한국전산구조공학회
1991년도 가을학술발표회 논문집

Simple Method of Analysis for Preliminary Design of the Composite Laminated Primary Structures for Civil Construction

Duk Hyun Kim
(Korea Composites)

ABSTRACT

In his recent book, D.H. Kim proposes to use the quasi-isotropic constants by Tsai for the preliminary design of the composite primary structures for the civil construction. Such structures generally require a large number of laminae layers.

Simple equations which can predict "exact" values of the buckling strength, the natural frequency of vibration, and the deflection for the special orthotropic laminates are presented. Many laminates with certain orientations have decreasing values of B_{16} and B_{26} as the number of plies increases. Such laminates, with $D_{16}=D_{26}\rightarrow 0$, including the laminates with anti-symmetric configurations can be solved by the same equation for the special orthotropic laminates. If the quasi-isotropic constants are used, the equations for the isotropic plates can be used. Use of some coefficients can produce "exact" value for laminates with such configuration.

1. Introduction

In his recent book, D.H. Kim proposes to use the quasi-isotropic constants by Tsai for the preliminary design of the composite primary structures for the civil construction. Such structures generally require a large number of laminae layers. This concept greatly simplifies the calculation effort at the early stage of design because

A. The classical mechanics and elasticity theories can be used.

B. There is no coupling between the bending and the mid-plane extension reducing the three simultaneous fourth order partial differential equations to one fourth order partial differential equation.

C. At the preliminary design stage, the orientations of laminae in a laminate are not known. This fact discourages the most of the engineers from the beginning. Use of the quasi-isotropic constants gives a guide-line toward a simple and accurate analysis.

Simple equations which can predict "exact" values of the buckling strength, the natural frequency of vibration, and the deflection for the special orthotropic laminates are presented. Many laminates with certain orientations have decreasing values of B_{16} and B_{26} as the number of plies increases. Such laminates, with $D_{16}=D_{26}\rightarrow 0$, including the laminates with anti-symmetric configurations can be solved by the same equation for the special orthotropic laminates. If the quasi-isotropic constants are used, the equations for the isotropic plates can be used. Use of some coefficients can produce "exact" value for laminates with such configuration. The most of the structures for civil construction require many layers of plies even though the ratio of the thickness to the length is small so that the effect of transverse shear deformation can be neglected.

2. Deflection of Laminated Plates

A. Deflection of Rectangular Specially Orthotropic Laminated Plates.

If all four edges are simply supported, the expression for the lateral deflection can be obtained as

$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (1)$$

where

$$w_{mn} = \frac{q_{mn}}{\pi^4 \cdot DEN4} \quad (2)$$

in which

$$DEN4 = D_1\left(\frac{m}{a}\right)^4 + 2\,D_3\left(\frac{m}{a}\right)^2\left(\frac{n}{a}\right)^2 + D_2\left(\frac{n}{b}\right)^4 \quad (3)$$

$$D_3 = D_{12} + 2D_{66}, \text{ and}$$

$$q_{mn} = \frac{4}{ab}\int_0^a\int_0^b q(x,y)\,\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy \quad (4)$$

in which $q(x,y)$ is the applied lateral loading.

B. Rectangular Antisymmetric Angle-Ply Laminated Plates.

For Laminates with such orientation,

$A_{16} = A_{26} = B_{11} = B_{22} = B_{66} = D_{16} = D_{26} = 0$.

The equation for the lateral deflection given by Ashton and Whitney (7-36) is

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\,\sin\frac{m\pi x}{a}\,\sin\frac{n\pi y}{b} \quad (5)$$

where

$$w_{mn} = \frac{q_{mn}R^4b^4}{\pi^4 Den}[(A_{11}m^2+A_{66}n^2R^2)(A_{66}m^2+A_{22}n^2R^2) -(A_{12}+A_{66})^2m^2n^2R^2] \quad (6)$$

where

$R = a/b$ and

$$Den. = [(A_{11}\,m^2+A_{66}\,n^2R^2)(A_{66}m^2+A_{22}n^2R^2) - (A_{12}+A_{66})^2m^2n^2R^2]\cdot[D_{11}m^4+2(D_{12}+2D_{66})\cdot m^2n\,R^2+D_{22}n^4R^4] + 2m^2n^2\,R^2(A_{12}+A_{66})\cdot(3B_{16}m^2+B_{26}n^2R^2)(B_{16}m^2+3B_{26}n^2R^2) - n^2R^2(A_{66}m^2+A_{22}n^2R^2)\cdot(3B_{16}m^2+B_{26}n^2R^2)^2 - m^2(A_{11}\,m^2+A_{66}\,n^2\,R^2)\cdot(B_{16}\,m^2+3B_{26}\,n^2R^2)^2 \quad (7)$$

If $B_{16} = B_{26} = 0$, we have $u^o = v^o = 0$, and Eqn(6) is exactly same as Eqn(2).

