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A TRACKING FILTER WITH PSEUDO-MEASUREMENTS IN LINE-OF-SIGHT CARTESIAN COORDINATE SYSTEM

TAE KYUNG SUNG JANG GYU LEE

Dept. of Control & Instrumentation Eng.

Seoul National University

ABSTRACT

This paper presents a tracking filter using pseudomeasurements in an estimated line-of-sight Cartesian coordinate system(ELCCS) whose x-axis is on the line-of-sight to an estimated target position. A target dynamics model and a measurement equation in the ELCCS are derived first and then a tracking filter in the ELCCS named moving coordinate tracking filter(MCTF) is proposed. It is shown that this MCTF is equivalent to a Kalman filter in the inertial Cartesian coordinate system which is widely used in the target tracking system. By approximating the MCTF for a pseudomeasurement noise and an error covariance matrix in the ELCCS, decoupling of three axes can be achieved. In this case, named decoupled moving coordinate tracking filter(DMCTF), computation time can be drastically reduced by utilizing its parallel structure. Finally, the stochastic properties of the MCTF and DMCTF are presented. Especially, a sufficient condition of nondestabilizing deviation for the DMCTF is proposed. The performance of the MCTF and DMCTF are compared with a conventional Kalman tracking filter.

1. INTRODUCTION

The target tracking problem, in which the position, velocity and acceleration of a maneuvering target are to be estimated from noisy measurement data, requires a nonlinear filtering since the measurements are collected in a polar coordinate system while target dynamics model is represented in a reference Cartesian coordinate system(RCCS). In real situation, however, a linear filter such as the Kalman filter is commonly employed for target tracking to ease computational burden. The conventional way to use the Kalman filter is to transform polar measurements to pseudomeasurements in the RCCS.[1] In that case, since the covariance matrix for pseudomeasurement noises is not diagonal, each axis is coupled to others and the filter requires a considerably large amount of computations.

Ailada[2,3] proposed a new pseudolinear tracking filter with bearing-only measurement in a new coordinate, *i.e.*, line-of-sight(LOS) coordinate. Generalizing the concept of Ailada to the three dimensional polar measurements, the Song *et al.*[4] proposed a tracking filter using pseudomeasurements in a line-of-sight Cartesian coordinate system(LCCS) to ensure the diagonal covariance matrix for the pseudomeasurement noise. In [4], the LCCS is defined so that the x-axis of the coordinate system is on the line-of-sight to the measured target position and target dynamics and pseudomeasurements in the LCCS are given. Using the pseudomeasurement and target dynamics equations in the LCCS, a new filter gain in the RCCS was derived in terms of an error covariance matrix in the LCCS, system matrices, and the coordinate transformation matrix between RCCS and LCCS. Since the filter equations of the error covariance matrix in the LCCS can be decoupled for each axis, computational burden of the tracking filter proposed by Song *et al.* should be much less than that of the conventional Kalman filter in the RCCS. But the tracking filter proposed by Song *et al.* is an approximation and therefore cannot guarantee its stability.

This paper presents a tracking filter with pseudomeasurements in a new LOS Cartesian coordinate system and it is shown that the proposed filter is completely equivalent to the Kalman filter in the RCCS. This tracking filter, named moving coordinate tracking filter(MCTF), employs the estimated line-of-sight Cartesian coordinate system(ELCCS) instead of the line-of-sight Cartesian coordinate system in [4] to guarantee its unbiasedness.[4-5] The ELCCS is constructed so that the x-axis of the coordinate system is on the line-of-sight to the estimated target position. Similar way to [4], pseudomeasurement and target dynamics equations in the ELCCS are derived. An MCTF in the ELCCS is constructed so that the x-axis of the coordinate system, that is generated at every sampling time, always tracks a target. In other words, from kth estimates in the (k-1)th ELCCS, a kth ELCCS, in which the kth estimates of position of y and z-axis are 0, is created first and then the filter informations in the (k-1)th ELCCS are transformed to the kth ELCCS. It can be shown that the MCTF is equivalent to the Kalman filter in the RCCS.

