An Optimal Dimensioning of ISDN and Simulation

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When various types of traffic streams are offered to a common trunk group, we develop an optimal dimensioning procedure using the alternate routing principle and verify our model by a simulation model. Our method reduces the total network cost by about 10%, compared with the full-mesh system.

1. Introduction

When various types of traffic streams are offered to a common trunk group the dimensioning problem is quite different from that of usual telephone network. In the case of usual telephone network, the traffic offered to a primary high usage route(PHU) is random and the mean and variance of overflow can be calculated exactly. Approximations are used in the intermediate high routes(IHU) and the final routes(FR). However, in the heterogeneous traffic stream case, the traffic offered to a PHU is peaked due to the superpositions, so that the load carried by the last trunk(LLT) which is the key quantity for optimizing the network cannot be computed by the 'Erlang loss formula' and also it seems to be impossible to calculate the peakness factor of the overflow exactly.

In this paper, the authors examine various types of approximations for the peakness factor of overflow from an arbitrary input process and the blocking probability for the G/M/s/s queueing system, and discuss how the classical dimensioning method, known as the 'cost ratio method' can be applied to the network with heterogeneous traffic streams. Next we develop an appropriate dimensioning procedure for the multiservice network using present circuit switched network. Finally we construct a simulation model using SIMAN to verify our dimensioning procedure. Some verification step is necessary since approximations are used in order to estimate key quantities which are difficult to obtain analytically.

2. Calculation of the load carried by the last trunk

Suppose that r types of traffic streams are offered to a common trunk group with s trunks and the i-th type has Poisson arrival rate λ_i and exponential service rate μ_i and requires d_i slots per call. Furthermore assume that the r input processes are stochastically independent. Then the mean and variance of the total traffic are given by[3,4]

$$A = \sum_{i=1}^{r} d_i a_i$$

$$V = \sum_{i=1}^{r} d_i^2 a_i$$

where $a_i = \lambda_i / \mu_i$. Therefore the total offered traffic is peaked unless $d_i = 1$ for all i.

The blocking probability of the i-th type, $B_i(s)$, is given by

$$B_i(s) = \sum_{n=s-d_i+1}^{s} Q(n) \quad \text{, where}$$

$$Q(n) = (1/G) \sum_{\sum k_i d_i = n} \prod_{i=1}^{r} (a_i^{k_i} / k_i!)$$

and G is the normalizing constant. Let $B^*(s,A)$ be the blocking probability when the number of trunks is s and the total offered traffic is A. If A is random, then $B^*(s,A)$ is the Erlang loss formula. In the case of our model, $B^*(s,A)$ is given by

$$B^*(s,A) = [\, \sum_{i=1}^{r} d_i a_i B_i(s) \,] / A$$

The load carried by the n-th trunk(l_n) in PHU can be calculated by using $B^*(s, A)$[2]:

$$l_n = A\ [\ B^*(n-1, A) - B^*(n, A)\ ].$$

The mean overflow traffic m is given by

$$m = A\ B^*(s, A).$$

It seems impossible to calculate the exact variance of the overflow traffic even though we can write the multidimensional birth-and-death recursive formula for the state probabilities

3. Approximations for the peakness factor and blocking probability for general input process

The peakness factor is another key quantity for the cost ratio method. In the usual telephone network, to estimate the peakness factor of the overflow, Wilkinson's equivalent random method is widely used. However there is a tendency to overestimate the number of trunks when the overflow is from non-Poisson inputs.

In this section, we introduce another approximation of the peakness factor for the overflow from peaked input traffic, based on the equivalent congestion model and in the subsequent sections, some dimensioning procedure and simulation models are considered to determine and verify which of the approximation models is suitable for the optimal dimensioning of a network with heterogeneous input traffic streams. In the Frederics' paper[5], Hayward-type approximation and its improvments are discussed. The peakness factor z_0 of the overflow traffic from the s server group which is offered A erlangs with peakness factor z can be approximated by

$$z_0 = z\ [1 - (A/z)\ E(s/z, A/z) + A\ /\ (s + z - A')\ ]$$

where A' is the total carried load on the s server group and $E(\cdot, \cdot)$ is the Erlang loss formula. The calculation of Erlang loss formula for the non-integer servers is performed by the interpolations. If we use the equivalent random method, z_0 can be approximated by

$$z_0 = 1 + A_e' - A_e + A_e\ /\ (\ N_e + s + 1 - A_e\ )$$

where A_e = equivalent random traffic in erlangs
N_e = imaginary number of servers
$A_e' = A_e\ [\ 1 - E(\ N_e + N\ ,\ A_e\ )\ ]$.

As mentioned earlier, the equivalent random method can be applied in the case that the non-random traffic is the sum of overflows from the independent Poisson inputs, which is called the simple overflow process(SOP). Notice that in the Hayward-type approximation there is no restrictions on the characteristic of the non-random input A.