C. Some Orientations with $B_{16} = B_{26} \rightarrow 0$.

For Angle-Ply laminates,

$(B_{16},B_{26}) = \Sigma\ (\bar{Q}_{16},\bar{Q}_{26})_k(h_k^2 - h_{k-1}^2)$ and

$$(\bar{Q}_{16})_{-\theta} = -(\bar{Q}_{16})$$

$$(\bar{Q}_{26})_{-\theta} = -(\bar{Q}_{26})$$

Therefore, as the number of layers is increased, $B_{16} \rightarrow 0$ and $B_{26} \rightarrow 0$.

Some other laminates with certain orientations will have similar situation with $D_{16} = D_{26} \rightarrow 0$. For such cases, the deflection equation is same as the case of the special orthotropic laminates.

D. DEN, The Denominator.

Consider DEN4 in Eqn(3).

$$DEN4 = D_1\left(\frac{m}{a}\right)^4 + 2\,D_3\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_2\left(\frac{n}{b}\right)^4$$

$$= \left(\frac{m}{a}\right)^4[D_1 + 2\,D_3\left(\frac{na}{mb}\right)^2 + D_2\left(\frac{na}{mb}\right)^4\ ]$$

$$= \left(\frac{m}{a}\right)^4[D_1+2D_3+D_2+2D_3(r^2-1)+D_2(r^4-1)]$$

$$= \left(\frac{m}{a}\right)^4\cdot\ DENGN \quad (8)$$

in which $r = \frac{na}{mb}$

and $DENGN = DEN4/(m/a)^4$ (10)

When the normalized stiffnesses are used,

$$DENNM = (12/h^3)DENGN$$
$$= D_1^*+2D_3^*+D_2^* + 2D_3^*(r^2-1)+D_2^*(r^4-1) \quad (11)$$

where $D^* = 12D/h^3$

Consider the i-th ply

$$D_{1,i}^* = \frac{h_i^3}{12}\,\frac{12}{h_i^3}\,[U_1+U_2\cos(2\theta_i)+U_3\cos(4\theta_i)]$$

$$D_{2,i}^* = U_1 - U_2\cos(2\theta_i) + U_3\cos(4\theta_i)$$

$$2D_{3,i}^* = 2[U_1-3U_3\cos(4\theta_i)]$$

where U_i's are as given by Eqns(5-43) and (5-44) of Ref(1).

Hence

$$D_{1,i}^*+2D_{3,i}^* + D_{2,i}^* = 4[U_1 - U_3\cos(4\theta_i)] \quad (12)$$

$$(DENNM)_i = 4[U_1-U_3\cos(4\theta_i)] + 2[U_1-3U_3\cos(4\theta_i)(r^2-1)] + [U_1-U_2\cos(2\theta_i)+U_3\cos(4\theta_i)](r^4-1) \quad (13)$$

If the ply thicknesses and material properties are equal, we can have

$$DENGN = (h^3/12)\cdot\ 4[U_1 - U_3CTH4) + 2(U_1-3U_3CTH4)(r^2-1) + (U_1-U_2CTH2 + U_3CTH4)(r^4-1)]$$

where

$$CTH2 = \left(\frac{h_o}{h}\right)^3\sum_{i=1}^{N}\cos(2\theta_i)[i^3-(i-1)^3)] \quad (14)$$

$$CTH4 = \left(\frac{h_o}{h}\right)^3\sum_{i=1}^{N}\cos(4\theta_i)[i^3-(i-1)^3)] \quad (15)$$

If $r=1$,

$$DENR1 = 4(U_1 - U_3CTH4) \quad (16)$$

In case of the special orthotropic laminates,

$$DENHR1 = 4(U_1-U_3) \quad (17)$$

E. Use of the Quasi-Isotropic Constants.

The quasi-isotropic constants, by Tsai, are

$$[Q]^{iso} = \begin{vmatrix} U_1 & U_4 & 0 \\ U_4 & U_1 & 0 \\ 0 & 0 & U_5 \end{vmatrix} \quad (18)$$