Under several assumptions, the covariance matrix of pseudomeasurement noises in the ELCCS becomes approximately diagonal and the submatrices of the error covariance matrix also become approximately diagonal through the filter equations. Then, the decoupling of three axes can be achieved, which is named by decoupled moving coordinate tracking filter(DMCTF). The structure of the DMCTF is suitable for parallel implementation and it is shown that the DMCTF is equivalent to the tracking filter proposed by Song *et al.* Because of decoupling into three axes, computations of the DMCTF for one filtering step is much less than that of the Kalman filter in the RCCS. Furthermore, if parallel processing for each axis is adopted, the filtering time can even be more reduced.

Finally, this paper presents stochastic properties of the MCTF and DMCTF. Since pseudomeasurement noises in the RCCS are not Gaussian, the Kalman filter in the RCCS may be suboptimal but stable and unbiased. Therefore, the MCTF is also a stable and unbiased estimator because the MCTF is equivalent to the Kalman filter in the RCCS. The DMCTF is an approximation of the MCTF and in general its stability cannot be guaranteed. In this paper, a theorem for a sufficient condition of nondestabilizing deviation for the DMCTF is proposed. From the condition, it can be said that if initial estimates are fairly accurate, the measurement sampling time is sufficiently small, and the target range is sufficiently long, the estimation errors of the DMCTF will be bounded.

2. MOVING COORDINATE TRACKING FILTER AND DECOUPLED MOVING COORDINATE TRACKING FILTER

2.1 Target Dynamics and Measurement

A target dynamics has to be simply modeled for real-time implementation while it accurately represents the target motion as much as possible. In 1970, Singer[6] proposed a target acceleration model as a Markov first order process with zero-mean and correlation time τ. The model describes three dimensional target motion in a reference Cartesian coordinate system(RCCS) as follows,

$$\dot{\underline{\mathrm{x}}}(t) = A\underline{\mathrm{x}}(t) + G\underline{w}(t) \tag{1}$$

where $\underline{\mathrm{x}}(t) = [x(t)\ y(t)\ z(t)\ \dot{x}(t)\ \dot{y}(t)\ \dot{z}(t)\ \ddot{x}(t)\ \ddot{y}(t)\ \ddot{z}(t)]^T$ is a state vector of three dimensional target position, velocity, and acceleration, $\underline{w}(t)$ is a three element zero-mean white noise vector with covariance $E\{\underline{w}(t)\underline{w}^T(\tau)\} = qI\ \delta(t-\tau)$, $G = [0\ 0\ I]^T$ is a 9×3 process noise gain matrix, and A is a 9×9 system matrix given by 3×3 submatrices as follows,

$$A = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}$$

For implementation in a digital computer, the target dynamics of Eq.(1) should be discretized. The discrete target dynamics for the $(k+1)$th time stage can be written by

$$\underline{\mathrm{x}}(k+1) = F\underline{\mathrm{x}}(k) + \underline{w}(k) \tag{2}$$

where $\underline{w}(k)$ is a 9×1 zero-mean white noise vector with covariance matrix $Q(k)$ described in details in [6], and F is a 9×9 system matrix which consists of 3×3 submatrices with a measurement time interval Δt and a maneuvering time constant τ as follows,

$$F = \begin{bmatrix} I & \Delta t I & \tau^2(-1 + \frac{\Delta t}{\tau} + exp\{-\frac{\Delta t}{\tau}\})I \\ 0 & I & \tau(1 - exp\{-\frac{\Delta t}{\tau}\})I \\ 0 & 0 & exp\{-\frac{\Delta t}{\tau}\}I \end{bmatrix}$$

The nonlinear polar measurements of range, azimuth, and elevation including noises can be written by

$$\begin{aligned} \underline{\mathrm{z}}(k) &= [r_m(k)\ \psi_m(k)\ \theta_m(k)]^T \\ &= \begin{bmatrix} r(k) \\ \psi(k) \\ \theta(k) \end{bmatrix} + \begin{bmatrix} v_r(k) \\ v_\psi(k) \\ v_\theta(k) \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{x^2(k)+y^2(k)+z^2(k)} \\ tan^{-1}(\frac{y(k)}{x(k)}) \\ tan^{-1}(\frac{z(k)}{\sqrt{x^2(k)+y^2(k)}}) \end{bmatrix} + \begin{bmatrix} v_r(k) \\ v_\psi(k) \\ v_\theta(k) \end{bmatrix} \end{aligned} \tag{3}$$

where the measurement noises $v_r(k)$, $v_\psi(k)$, and $v_\theta(k)$ are assumed to be mutually uncorrelated and zero-mean white Gaussian noises with variances $\sigma_r^2(k)$, $\sigma_\psi^2(k)$, and $\sigma_\theta^2(k)$ respectively.

