frac{dZ}{dl} = 0. \tag{2}$$

For all l, Eqs.(1) and (2) can be rearranged by Eqs.(3) and (4) respectively

$$JZ = 0 \tag{3}$$

and

$$J\frac{dZ}{dl} = -\begin{bmatrix} Z^T\nabla^2 F_1 Z \\ Z^T\nabla^2 F_2 Z \\ \vdots \\ Z^T\nabla^2 F_m Z \end{bmatrix} \in R^m, \tag{4}$$

where

$$J \triangleq \begin{bmatrix} (\nabla F_1)^T \\ (\nabla F_2)^T \\ \vdots \\ (\nabla F_m)^T \end{bmatrix} \in R^{m\times n}.$$

Here, ∇F_i and $\nabla^2 F_i$ denote $\nabla F_i(\theta(l))$ and $\nabla^2 F_i(\theta(l))$, respectively. Similarly, ∇H and $\nabla^2 H$ denote $\nabla H(\theta(l))$ and $\nabla^2 H(\theta(l))$, respectively.

With above equations, we can derive the optimality condition using Taylor series expansion of H about the equilibrium state θ^*.

$$H(\theta(l)) = H(\theta^* + \varepsilon Z)$$
$$= H(\theta^*) + \varepsilon \frac{d}{dl}H(\theta(l))\bigg|_{l=0} + \frac{1}{2}\varepsilon^2 \frac{d^2}{dl^2}H(\theta(l))\bigg|_{l=0} + O(\varepsilon^3) \tag{5}$$

where $O(\cdot)$ denotes the order, ε is a small positive value, and $Z \in R^n$ is a null basis vector of the Jacobian matrix J, which is tangential to the self-motion manifold.

From Eq.(5), a sufficient condition of the optimal state θ^* for maximizing the performance measure $H(\theta)$ subject to the kinematic equations $F_i(\theta)=0,\ l = 1,\cdots,m$ is derived as follows.

$$\frac{d}{dl}H^* \triangleq \frac{d}{dl}H(\theta(l))\bigg|_{l=0} = 0 \tag{6a}$$

and

$$\frac{d^2}{dl^2}H^* \frac{d^2}{dl^2}H(\theta(l))\bigg|_{l=0} < 0. \quad (6b)$$

From Eq.(6), the derivative H with respect to l is given by

$$\frac{d}{dl}H^* = \left[\nabla H(\theta^*)\right]^T Z = 0. \quad (7)$$

Eq.(7) is equivalent to the following condition; $\nabla H^* \triangleq \nabla H(\theta^*)$ must be a linear combination of the rows of J in Eq.(3).

$$\left[\nabla H^*\right]^T = \lambda^T J \quad \text{for some } m\times 1 \text{ vector } \lambda. \quad (8)$$

Therefore,

$$\lambda^T = \left[\nabla H^*\right] J^+ \quad \text{where } J^+ \triangleq J^T (JJ^T)^{-1} \in R^{n\times m}. \quad (9)$$

Using Eqs. (4), (6), and (9),

$$\frac{d^2}{dl^2}H^* = \left[\frac{d}{dl}\left[\nabla H^*\right]^T\right] Z + \left[\nabla H^*\right]^T \frac{dZ}{dl}$$

$$= Z^T \nabla^2 H^* Z + \lambda^T J \frac{dZ}{dl}$$

$$= Z^T \left[\nabla^2 H^* + \sum_{i=1}^{m} \lambda_i \nabla^2 F_i\right] Z$$

$$= Z^T B Z < 0 \quad (10)$$

where $B = \nabla^2 H + \sum_{i=1}^{m} \lambda_i \nabla^2 F_i$ and λ_i is ith component of the vector λ.

Since the basis of the null space of J can be represented by the following equation[5]

$$Z = [J_{n-m} J_m^{-1} : -I_{n-m}]^T, \quad (11)$$

Eq.(10) can be directly computed. When the redundancy is one, Z is represented as an n dimensional vector.

Therefore, a sufficient condition for the optimization problem can be summarized by Eqs.(7) and (10). Note that if the redundancy is more than one, the left hand side of Eq.(10) must be a negative definite matrix.

III. ALGORITHMIC SINGULARITIES AND SINGULARITY-FREE SPACE

We can explain singularities consisting of kinematic singularities and algorithmic singularities in the Configuration space. Configurations satisfying inflection condition that is described by $Z^T BZ = 0$ make inflection curves in the Configuration space. Therefore, inflection points while satisfying Eq.(7) are, so called, algorithmic singularities for optimality-based methods.

Since we deal with singularities consisting of kinematic singularities and algorithmic singularities, we define a singularity-free space to characterize the performance of a secondary task.

Definition : A Singularity-Free Space (SFS) is a set of regions classified by the inequality constraint for manipulability measure in the Configuration space such that

$$\Lambda = \{\ \theta : Z^T B Z < 0, \ \text{ for } \theta \in T^n\ \} \quad (12)$$

where T^n is an n-torus which forms the Configuration space.