When the quasi-isotropic constants are used,

$$D_{11} = D_{22} = D_{12} + 2D_{66} = D_3 = (h^3/12)Q_{11}^{iso} \quad (19)$$

and the Eqn(3) reduces to

$$DEN4^{iso} = D_{11}[\left(\frac{m}{a}\right)^4 + 2\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + \left(\frac{n}{b}\right)^4\ ] \quad (20)$$

which is identical to the case of an isotropic plate

Eqn(8) can be written as

$$DEN4^{iso}=(\frac{m}{a})^4 \frac{h^3}{12} U_1[4 + 2(r^2-1) + (r^4-1)] \quad (21)$$

If r = 1

$$DEN4^{iso} = (\frac{m}{a})^4 \frac{h^3}{12} (4U_1)$$

One may obtain the initial deflection for the preliminary design by the use of Eqns(1), (2), and(21).

F. Use of Correction Factor to Obtain the "Exact" Solution.

When thick laminates are used for civil construction, considerable number of orientations will have rapidly decreasing quantities of B_{ij} as the number of layers increases for which cases Eqn(2) can be used with good accuracy. For such cases one may start use to Eqn(21). Relatively "exact" value can be obtained from the preliminary design stage by the use of the formulas proposed as follows.

With

$$w_{mn}^{iso} = \frac{q_{mn}}{\pi^4 DEN4^{iso}} \quad (22)$$

we define

$$w_{mn} = w_{mn}^{iso}/FRC^2 \quad (23)$$

and $MDEN4^{iso} = DEN4^{iso} \times (a/m)^4$

where

$$FRC(1)^2 = \frac{[D_1+2D_3+D_2+2D_3(r^2-1)+D_2(r^4-1)]}{(h^3/12)U_1[4 + 2(r^2-1) + (r^4 -1)]} \quad (24)$$

$$= \frac{DENGN}{MDEN4^{iso}}$$

$$FRC(2)^2=\frac{[D_1^*+2D_3^*+D_2^*+2D_3^*(r^2-1)+D_2^*(r^4-1)]}{U_1[4+2(r^2-1)+(r^4-1)]} \quad (25)$$

$$FRC(3)^2 = [4(U_1-U_3CTH4) + 2(U_1-3U_3CTH4) \cdot(r^2-1)+(U_1- U_2CTH2 + U_3CTH4)(r^4-1)] \times 1/\{U_1[4+2(r^2-1)+(r^4-1)]\} \quad (26)$$

At the preliminary design stage, the orientation of each ply is not known. In such case, one may use the invariants only

$$FRC(4)^2 = \frac{[4(U_1-U_3)+2(U_1-3U_3)(r^2-1)+(U_1-U_2+U_3)(r^4-1)]}{U_1[4+2(r^2-1)+(r^4-1)]} \quad (27)$$

G. Design Steps Using the Correction Factors.

1) Decide the material properties and obtain the U_i's.

2) Obtain w_{mn}^{iso} by Eqn(2) and (21) and $w_{mn} = w_{mn}^{iso}/FRC(4)^2$ given by Eqn(27).

3) Proceed to analyse the whole structural system.

4) With the result of 3), design the orientations.

5) Obtain the exact deflection by

$w_{mn} = w_{mn}^{iso}/FRC(1)^2$,

$w_{mn} = w_{mn}^{iso}/FRC(2)^2$, or

$w_{mn} = w_{mn}^{iso}/FRC(3)^2$.

In reality, use of FRC(1), at this stage, may be simpler.

3. Eigenvalue Problems.

The concept developed in the previous article can be extended to the eigenvalue problems of laminated composite primary structures of civil construction as long as $D_{16} = D_{26} = 0$, and $B_{16} \rightarrow 0, B_{26} \rightarrow 0$ as the number of plies increases.