Now, an estimated line-of-sight Cartesian coordinate system(ELCCS) which is constructed using an estimated target position is introduced and the variables represented with respect to the ELCCS will be denoted by subscript L. The x-axis of the ELCCS is toward the estimated target position, y-axis is on the x-y plane of the RCCS and z-axis is determined from the right-hand law. The relationship between the RCCS and ELCCS is shown in Figure 1 and a coordinate transformation matrix $T_3(k)$ from the RCCS to ELCCS is given by

$$T_3(k) = \begin{bmatrix} \cos\hat{\psi}(k)\cos\hat{\theta}(k) & \sin\hat{\psi}(k)\cos\hat{\theta}(k) & \sin\hat{\theta}(k) \\ -\sin\hat{\psi}(k) & \cos\hat{\psi}(k) & 0 \\ -\cos\hat{\psi}(k)\sin\hat{\theta}(k) & -\sin\hat{\psi}(k)\sin\hat{\theta}(k) & \cos\hat{\theta}(k) \end{bmatrix} \tag{4}$$

where $\hat{\psi}(k)$ and $\hat{\theta}(k)$ denote estimated Euler angles between the two coordinate systems obtained from

$$\hat{\psi}(k) = \tan^{-1}(\frac{\hat{y}(k)}{\hat{x}(k)})$$

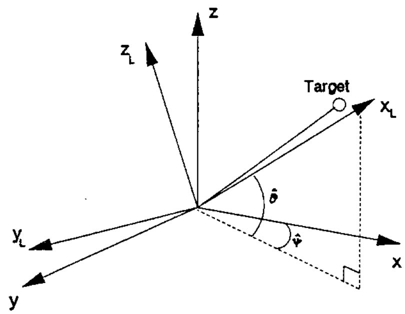


Figure 1: Relationship between the RCCS and ELCCS

$$\hat{\theta}(k) = \tan^{-1}(\frac{\hat{z}(k)}{\sqrt{\hat{x}^2(k)+\hat{y}^2(k)}}) \tag{5}$$

As noticed in [4], the pseudomeasurements in the line-of-sight Cartesian coordinate system that is generated from the true measurements may cause the filter biasedness. In this paper, to prevent this problem, the ELCCS is used that is generated from the estimates to guarantee the filter unbiasedness.[4,5]

The measurements can be transformed to the ELCCS by transforming the polar measurements to the RCCS first and then to the ELCCS as follows,

$$\begin{aligned} \underline{\mathrm{z}}_L(k) &= T_3(k)\underline{\mathrm{z}}_R(k) \\ &= T_3(k)\begin{bmatrix} r_m(k)\cos\psi_m(k)\cos\theta_m(k) \\ r_m(k)\sin\psi_m(k)\cos\theta_m(k) \\ r_m(k)\sin\theta_m(k) \end{bmatrix} \\ &\equiv H\underline{\mathrm{x}}_L(k) + \underline{v}_L(k) \end{aligned} \tag{6}$$

where $\underline{\mathrm{z}}_R(k)$ is a pseudomeasurement in the RCCS, $H = [I\,0\,0]$, and the three elements of $\underline{v}_L(k)$ are written by

$$\underline{v}_L(k) = [v_x(k)v_y(k)v_z(k)]^T \tag{7}$$

$$\begin{aligned} v_x(k) &= r_m(k)\cos(\theta_m(k)-\hat{\theta}(k)) - r(k)\cos(\theta(k)-\hat{\theta}(k)) \\ &\quad -2\cos\hat{\theta}(k)[r_m(k)\cos\theta_m(k)\sin^2(\frac{\psi_m(k)-\hat{\psi}(k)}{2}) \\ &\quad -r(k)\cos\theta(k)\sin^2(\frac{\psi(k)-\hat{\psi}(k)}{2})] \\ v_y(k) &= r_m(k)\cos\theta_m(k)\sin(\psi_m(k)-\hat{\psi}(k)) \\ &\quad -r(k)\cos\theta(k)\sin(\psi(k)-\hat{\psi}(k)) \\ v_z(k) &= r_m(k)\sin(\theta_m(k)-\hat{\theta}(k)) - r(k)\sin(\theta(k)-\hat{\theta}(k)) \\ &\quad -2\sin\hat{\theta}(k)[r_m(k)\cos\theta_m(k)\sin^2(\frac{\psi_m(k)-\hat{\psi}(k)}{2}) \\ &\quad -r(k)\cos\theta(k)\sin^2(\frac{\psi(k)-\hat{\psi}(k)}{2})] \end{aligned} \tag{8}$$

