

CORRELATION SEARCH METHOD WITH THIRD-ORDER STATISTICS FOR COMPUTING VELOCITIES FROM MEDICAL IMAGES

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ABSTRACT

The correlation search method yields velocity information by tracking scatter patterns between medical image frames. The displacement vector between a target region and the best correlated search region indicates the magnitude and direction of the inter-frame motion of that particular region. However, if the noise sources in the target region and the search region are correlated Gaussian, then the cross-correlation technique fails to work well because it estimates the cross-correlation of both signals and noises.

In this paper we develop a new correlation search method which seeks the best correlated third-order statistics between a target and the search region to suppress the effect of correlated Gaussian noise sources. Our new method yields better estimations of velocity than the conventional cross-correlation method.

1. Introduction

Over the last several years, research on the analysis of visual motion has come to play an important role within the computer vision community.

The analysis of visual motion can be applied to diagnostic ultrasound. Mailloux et al [1] quantify the motion of the heart from ultrasonic image sequences. Meunier et al [2] study the capability and the diagnostic value of a method to extract parameters describing tissue dynamics by tracking myocardial speckle pattern motion during the cardiac cycle. Trahey et al. [3] present a new technique using correlation search method for blood velocity imaging based on tracking the motion of the scatter pattern produced by blood.

The analysis of motion from image sequences can be roughly classified into two approaches. The first approach is the optical flow (OF) method. The OF is the distribution of apparent velocities of movement of brightness patterns in an image. The OF cannot be computed at a point in the image independently of neighboring points without introducing additional constraints, because the velocity field at each image point has two components while the change in image brightness at a point in the image plane due to motion yields only a scalar [4]. The second approach is the correlation search method based on comparisons of gray levels. The correlation search method yields velocity information by tracking scatter patterns between image sequences. This idea is quite straight-forward, and the first application of this idea to motion detection in ultrasound images appeared in the literature in 1987 [3].

Trahey et al [5] have reported the stability of scatter patterns with small target displacements and

complex tissue deformation. The determination of a good measure of match is very important in the correlation search method. The normalized correlation coefficient is considered as a good measure of match between the target region and the search region [3,6]. In practical applications, ultrasound image sequences can contain correlated Gaussian noises. If the noise sources in a target region and the search region are correlated Gaussian, then the correlation search method fails to work well because it estimates the correlation of both signals and noises.

One of the important properties of higher-order spectra is that all higher-order spectra of order greater than two are identically zero for Gaussian processes [7,8]. Thus, in those signal processing settings where the additive noise to the signal is spatially correlated Gaussian, there might be advantages in computing correlation coefficients in higher-order spectrum domains. In this paper we develop a new correlation search method which seeks the best correlated third moment sequences between a target and the search region to suppress the effect of correlated Gaussian noise sources.

Our new correlation search method requires the calculation of correlation coefficients for selected subregions in the search region using the assumption that the brightness of a particular point remains constant following displacement by a small distance over a small time interval. Such assumption is also used in the optical flow method [4].

Experimental results with images of ultrasound phantoms confirm that a new correlation search method yields better estimations than the conventional correlation method. When the target region size was 5x5 pixels, and the search region size was 11x11 pixels, the previous correlation search method requires the calculation of 49 correlation coefficients between a target region and subregions in the search region. However, the new correlation search method in this paper requires the calculation of 14 correlation coefficients at most.

11. Correlation Search Method Using the Third Moment

Transformation into the Third Moment Domain

The motion of small, non-overlapping target regions between two consecutive image frames, X and Y, may be found by calculating the correlation coefficients between a rectangular region in image X and a subregion in image Y. The displacement vector between a target region in X and the best correlated region in Y is assumed to indicate the magnitude and the direction of the inter-frame motion of that particular target region.

The correlation coefficients used by the conventional correlation search method is given by [3]

$$\rho_{mn} = \frac{\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (X_{ij} - \bar{X})(Y_{i+m, j+n} - \bar{Y})}{\sqrt{\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (X_{ij} - \bar{X})^2 \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (Y_{i+m, j+n} - \bar{Y})^2}} \quad (1)$$

where the region size is $N_1 \times N_2$, \bar{X} is the mean pixel value of the corresponding image region X , and \bar{Y} in the mean pixel value of the corresponding image region Y . The position of the maximum value of ρ_{mn} is assumed to correspond to the new target position in Y . However, if the noise sources in a target region and the search region are correlated Gaussian, the maximum value of ρ_{mn} does not always correspond to the best correlated position.

In this section we introduce the transformation of images X and Y with correlated Gaussian noises into noise free third moment domain. Then we can calculate the correlation coefficients without noises in the third moment domain.