To uniquely determine an inverse kinematic solution in a practical sense, a solution performing a primary given task while optimizing an objective function is determined by overlapping the MCL with self-motion manifolds in the SFS. Intersecting points of the MCL and inflection curves in the SFS are the algorithmic singularities in the optimality-based methods. These points can be obtained by an alternative method. In algorithmic singular points, the row vector generated from the optimality constraint[2], $\frac{\partial}{\partial\theta}(Z^T \nabla H)$, is a linear combination of the rows of Jacobian matrix J. That is, for some $m\times 1$ vector α

$$y \triangleq \frac{\partial}{\partial\theta}(Z^T\, \nabla H) = J^T \alpha. \quad (13)$$

If both sides are multiplied by a null basis vector, Z, of J,

$$Z^T y = 0. \quad (14)$$

This equation indicates that in algorithmic singular configuration the tangential vectors of the MCL lie in the tangential space of self-motion manifolds. This condition is the same as satisfying Eqs.(7) and (10). Through the numerical example, the inverse algorithms based on the optimality constraint in the SFS will be analyzed.

IV. AN EXAMPLE

There are many dexterity measures proposed to avoid singularities. To show a numerical example, we adopt the manipulability measure[7] as a secondary task which has been widely used as a secondary task to avoid singular configurations in literatures.[1,2] The manipulability measure as a secondary task is given by

$$H \triangleq \sqrt{det(JJ^T)}. \quad (15)$$

Consider a three-link planar robot manipulator in x_1–x_2 plane shown in Fig. 1. Operational space is two dimensional plane, accordingly $x \in R^2$ so that the degree of redundancy at kinematically nonsingular points is equal to one. Tracking a circle is the primary task as shown in Fig. 1. It is assumed that the manipulator has the following link lengths, $l_1 = 3$, $l_2 = 2.5$, and $l_3 = 2.0$ *units*. If we denote $s_1 = sin(\theta_1)$, $c_1 = cos(\theta_1)$, $s_{12} = sin(\theta_1 + \theta_2)$, $c_{12} = cos(\theta_1 + \theta_2)$, $s_{123} = sin(\theta_1 + \theta_2 + \theta_3)$, and $c_{123} = cos(\theta_1 + \theta_2 + \theta_3)$, the kinematic equations are

$$x = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \end{bmatrix}, \quad (16)$$

and the Jacobian matrix becomes

$$J = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12} + l_3 s_{123}) & -(l_2 s_{12} + l_3 s_{123}) & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \end{bmatrix}. \quad (17)$$

In the numerical example, the cyclic task is described as

$$x = \begin{bmatrix} -r\cos(2\pi t) + c \\ -r\sin(2\pi t) \end{bmatrix}, \quad (18)$$

where r is the radius and c is the x_1–axis position of the center of the circle. The task is to rotate the circle of r unit radius, centered at $(c, 0)$, in unit time, in counterclockwise. Thus the initial position at $t_0 = 0$ is $(c-r, 0)$ and the final position at $t_1 = 1$ is the same as the initial position. And the farthest position from the base of the manipulator is at $t = 1/2$. Therefore, we denote this point as the mid-position of a cyclic task, which is $(c+r, 0)$.

As mentioned before, the MCL is the set of configurations satisfying optimality constraint. Fig. 2. shows the MCL and self-motion manifolds. For the case of a three-link planar manipulator, the Configuration space is a product space formed by the individual joint manifolds such that a 3-torus as T^3. However the singularity-free space and the basis of the null space of J are not functions of θ_1 in a planar manipulator with rotary joints, so we use the θ_2–θ_3 plane as the Configuration space.

Using Eq.(12), the singularity-free space for the three-link planar manipulator is obtained as shown in Fig. 3. The shaded area indicates the region of minima including kinematic singular points, and the unshaded area indicates the region of maxima in the θ_2–θ_3 space. In the SFS, the intersecting points of the MCL and curves satisfying the condition of $Z^T B Z = 0$ are algorithmic singular points. Also in Fig.2 the points where the tangential vector of MCL lies in the tangential space of self-motion manifolds are algorithmic singularities.

Consider a task where $r = 1$ and $c = 3$ *units*. Dot lines in Fig.3 are self-motion manifolds for initial position (near to the origin) and mid-position (far to the origin). Six points are obtained from the necessary condition as the initial points in the initial position of the Operational space. Let us take three points (A, B, C) of them as initial points. Therefore, if the point B is chosen as an initial point, the joint trajectory meets with singular point as shown in Fig. 4. Using the sufficient condition we proposed, we easily determine that the point A and C are maximum points. If a task is carried out using the extended Jacobian technique among optimality constraint methods, we can obtain optimal joint trajectories from maximum point A if we select it as an initial point. But the point C chosen as an initial point meets with algorithmic singularity. Fig. 4. results show that according to the selection of the initial configuration of a redundant manipulator, the optimality-based methods may meet singular points during the kinematic control.