Define a 9×9 matrix $T_9(k)$ which consists of 3×3 matrix $T_3(k)$ as follows,

$$T_9(k) = \begin{bmatrix} T_3(k) & 0 & 0 \\ 0 & T_3(k) & 0 \\ 0 & 0 & T_3(k) \end{bmatrix}$$

Utilizing the fact that $FT_9(k) = T_9(k)F$, the target dynamics model in the ELCCS is obtained from Eq.(2) as

$$\underline{\mathrm{x}}_L(k+1) = F\underline{\mathrm{x}}_L(k) + \underline{w}_L(k) \tag{9}$$

where $\underline{\mathrm{x}}_L(k)$ and $\underline{w}_L(k)$ are respectively a state vector and a zero-mean white process noise vector in the ELCCS which are transformed from $\underline{\mathrm{x}}(k)$. Note that the covariance matrix of $\underline{w}_L(k)$ is same to that of $\underline{w}(k)$ since $T_9(k)Q(k)T_9^T(k) = Q(k)$.

2.2 Moving Coordinate Tracking Filter

Using the linear pseudomeasurements of Eqs.(6)-(8) and target dynamics of Eq.(9) derived in the ELCCS, the moving coordinate tracking filter(MCTF) can be established. In the MCTF proposed in this paper, a filtering step is executed first in the ELCCS which was constructed from prior estimates of a target position. Using new estimates, a next ELCCS is generated so that the x-axis of the new coordinate system coincides with the line-of-sight to the estimated target position at that time. And it is assumed that the coordinate systems that are generated at every time are fixed and treated as an inertial coordinate system. Therefore, as the filter tracks the target, the coordinate transformation from one ELCCS to the next ELCCS is required in order to keep the assumption of pseudomeasurement noise in the ELCCS that the true target is located around the x-axis of the ELCCS, *i.e.*, the line-of-sight to the estimated target position.

A Kalman filter is commonly employed for target tracking and consists of two procedures, *i.e., System Propagation* and *Measurement Update*.[7] In the MCTF, as described above, the coordinate transformation from one ELCCS to the next ELCCS is required and this will be called, *Coordinate Update*. The algorithm of the MCTF is given as follows.

System Propagation

$$\hat{\underline{x}}_L^{k-1}(k/k-1) = F\hat{\underline{x}}_L^{k-1}(k-1/k-1) \tag{10}$$

$$P_L^{k-1}(k/k-1) = FP_L^{k-1}(k-1/k-1)F^T + Q(k) \tag{11}$$

Coordinate Update

$$\Delta T_9(k/k-1) \equiv T_9(\Delta\hat{\psi}(k), \Delta\hat{\theta}(k)) \tag{12}$$

$$\Delta\hat{\psi}(k) = \tan^{-1}\left(\frac{\hat{y}_L^{k-1}(k/k-1)}{\hat{x}_L^{k-1}(k/k-1)}\right) \tag{13}$$

$$\Delta\hat{\theta}(k) = \tan^{-1}\left(\frac{\hat{z}_L^{k-1}(k/k-1)}{\sqrt{\{\hat{x}_L^{k-1}(k/k-1)\}^2 + \{\hat{y}_L^{k-1}(k/k-1)\}^2}}\right) \tag{14}$$

$$\hat{\underline{x}}_L^{k}(k/k-1) = \Delta T_9(k/k-1)\hat{\underline{x}}_L^{k-1}(k/k-1) \tag{15}$$

$$P_L^{k}(k/k-1) = \Delta T_9(k/k-1)P_L^{k-1}(k/k-1)\Delta T_9^T(k/k-1) \tag{16}$$