A. Special Orthotropic Rectangular Laminates

The natural frequency of vibration and the critical buckling strength are given as

$$\omega_n^2 = \frac{\pi^4}{\rho} DEN4 \quad (28)$$

$$N_{x_{cr}} = -(\pi a/m)^2 DEN4 \quad (29)$$

B. Rectangular Antisymmetric Angle-Ply Laminated Plates

Consider Eqn(7-113) and (7-86) of Ref(1)which can be expressed as

$$\omega_n^2 = \frac{1}{\rho} T123 \quad (30)$$

$$N_{x_{cr}} = -(a/m\pi)^2 T123 \quad (31)$$

where

$$T_{123} = T_{33} + \frac{2T_{12}T_{23}T_{13} - T_{22}T_{13}^2 - T_{11}T_{23}^2}{T_{11}T_{22} - T_{12}^2} \quad (32)$$

in which

$$T_{11} = A_{11} (\frac{m\pi}{a})^2 + A_{66}(\frac{n\pi}{b})^2$$

$$T_{12} = (A_{12} + A_{66})(\frac{m\pi}{a})(\frac{n\pi}{b})$$

$$T_{13} = - [3B_{16}(\frac{m\pi}{a})^2 + B_{26} (\frac{n\pi}{b})^2] (\frac{n\pi}{b})$$

$$T_{22} = A_{22}(\frac{n\pi}{b})^2 + A_{66}(\frac{m\pi}{a})^2$$

$$T_{23} = - [B_{16}(\frac{m\pi}{a})^2 + 3B_{26} (\frac{n\pi}{b})^2](\frac{m\pi}{a})$$

$$T_{33} = D_{11} (\frac{m\pi}{a})^4 + 2(D_{12} + 2D_{66}) (\frac{m\pi}{a})^2(\frac{n\pi}{b})^2 + D_{22}(\frac{n\pi}{a})^4$$

$$= \pi^4(DEN4) \quad (33)$$

C. Laminats with some orientations may have $D_{16} = D_{26} = 0$, and $B_{16} \rightarrow 0$ and $B_{26} \rightarrow 0$ as the number of plies are increased. For such cases, the solutions for the eigenvalue problems will be similar to the case of the special orthotropic claminates.

D. The DEN4 terms are same as the case of deflection.

E. The quasi-isotropic constants given by Eqn(18) are used. $DEN4^{iso}$ is as given by Eqn(20), (21), and(22). One may obtain the initial values by

$$\omega_n^2 = \frac{\pi^4}{\rho} DEN4^{iso} \quad (34)$$

$$N_{x_{cr}} = -(\pi a/m)^2 DEN4^{iso} \quad (35)$$

Eqn(21) can be used for $DEN4^{iso}$.

F. Use of Correction Factor to Obtain the "exact" Solutions.

With (ω_n^{iso}) and Nx_{cr}^{iso} defined by Eqns(34) and (35), we define the "exact" solutions as

$$(\omega_n)^2 = (\omega_n^{iso})^2 \cdot FRC^2 \quad (36)$$

$$Nx_{cr} = Nx_{cr}^{iso} \cdot FRC^2 \quad (36)$$

The FRC's defined in the previous article are used for the eigenvalue problems also.

G. Design steps using "exact" values from the preliminary stage are the same except that the $(\omega_n^{iso})^2$ and Nx_{cr}^{iso} are multiplied by FRC^2. In case of deflection, w_{mn}^{iso}, is devided by FRC^2.

4. Numerical Example

The material properties used are as follows.

E_1 = 38.6 GPa

E_2 = 8.27 GPa

ν_{12} = 0.26

ν_{21} = 0.0557

h_o =0.00125 m

G_{12} = 4.14 GPa

The results of calculation are as shown in Table (1) to (4)

CONCLUSION

Designing with composite materials is very much complicated. Not only the anisotropy of materials but the several kinds of coupling terms also make the analysis so much messed up. Even after obtaining the stress and strain for each ply,the strength theory involves six equations for each ply. Thus,if both maximum stress and strain theories are used,each ply requires 12 equations. These has to be done for each lamina and a laminate with N laminae requires 12N equations for failure conditions. Use of quadratic equations especially for the strain space can reduce the number of equations significantly but use of "exact" theory for the preliminary design of large scale composite structures is too difficult if not impossible. D.H.Kim proposed in his recent book[1] to use quasi-isotropic constants for the preliminary design.