Let us assume that $X(i,j)$ and $Y(i,j)$ are pixel values at (i,j) satisfying

$$X(i,j) = S(i,j) + W_1(i,j), \quad (2)$$

$$Y(i,j) = S(i-m, j-n) + W_2(i,j), \quad (3)$$

where $S(i,j)$ is a pixel value without noise, $S(i-m, j-n)$ is the same signal, changed in position, and $W_1(i,j)$ and $W_2(i,j)$ are unknown noise sources. Assuming that the noise sources are zero-mean independent stationary processes, the conventional operation adopted to search for a change in position between $X(i,j)$ and $Y(i,j)$ is the cross-correlation which is:

$$\begin{aligned} \gamma_{xy}(k, \ell) &= E\{X(i,j) Y(i+k, j+\ell)\} \\ &= \gamma_{ss}(k-m, \ell-n) \end{aligned} \quad (4)$$

where $E\{\cdot\}$ denotes the expectation operation, and

$$\gamma_{ss}(k-m, \ell-n) = E\{S(i,j) S(i+k-m, j+\ell-n)\}. \quad (5)$$

Thus, $\gamma_{xy}(K, \ell)$ peaks at $k=m, \ell=n$.

In practical application problems, the noise sources are correlated. Then, cross-correlation of $X(i,j)$ is given by

$$\gamma_{xy}(k, \ell) = \gamma_{ss}(k-m, \ell-n) + \gamma_{12}(k, \ell) \quad (6)$$

where $\gamma_{12}(k, \ell) = E\{W_1(i,j) W_2(i+k, j+\ell)\}$.

$S(i,j)$ is assumed to be zero mean non-Gaussian stationary random process. The assumption that $S(i,j)$ is zero mean is a reasonable practical assumption because a filter can be used to remove the dc component. Additionally, the assumption that $S(i,j)$ is a non-Gaussian process is also reasonable because $S(i,j)$ is obtained from ultrasound signals propagating through an attenuating medium. Any signals propagating through a fading communication channel are often assumed to be a non-Gaussian signal [9]. It follows from Eq.(6) that all the cross-correlation-based methods with second-order statistics will generally fail to work well without some knowledge of $\gamma_{12}(k, \ell)$. However, it is very difficult to compute the cross-correlation of the noise sources.

On the other hand, the following relationship holds in the third moment domain for the data given by Eqs.(2) and (3).

$$\begin{aligned} R_{xxx}(k, \ell) &= E\{X(i,j) X^2(i+k, j+\ell)\} \\ &= R_{sss}(k, \ell). \end{aligned} \quad (7)$$

$$\begin{aligned} R_{yxx}(k-u, \ell-v) &= E\{Y(i+u, j+v) X^2(i+k, j+\ell)\} \\ &= R_{sss}(k+m-u, \ell+n-v). \end{aligned} \quad (8)$$

where $W_1(i,j)$ and $W_2(i,j)$ are zero mean, possibly correlated, Gaussian process, and

$$R_{sss}(k, \ell) = E\{S(i,j) S^2(i+k, j+\ell)\}. \quad (9)$$

This is because the third moment sequence of a zero mean Gaussian process is identically zero in theory [7,8,9]. It follows from Eqs. (7) and (8) that the target region and the search region in the third moment domain are best correlated when $u=m$ and $v=n$. Additionally, we can suppress the effect of correlated Gaussian noise sources by calculating the correlation coefficients between $R_{xxx}(k, \ell)$ and $R_{yxx}(k-u, \ell-v)$. To calculate the correlation coefficients in the third moment domain, we define the following transformation of the target region X and the search region Y into the third moment domain.

$$XT(k, \ell) = E\{X(i,j) X^2(i+k, j+\ell)\} \quad (10)$$

$$YT(k-u, \ell-v) = E\{Y(i+u, j+v) X^2(i+k, j+\ell)\}. \quad (11)$$

The new correlation coefficient defined in this paper is

$$\begin{aligned} \rho_{uv} &= \\ &= \frac{\sum_{k=1}^{N_1} \sum_{\ell=1}^{N_2} (XT(k, \ell) - \overline{XT})(YT(k-u, \ell-v) - \overline{YT})}{\sqrt{\sum_{k=1}^{N_1} \sum_{\ell=1}^{N_2} (XT(k, \ell) - \overline{XT})^2 \sum_{k=1}^{N_1} \sum_{\ell=1}^{N_2} (YT(k-u, \ell-v) - \overline{YT})^2}} \end{aligned} \quad (12)$$

where \overline{XT} and \overline{YT} are the mean pixel values of the corresponding image region XT and YT .