$$T_9(k) = \Delta T_9(k/k-1)T_9(k-1) \tag{17}$$

Measurement Update

$$\underline{z}_L^k(k) = T_3(k)\underline{z}_R(k) \tag{18}$$

$$\hat{\underline{x}}_L^k(k/k) = \hat{\underline{x}}_L^k(k/k-1) + K_L(k)[\underline{z}_L^k(k) - H\hat{\underline{x}}_L^k(k/k-1)] \tag{19}$$

$$P_L^k(k/k) = [I - K_L(k)H]P_L^k(k/k-1) \tag{20}$$

$$K_L(k) = P_L^k(k/k-1)H^T[HP_L^k(k/k-1)H^T + R_L(k)]^{-1} \tag{21}$$

where the subscript L denotes the ELCCS, the superscript k means the kth ELCCS which is constructed from the kth estimates of the target position. The $\underline{z}_R(k)$ is a pseudomeasurement vector in the RCCS which is transformed from polar measurements and $R_L(k)$ is a covariance matrix of the pseudomeasurement noise in the ELCCS given by Eqs.(7)-(8). Note that the initial values of $T_9(k)$ can be computed from the initial estimates so that the x-axis of the initial ELCCS is toward the initial target position. In real implementation, the state estimate in the RCCS is required. It can be obtained from

$$\hat{\underline{x}}(k/k) = T_9^T(k)\hat{\underline{x}}_L^k(k/k) \tag{22}$$

Next, examine a relationship between the Kalman filter in the RCCS and the MCTF in the ELCCS. From Eqs.(10), (15), and (19),

$$\begin{aligned}\hat{\underline{x}}_L^k(k/k) &= \Delta T_9(k/k-1)F\hat{\underline{x}}_L^{k-1}(k-1/k-1) + K_L(k)[\underline{z}_L^k(k) \\ &\quad -H\Delta T_9(k/k-1)F\hat{\underline{x}}_L^{k-1}(k-1/k-1)]\end{aligned} \tag{23}$$

And, from Eqs.(11), (16), and (20),

$$\begin{aligned}P_L^k(k/k) &= [I - K_L(k)H]\Delta T_9(k/k-1)[FP_L^{k-1}(k-1/k-1)F^T \\ &\quad +Q(k)]\Delta T_9^T(k/k-1)\end{aligned} \tag{24}$$

Premultiply $T_9^T(k)$ to Eqs.(23) and (24), postmultiply $T_9(k)$ to Eq.(24). Rearrange them using Eqs.(17)-(18), (22) and facts that $FT_9(k) = T_9(k)F$, $HT_9(k) = T_3(k)H$, $T_9(k)Q(k)T_9^T(k) = Q(k)$, and $T_9(k)T_9^T(k) = I$. Then they become

$$\hat{\underline{x}}(k/k) = F\hat{\underline{x}}(k-1/k-1) + K(k)[\underline{z}_R(k) - HF\hat{\underline{x}}(k-1/k-1)] \tag{25}$$

$$P(k/k) = [I - K(k)H][FP(k-1/k-1)F^T + Q(k)] \tag{26}$$

where $P(k/k)$ is an error covariance matrix in the RCCS and

$$\begin{aligned}K(k) &= T_9^T(k)K_L(k)T_3(k) \\ &= T_9^T(k)P_L^k(k/k-1)H^T[HP_L^k(k/k-1)H^T + R_L(k)]^{-1}T_3(k) \\ &= P(k/k-1)H^T[HP(k/k-1)H^T + R(k)]^{-1}\end{aligned} \tag{27}$$

The above Eqs.(25)-(27) are well-known Kalman filter equations in the RCCS. Thus, the MCTF in the ELCCS is exactly equivalent to the Kalman filter in the RCCS.

2.3 Decoupled Moving Coordinate Tracking Filter

To ease computational burden, decoupling of three axes is desirable. But non-diagonality of *R(k)* of the Kalman filter in the RCCS makes it impossible. After some approximations in the MCTF, decoupling of three axes can be achieved. This filter is named as decoupled moving coordinate tracking filter(DMCTF). Detailed derivations are as follows.