This report proves that his concept is good for the deflections and eigenvalue problems of the special orthotropic and antisymmetric angle ply laminates for which closed form solutions are available. This concept is good for many other laminates with different orientations especially when the increase of the number of laminae reduces the bending-extension coupling terms. The most of the structures used in civil construction are large in sizes and such structures require large number of plies even though the ratios of the thicknesses to the lengths are small so that the effects of shear deformation can be neglected.

It should be borne in mind that the "exact" analysis is the "must" at the stage of final analysis. The maximun strains at failure of the most of the composites are much smaller than those of metals. In case of advanced carbon fiber reinforcements, maximum elongation to failure reported to date is only 2%.

The use of formulas, proposed by D.H. Kim , to modify the result by the simple method, to obtain "exact" solution is proved to be effective.

Table 1. [$\pm\theta$] θ = $\pm 15^\circ$ (Antisymmetry Angle-Ply)

r(N)	3(6)	6(12)	9(18)	12(24)	15(30)	21(42)	27(54)
$\omega_n^{(exact)}$	501	1430	2632	4055	5668	9392	13694
$\omega_n^{(orth)}$	507	1434	2635	4058	5671	9395	13696
$\omega_n^{(iso)}$	529	1497	2750	4234	5917	9802	14290
B_{16}^*/D_{11}^*	0.0316	0.0158	0.0104	0.0079	0.0062	0.0045	0.0033
B_{26}^*/D_{11}^*	0.0045	0.0023	0.0015	0.0011	0.0009	0.0006	0.00048
$FRC^2(1)$	0.9149	0.9149	0.9149	0.9149	0.9149	0.9149	0.9149
$FRC^2(2)$	0.9582	0.9582	0.9582	0.9582	0.9582	0.9582	0.9582
$\omega_n^{iso}\cdot FRC\ (2)$	506	1434	2635	4057	5669	9392	13692
$\omega_n^{iso}\cdot FRC\ (1)$	484	1369	2516	3874	5413	8968	13074
$\frac{\omega_n^{(exact)}}{\omega_n^{(orth)}}$	0.998	0.999	0.999	0.999	0.999	0.999	0.999
$\frac{\omega_n^{(exact)}}{\omega_n^{(iso)}}$	0.956	0.958	0.958	0.958	0.958	0.958	0.958

Table 2. [ABBAAB] A=15° B=−15° (Antisymmetry Angle-Ply)

r(N)	3(6)	6(12)	9(18)	12(24)	15(30)	21(42)	27(54)
$\omega_n^{(exact)}$	506	1434	2635	4057	5671	9394	13696
$\omega_n^{(orth)}$	507	1434	2635	4058	5671	9395	13696
$\omega_n^{(iso)}$	529	1497	2750	4234	5917	9802	14290
B_{16}^*/D_{11}^*	0.0316	0.0158	0.0104	0.0079	0.0062	0.0045	0.0033
B_{26}^*/D_{11}^*	0.0045	0.0023	0.0015	0.0011	0.0009	0.0006	0.00048
$FRC^2(1)$	0.9149	0.9149	0.9149	0.9149	0.9149	0.9149	0.9149
$FRC^2(2)$	0.9582	0.9582	0.9582	0.9582	0.9582	0.9582	0.9582
$\omega_n^{iso}\cdot FRC\ (2)$	506	1434	2635	4057	5669	9393	13693
$\omega_n^{iso}\cdot FRC\ (1)$	484	1369	2516	3874	5413	8968	13074
$\frac{\omega_n^{(exact)}}{\omega_n^{(orth)}}$	0.998	0.999	0.999	0.999	0.999	0.999	0.999
$\frac{\omega_n^{(exact)}}{\omega_n^{(iso)}}$	0.956	0.958	0.958	0.958	0.958	0.958	0.958

FRC(1) : By Eqn(26) FRC(2) : By Eqn(24)

Table 3. [$\pm\theta$] θ = $\pm 15^\circ$ (Antisymmetry Angle-Ply)