Next, we consider the blocking probabilty for the G/M/s/s queueing system. Assuming that the arrival process is characterized by offered load A and the peakness factor z, a generalized Hayward-type approximation for the blocking probability, $B_H(s, A)$, is given by

$$B_H(s, A) = E(s/z, A/z).$$

See reference[5].

4. Optimal Dimensioning Procedure

There have been some researches on the dimensioning of multi-slot traffic system[7,8]. However they are focused mainly on the dimensioning of direct route(DR). Therefore their results can be applied only to the full-mesh network structure, which is inefficient.

In this section we develop a procedure for the optimal dimensioning of a network with heterogeneous traffic streams using the alternate routing principle. For simplicity, we consider the terminating-sector tandem rule shown in the Fig.1

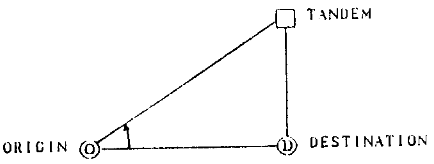

Fig.1 Terminating-sector tandem rule

Therefore there are 2 types of routes in our model, PHU and FR. In the next subsections, the dimensioning methods for DR, PHU, and FR are discussed.

4-1. Dimensioning of DR(Direct Route)

We consider DR in order to compare full-mesh with alternate routing.

Given total offered load A and the grade of service(GOS) the number N of trunks in DR is computed by the following equation:

$$B^*(N, A) = GOS.$$

4-2. Dimensioning of PHU(Primary High usage Route)

In the derivation of the optimizing equations for the high usage routes, the characteristic of offered traffic does not matter. Therefore the classical cost ratio method can be applied as follows.
Given offered load A with peakness z and cost ratio C_R, the number N of trunks in PHU satisfies the following equation:

$$\ell_{N+1} = A\ [\ B^{*}(N,A) - B^{*}(N+1,A)\ ] = \beta\ /\ C_R$$

where β is the marginal capacity[6]. In the classical cost ratio method, Y. Rapp's approximation for β is given by $\beta = 1 - 0.3(\ 1 - 1\ /\ C_R^2\ )$ and $\beta = 0.778$ erlangs in the ECCS method. The overflow m from PHU is given by $m = A\ B^{*}(N,A)$ and the variance of overflow v is given by $v = m\ z_0$, where

$$z_0 = z[1-(A/z)E(N/z,A/z)+A/(N+z-(A-m))]$$

Notice that we use the Hayward-type approximation to obtain the variance of overflow from PHU and that if
z=1
then our method for PHU coincides with the calssical cost ratio method.

4-3. Dimensioning of FR(Final Route)

Let M and V be the total offered load and variance respectively. We can not use $B^{*}(\cdot,M)$ to dimension FR subject to a given GOS as in DR since overflows from various PHU are superposed in FR. Therefore we use an approximation $B_H(\cdot,\cdot)$ instead of $B^{*}(\cdot,\cdot)$ discussed in section 3 to overcome this difficulty.
Given the total offered load M with peakness z and GOS, the number N of trunks in FR satisfies

$$B_H(N,M) = E(N/z,M/z) = GOS.$$

The drawback of this method is that we can not estimate the blocking probability of each type, which must be balanced using the concept of trunk reservation[8].

5. Numerical Example

We compare the optimal dimensioning procedure developed in the section 4 with the full-mesh structure by a numerical example. We consider the network in the Fig.2. The number on each link represents the distance between exchanges.

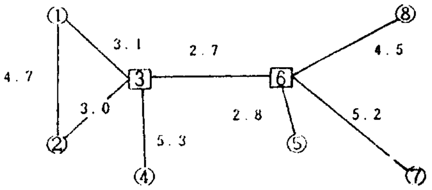

Fig.2 Network for a numerical example

In this example, the number of exchanges is 8, including 2 tandems, numbered by 3 and 6. The sons of each tandem are determined by shortest distances. The exchanges 1, 2, 4 and exchanges 5, 7, 8 are the sons of tandem 3 and tandem 6, respectively. We assume that there are 3 types of traffic streams; 1 slot, 6 slots, and 12 slots. The offered traffic for each type is given in the Table 1. of the appendix.

The proportion of each type of traffic streams in the Table 1 is $A_1 : 6A_2 : 12A_3 = 69.7 : 18.6 : 11.7$, where A_i is total traffic of type $i = 1, 2, 3$, and the overall total traffic is $A_1 + 6A_2 + 12A_3 = 2122.24$ erl.

We use Y. Rapp's approximation for the marginal efficiency β and the values of the cost ratio C_R vary between 1.70 and 1.97.

The trunk matrices for two methods are given in the Table 2 and Table 3, and the summary of calculations is shown in the Table 4 below.

Table 4 The summary of calculations

Methods	# of trunks	Load/trunk	Total cost
Full-mesh	3821	0.56	1
Alt. route	3462	0.61	0.88

Notice that our method developed in the section 4 reduces the total cost by 12%, compared with the full-mesh structure.