The displacement vector between a target region and the search region is found by finding (u,v) which has the maximum value of ρ_{uv} in the search region.

Correlation Search Algorithm

If the conventional correlation search method is applied to a 5×5 grid of target regions and 11×11 grid of search regions, the set of displacement vector (u,v) to find the best correlated region in the search region is shown in Fig. 1a. However we can reduce the number of displacement vectors (u,v) required to find the correlation coefficients by using the property that the brightness of a particular point in the image is constant.

Let $E(x,y,t)$ be the image brightness at the image point (x,y) at time t . Then, if $u(x,y)$ and $v(x,y)$ are the x and y components of the displacement vector at that point, we expect that the brightness will be the same at time $t+\delta t$ at the point $(x+\delta x, y+\delta y)$ [4]. Horn and Schunk [4] show that the brightness of a particular point in the image is constant, so that

$$E_x u + E_y v + E_t = 0. \quad (13)$$

where E_x , E_y and E_t are the partial derivatives of image brightness with respect to x, y , and t , respectively. Therefore, we need to calculate the correlation coefficient ρ_{uv} at the set of (u, v) which satisfies Eq. (13). Figure 1b shows the reduced set of (u, v) to find the best correlated region in the search region. The estimates E_x , E_y , and E_t are the average of four first differences taken over adjacent measurements [4].

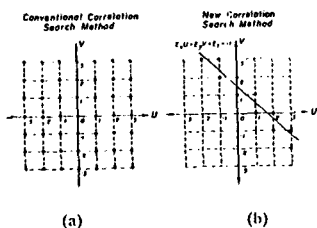


Fig. 1. The sets of (u, v) for the conventional and new correlation search methods.

The new correlation search method in this paper adopts the same constraint of the local linearity of the displacement vector as described in [10]. Any vector field can be considered as locally linear, that is, describable by linear equations in the form

$$\begin{bmatrix} u(x, y) \\ v(x, y) \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (14)$$

An important property of linear fields is that they can be described as the sum of translational, rotational, divergent, and shear components [10]. The parameters a, b, A, B, C, D are found using the method of least squares in [10].

III. Experimental Results

Ultrasound images of a tissue equivalent phantom were obtained before and after it was rotated one degree around its center axis. Figure 2 shows the before and after image frames used for experiment. Computer-generated Gaussian noises are added to the before and after image frames. We select the target region size as 5×5 pixels, and the search region size as 11×11 pixels. Figure 3 shows that computed inter-frame displacement vector field with the new correlation search method and the conventional correlation search method. In this case, signal-to-noise-ratio (SNR) is 24 dB. Figure 4 shows the computed inter-frame displacement vector field with SNR=4 dB.

It follows from Figs.3 and 4 that the new correlation search method with third order statistics yields a more clear rotational vector field than the conventional correlation search method even in the case of low SNR. Thus, the new correlation search method has a capability of suppressing the effect of Gaussian noises.



Fig.2. Ultrasound phantom images. (They are obtained before and after it is rotated one degree around its center axis.)

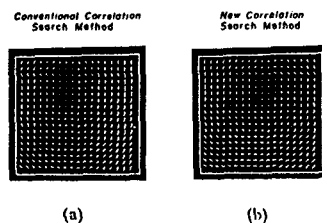


Fig.3. The computed inter-frame displacement vector with the conventional correlation search method and with the new correlation search method (SNR = 24 dB).

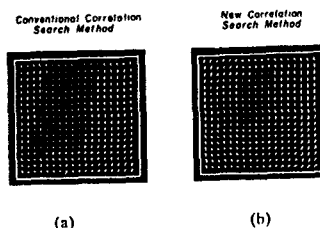


Fig.4. The computed inter-frame displacement vector with the conventional correlation search method and with the new correlation search method (SNR = 4 dB)

IV. Conclusion

In this paper, we developed the new correlation search method with a target region and the search region transformed into the third moment domain. By using these third-order statistics, we show that the new correlation search method can suppress the effect of Gaussian noise sources, possibly correlated Gaussian noises. We also propose the method to reduce the set of (u, v) to find the best correlated region in the search region by using the property that the brightness of a particular point in the image is constant.

The most important limitation of the correlation search method is that motion information is obtained for only two dimensions. However, the actual dynamics of biological structures such as myocardium is three dimensional. Thus, this algorithm fails to fully characterize the velocity field of the tissue. The developed algorithm has utility as long as the frame rates are high enough and views are chosen so as to minimize the motion normal to the perpendicular plane. Techniques for restoring the full three-dimensional vector fields are currently being investigated in our lab.

VI. References

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