r(N)	3(6)	6(12)	9(18)	12(24)	15(30)	21(42)	27(54)
$Nx_{cr}^{(exact)}$	-25707	-207848	-702858	-1667169	-3257215	-8940249	-19003430
$Nx_{cr}^{(orth)}$	-26072	-208578	-703952	-1668628	-3259040	-8942803	-19006710
$Nx_{cr}^{(iso)}$	-28382	-227062	-766335	-1816493	-3547847	-9735293	-20691040
B_{16}^*/D_{11}^*	0.0316	0.0158	0.0104	0.0079	0.0062	0.0045	0.0033
B_{26}^*/D_{11}^*	0.0045	0.0023	0.0015	0.0011	0.0009	0.0006	0.00048
$FRC^2(1)$	0.8372	0.8372	0.8372	0.8372	0.8372	0.8372	0.8372
$FRC^2(2)$	0.9181	0.9181	0.9181	0.9181	0.9181	0.9181	0.9181
$Nx_{cr}^{iso}\cdot FRC^2(2)$	-26057	-208465	-703572	-1667726	-3257278	-8937972	-18996443
$Nx_{cr}^{iso}\cdot FRC^2(1)$	-23761	-190096	-641575	-1520772	-2970257	-8150389	-17322538
$\frac{Nx_{cr}^{(exact)}}{Nx_{cr}^{(orth)}}$	0.986	0.996	0.998	0.999	0.999	0.999	0.999
$\frac{Nx_{cr}^{(exact)}}{Nx_{cr}^{(iso)}}$	0.901	0.915	0.917	0.918	0.918	0.918	0.918

Table 4. [ABBAAB] A=15° B=−15° (Antisymmetry Angle-Ply)

r(N)	3(6)	6(12)	9(18)	12(24)	15(30)	21(42)	27(54)
$Nx_{cr}^{(exact)}$	-26031	-208431	-703831	-1668466	-3258837	-8942519	-19006350
$Nx_{cr}^{(orth)}$	-26072	-208579	-703953	-1668628	-3259040	-8942803	-19006710
$Nx_{cr}^{(iso)}$	-28382	-227062	-766335	-1816493	-3547847	-9735293	-20691040
B_{16}^*/D_{11}^*	0.0104	0.005	0.0033	0.0026	0.0021	0.0015	0.0011
B_{26}^*/D_{11}^*	0.0015	0.00073	0.00048	0.00036	0.0003	0.00021	0.00016
$FRC^2(1)$	0.8372	0.8372	0.8372	0.8372	0.8372	0.8372	0.8372
$FRC^2(2)$	0.9181	0.9181	0.9181	0.9181	0.9181	0.9181	0.9181
$Nx_{cr}^{iso}\cdot FRC^2(2)$	-26057	-208465	-703572	-1667726	-3257278	-8937972	-18996443
$Nx_{cr}^{iso}\cdot FRC^2(1)$	-23761	-190096	-641575	-1520772	-2970257	-8150389	-17322538
$\frac{Nx_{cr}^{(exact)}}{Nx_{cr}^{(orth)}}$	0.998	0.999	0.999	0.999	0.999	0.999	0.999
$\frac{Nx_{cr}^{(exact)}}{Nx_{cr}^{(iso)}}$	0.917	0.918	0.918	0.919	0.919	0.919	0.919

FRC(1) : By Eqn(26)
FRC(2) : By Eqn(24)

REFERENCES

1. Kim,D.H., Composite Structures for Civil and Architectural Engineers,Elsevier Science Publishers Ltd,to be published Dec.,1991.

2. Verchery,G.,et al.,"A Quantitative Study of the Influence of Anisotropy on the Bending Deformation of Laminates",Proc.of 8th International Conference of Composite Materials,July,1991.

3. Kim,D.H., "A Method of Vibration Analysis of Irregulary Shaped Structural Elements", International Symposium on Engineering Problem in Creating Industrial Sites, Seoul, Korea, 1974.

4. Kim, D.H., Hwang, J.W., Chun, D.S., "A Simple Method of Vibration Analysis of Irregularly Shaped Composite Structural Elements", 1st Japan Int'l SAMPE Symposium, Nov. 28-Dec.1, 1989.

5. Kim, D.H., Hwang, J.W., Chun, D.S.,"Vibration Analysis of Irregularly Shaped Composite Structural Members-For Higher Medes", 8th Structural Congress, American Society of Civil Engineers, Baltimore, MD, U.S.A., Apr.30-Mar.3, 1990.

6. Kim,D.H., "Vibration Analysis of Irregularly Shaped Laminated Thick Composite Plates", ICCM 8, Honolulu, Hawaii, July, 1991.

7. Kim,D.H.,Park,J.S.,Kim,K.J., "Vibration Analysis of Irregularly Shaped Laminated Thick Composite Plates(Ⅱ)", JISSE 2, Dec.,1991.