First, consider the pseudomeasurement noise in the ELCCS given by Eqs.(7)-(8). At sufficiently long range, it can be assumed that $\sigma(k) = \sigma_\psi(k) \simeq \sigma_\theta(k)$ and $\sigma_r(k) \simeq r(k)\sigma(k)$. If we assume that $\psi(k) - \hat{\psi}(k)$ and $\theta(k) - \hat{\theta}(k)$ are sufficiently small in the order of $\sigma(k)$, $\underline{v}_L(k)$ in Eq.(7) can be approximated to $\overline{\underline{v}}_L(k)$ whose covariance matrix is diagonal as follows,

$$\underline{v}_L(k) \simeq \overline{\underline{v}}_L(k) = \begin{bmatrix} v_r(k) \\ r(k)\cos\theta(k)v_\psi(k) \\ r(k)v_\theta(k) \end{bmatrix} \tag{28}$$

$$\begin{aligned}\overline{R}_L(k) &\equiv E\{\overline{\underline{v}}_L(k)\overline{\underline{v}}_L^T(k)\} \\ &= \begin{bmatrix} \sigma_r^2(k) & 0 & 0 \\ 0 & r^2(k)\cos^2\theta(k)\sigma_\psi(k) & 0 \\ 0 & 0 & r^2(k)\sigma_\theta^2(k) \end{bmatrix}\end{aligned} \tag{29}$$

Note that $E\{(\underline{v}_L(k)-\overline{\underline{v}}_L(k))(\underline{v}_L(k)-\overline{\underline{v}}_L(k))^T\} \simeq r^2(k)\sigma^4(k)$ is negligible compared with $E\{\underline{v}_L(k)\underline{v}_L^T(k)\} \simeq r^2(k)\sigma^2(k)$. Therefore, $\overline{\underline{v}}_L(k)$ is approximately equivalent to $\underline{v}_L(k)$ in mean square sense.

Next, examine the behavior of the error covariance matrix. If the initial covariance matrix $P_L^0(0)$ consists of 3×3 diagonal submatrices $P_{ij}^0(0)(i,j = 1,3)$, $P_L^{k-1}(k/k-1)$ computed from Eq.(11) also has diagonal submatrices. Define this 9×9 matrix $P_L^{k-1}(k/k-1)$ as

$$P_L^{k-1}(k/k-1) \equiv \begin{bmatrix} P_{11}^{k-1}(k/k-1) & P_{12}^{k-1}(k/k-1) & P_{13}^{k-1}(k/k-1) \\ P_{12}^{k-1}(k/k-1) & P_{22}^{k-1}(k/k-1) & P_{23}^{k-1}(k/k-1) \\ P_{13}^{k-1}(k/k-1) & P_{23}^{k-1}(k/k-1) & P_{33}^{k-1}(k/k-1) \end{bmatrix} \tag{30}$$

where 3×3 matrix $P_{ij}^{k-1}(k/k-1) = diag(p_{ij_1}^{k-1}(k/k-1), p_{ij_2}^{k-1}(k/k-1), p_{ij_3}^{k-1}(k/k-1))$. Then, every 3×3 submatrix of Eq.(16) is written by

$$\begin{aligned}P_{ij}^k(k/k-1) &= \Delta T_3(k/k-1)P_{ij}^{k-1}(k/k-1)\Delta T_3^T(k/k-1) \qquad (31) \\ &\equiv \begin{bmatrix} p_{ij_{11}}^k(k/k-1) & p_{ij_{12}}^k(k/k-1) & p_{ij_{13}}^k(k/k-1) \\ p_{ij_{12}}^k(k/k-1) & p_{ij_{22}}^k(k/k-1) & p_{ij_{23}}^k(k/k-1) \\ p_{ij_{13}}^k(k/k-1) & p_{ij_{23}}^k(k/k-1) & p_{ij_{33}}^k(k/k-1) \end{bmatrix}\end{aligned}$$