Therefore, the SFS gives a clue in the study of the problems associated with inverse kinematic algorithms based on the optimality constraint such as the extended Jacobian method or a closed-form solution.[5] Through the SFS, we showed that the invertible workspace is limited according to the initial configuration, and why the optimality constraint-based schemes can blunder into the algorithmic singular configurations during performing the task. As mentioned in [6] we can obtain the inverse kinematic solution in two stages. First stage is to select a proper initial configuration on the self-motion manifolds for the initial position to achieve a globally optimized configuration.[11] Second stage is to solve inverse kinematic algorithm by optimality-based methods[2,5-8] with a global maximum as the initial configuration. By doing this, a globally maximized, conservative and singularity-free joint trajectory for almost entire workspace can be obtained.

V. CONCLUSIONS

Optimality-based methods may lead to undesirable effects such as algorithmic singularities and limitations of invertible workspace. In this paper a sufficient condition of the optimal solution while avoiding singular points was derived. With the compact form, Z, of null basis of Jacobian matrix, the condition is easily checked. From this condition, a SFS which characterizes the performance of a secondary task was defined. Through an example, we showed that the invertible workspace is limited according to the initial configuration, and that trajectories from the optimality constraint-based schemes can blunder into singular configurations during performing the primary task. Specifically it shows that in $Z^T y = 0$ the tangential vector of MCL lies in the tangent space of self-motion manifolds in algorithmic singular configuration. Therefore, by a proper selection of initial configuration of a redundant manipulator on the self-motion manifolds, singularities can be avoided during the kinematic control and globally optimal trajectories for a secondary task can be generated.

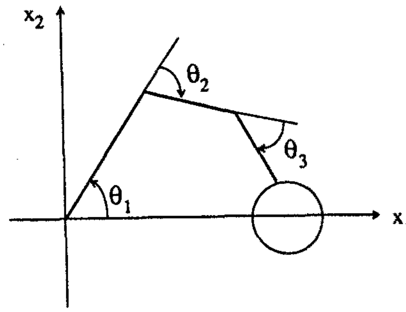

Fig. 1. Geometry of a three link planar manipulator

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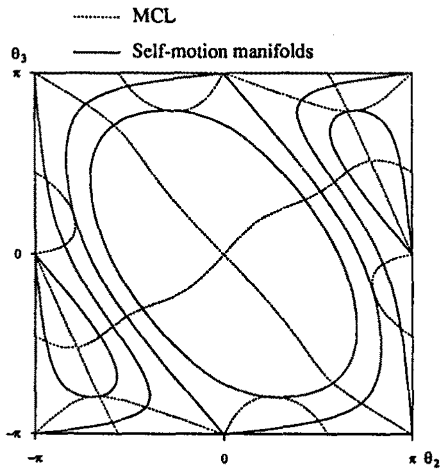

Fig. 2. Measure constraint locus (dotted line) with self-motion manifolds (solid) in Configuration space

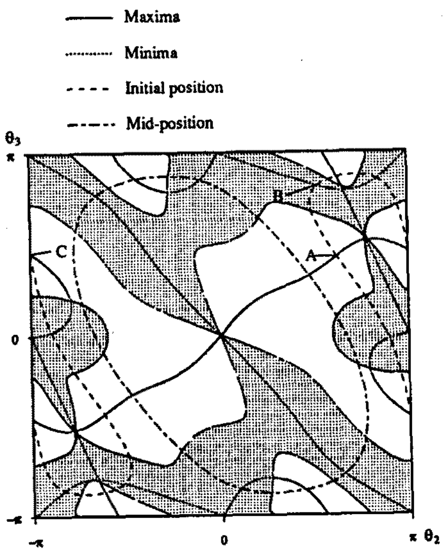

Fig. 3. Singularity-free space (unshaded region)

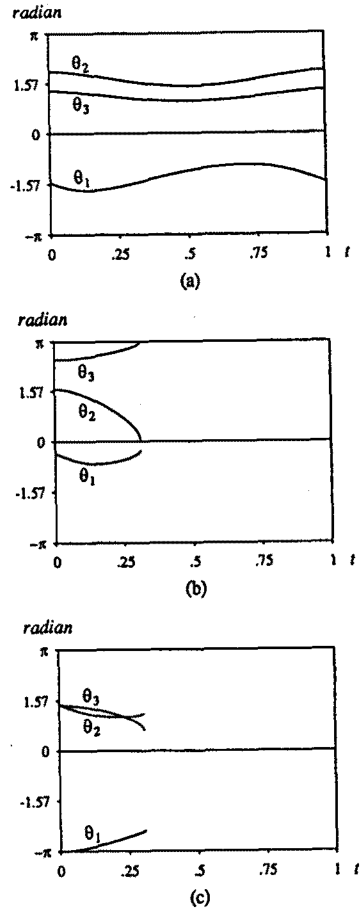

Fig. 4. Joint trajectories with different initial points; (a) a maximum point: A (b) a minimum point: B (c) another maximum: C