Data Structures and Sparse Matrix Techniques for the Implementation of Projector-Based Optimization Algorithms

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ABSTRACT

The goal of our research is to incorporate recent advances in the efficient solution of large, sparse linear systems in implementations of projector-based optimization algorithms. All such algorithms have, as thier main computational requirement, the solution of a sequence of symmetric positive definite systems of linear equations having coefficient matrix, AD^2A^T, where A is randomly sparse and D is diagonal.

Quoting the 1989 SIAM article by M.H. Wright: "more than $100 million in human and computer time is invested yearly in the formation and solution of linear programming(LP) problems. " Two new LP algorithms based on dynamic projection over constraints have been recently introduced: one is the (Karmarkar) Affine Variant Algorithm and the other is the Proximity Map (Exterior Point) Algorithm. Researchers are actively engaged in developing versions of both algorithms which are suitable for solving nonlinear problems.

The crucial, time consuming (about 90% in our experience), computation in these new mathematical programming algorithms requires the repeated solution of large, sparse linear systems. In our research, this is done by a Cholesky factorization and sparse matrix techniques incorporated in BCSLIB_EXT (from Boeing Computer Services) are then utilized to reduce execution time and storage requirements in all factorizations. All these implementations are done on CRAY-2S in SERI, and when making a comparison between the implementations with and without sparse treatment, we have obtained dramatic speed gains from the former over the latter.

In numerical experiments utilizing the affine approach it has been observed that, initially, large gains are achieved, followed by relatively slow convergence to the solution. By contrast, when using the Proximity Map approach, it appears that convergence accelerates as the process nears a solution. In addition to adapting vectorized, parallelized sparse matrix techniques for projector-based optimization, our research will seek to determine whether or not a combination (i.e., polyalgorithm) approach is superior to exclusive use of either algorithm.