6. Simulation using SIMAN

We verify our dimensioning method by a simulation model. As mentioned in the introduction and the section 4, approximations are used in FR to obtain total offered traffic M and total variance V and to calculate the blocking probability distribution for the input process characterized by M and V.

6-1. Simulation model description

We construct the simulation model under the following conditions:

- 3 types of traffic streams (type 1, type 2, type 3)
- arrival rates of type 1, type 2, and type 3 traffic streams are λ_1, λ_2, and λ_3, respectively
- arrival rate of total offered traffic is $\lambda(=\lambda_1+\lambda_2+\lambda_3)$
- numbers of slots of type 1, type 2, and type 3 traffic streams are 1, 6, and 12, respectively
- basic triangle network segment (see Fig. 1) is considered

The traffic amounts of type 1, type 2, and type 3 traffic streams are generated at the rates of λ_1/λ, λ_2/λ, and λ_3/λ, respectively during the unit time (1 hr.). These traffic amounts are offered to a common trunk group whose number of trunks is calculated by our dimensioning method.

6-2. Results of simulation

We simulate many numerical examples varing the proportion of traffic amounts of type 1, type 2, and type 3. As a result, the average blocking probability of the whole network is less than 0.01. It seems that our dimensioning method has the tendency to overestimate the number of trunks. Therefore if we consider the average GOS only, our dimensioning method can be applied to real situations.

7. Conclusions and Comments

We construct a large number of examples, varying the proportions of each type, for the comparison of our method with the full-mesh method, which show that our method reduces the total network cost by about 10%. We also try to find a new ECCS for our model, but the result is not significantly different from the old one, which shows that the marginal capacity β is not seriously affected by the characteristic of input traffic.

Throughout the development of dimensioning method for heterogeneous traffic streams using the concept of the cost ratio, we considered the average GOS only, as the first attempt to dimension an ISDN optimally. However, if a network is dimensioned subject to the average GOS only, then the blocking probabilities for various types of traffic streams are unbalanced. Therefore the new concepts of GOS, or some trunk reservation policy must be taken into account to overcome this problem[7,8]. Also further research is required to improve the method of dimensioning FR ,which enables us to estimate the blocking probability of each traffic stream.

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[APPENDIX]

Table 1 The offered traffic for each type(erl)

TYPE 1

0.00	22.46	17.31	16.38	33.48	24.82	21.77	6.44
35.55	0.00	15.27	13.11	18.30	25.56	23.12	16.20
9.03	9.56	0.00	26.03	7.66	27.83	4.44	25.76
6.62	42.59	12.51	0.00	29.38	25.14	20.34	14.79
7.45	28.23	22.59	16.27	20.14	13.11	43.75	76.41
14.26	9.57	47.36	27.42	14.88	0.00	21.10	10.42
56.55	22.77	37.42	76.86	23.41	29.47	0.00	26.59
33.66	24.87	46.71	56.73	52.12	49.56	37.58	0.00

TYPE 2

0.00	1.36	2.31	2.38	1.48	0.82	1.77	0.44
1.55	0.00	0.27	1.11	1.30	1.56	1.12	0.20
1.03	1.02	0.00	1.03	1.10	1.07	1.07	1.14
2.45	2.00	1.58	0.00	2.14	1.19	0.34	4.79
0.48	0.48	0.15	1.25	0.97	1.01	0.75	0.41
0.26	0.57	1.36	1.02	1.08	0.00	1.10	0.42
1.02	0.77	1.20	1.86	1.41	0.47	0.00	1.13
0.66	0.87	1.71	1.01	1.12	1.56	1.07	0.00

TYPE 3

0.00	0.05	0.05	0.07	0.04	0.07	0.21	0.81
0.15	0.00	0.07	0.25	0.48	0.47	0.12	0.20
1.03	0.01	0.00	0.03	0.66	0.84	0.44	1.04
0.04	0.05	0.01	0.00	0.04	0.34	0.27	0.25
0.15	0.14	0.91	0.14	0.00	0.18	0.25	0.14
0.89	0.94	0.58	0.48	0.14	0.00	0.40	0.14
0.54	0.47	0.54	0.25	0.08	0.58	0.00	0.58
0.58	0.87	0.89	0.48	0.25	0.48	0.58	0.00

Table 2 The trunk matrix for the full-mesh

0	55	60	61	66	52	65	60
73	0	37	52	66	74	56	44
72	37	0	54	61	82	52	87
52	81	46	0	69	67	52	86
37	55	73	52	0	49	77	102
66	66	96	72	49	0	64	38
101	65	84	120	58	72	0	75
77	79	106	99	89	97	84	0

Table 3 The trunk matrix for the altrenate route

0	36	73	35	47	91	38	8
52	0	54	24	35	95	33	19
117	98	0	110	25	120	5	50
25	60	64	0	46	92	24	57
10	35	87	27	0	61	55	86
29	26	126	43	[illegible]	0	122	108
73	34	113	96	36	86	0	43
47	43	132	76	67	112	55	0