$$
\begin{aligned}
p^k_{ij_{11}}(k/k-1) &= [p^{k-1}_{ij_1}(k/k-1)\cos^2\Delta\hat{\psi}(k) + p^{k-1}_{ij_2}(k/k-1) \\
&\quad \sin^2\Delta\hat{\psi}(k)]\cos^2\Delta\hat{\theta}(k) + p^{k-1}_{ij_3}(k/k-1)\sin^2\Delta\hat{\theta}(k) \\
p^k_{ij_{12}}(k/k-1) &= [p^{k-1}_{ij_2}(k/k-1) - p^{k-1}_{ij_1}(k/k-1)]\sin\Delta\hat{\psi}(k) \\
&\quad \cos\Delta\hat{\psi}(k)\cos\Delta\hat{\theta}(k) \\
p^k_{ij_{13}}(k/k-1) &= -[p^{k-1}_{ij_1}(k/k-1)\cos^2\Delta\hat{\psi}(k) + p^{k-1}_{ij_2}(k/k-1) \\
&\quad \sin^2\Delta\hat{\psi}(k)]\sin\Delta\hat{\theta}(k)\cos\Delta\hat{\theta}(k) + p^{k-1}_{ij_3}(k/k-1) \\
&\quad \sin\Delta\hat{\theta}(k)\cos\Delta\hat{\theta}(k) \\
p^k_{ij_{22}}(k/k-1) &= p^{k-1}_{ij_1}(k/k-1)\sin^2\Delta\hat{\psi}(k) + p^{k-1}_{ij_2}(k/k-1)\cos^2\Delta\hat{\psi}(k) \\
p^k_{ij_{23}}(k/k-1) &= [p^{k-1}_{ij_1}(k/k-1) - p^{k-1}_{ij_2}(k/k-1)]\sin\Delta\hat{\psi}(k) \\
&\quad \cos\Delta\hat{\psi}(k)\sin\Delta\hat{\theta}(k) \\
p^k_{ij_{33}}(k/k-1) &= [p^{k-1}_{ij_1}(k/k-1)\cos^2\Delta\hat{\psi}(k) + p^{k-1}_{ij_2}(k/k-1) \\
&\quad \sin^2\Delta\hat{\psi}(k)]\sin^2\Delta\hat{\theta}(k) + p^{k-1}_{ij_3}(k/k-1)\cos^2\Delta\hat{\theta}(k)
\end{aligned} \tag{32}
$$

If the tracking filter is stable and the sampling interval is sufficiently small, $\Delta\hat{\psi}(k)$ and $\Delta\hat{\theta}(k)$ of Eqs.(13)-(14) may have small values. In this case, Eq.(31) can be approximated to

$$
P^k_{ij}(k/k-1) \simeq \begin{bmatrix} p^{k-1}_{ij_1}(k/k-1) & 0 & 0 \\ 0 & p^{k-1}_{ij_2}(k/k-1) & 0 \\ 0 & 0 & p^{k-1}_{ij_3}(k/k-1) \end{bmatrix} \tag{33}
$$

This means that Eq.(16) can be omitted and subsequently the diagonality of the submatrix is preserved. Also, $P^k_L(k/k)$ computed from Eq.(20) has the diagonal submatrices if the submatrices of $P^k_L(k/k-1)$ are diagonal.

In case that the pseudomeasurement noise and the error covariance matrix are approximated as in Eqs.(28) and (33), decoupling of three axes can be achieved. The structure of this approximate filter named DMCTF is shown in Figure 2 and is suitable for parallel implementation. It is clear that the DMCTF is equivalent to the tracking filter proposed by Song *et al.* if the coordinate transformation matrix in [4] is computed using the estimates instead of the measurements. The advantage of the DMCTF is that the computations of a filtering step for the DMCTF is much less than that of the Kalman filter in the RCCS since the DMCTF requires only 18 storages for the error covariance matrix while the Kalman filter in the RCCS does 45 storages. Furthermore, if parallelism is adapted for each axis as shown in Figure 2, computation time can be reduced further. The DMCTF is an approximate form of the MCTF and the global stability of the filter cannot be guaranteed. Since $\psi(k) - \hat{\psi}(k)$ and $\theta(k) - \hat{\theta}(k)$ have been assumed to be sufficiently small in the order of $\sigma_\psi(k)$ or $\sigma_\theta(k)$, the initial estimate should be given as accurately as possible. Under the assumptions that the initial target range is sufficiently large, the measurement sampling interval is sufficiently small and the target motion maintains constant velocity, the initial estimates is roughly obtained using the least square estimation method with several initial measurement sequences.[10]

3. STOCHASTIC PROPERTIES OF MCTF AND DMCTF

It is known that a Kalman filter is an unbiased, stable, and optimal estimator with minimum variance if the system which the Kalman filter is based upon is stochastically controllable and observable and some noise assumptions are satisfied.[8-9] But if the Gaussian noise assumption is dropped, the filter is the best estimator among a limited class of linear filters which will produce an estimate minimizing mean square error.[9] In case of the Kalman filter in the RCCS, since pseudomeasurement noises are not exactly Gaussians, the filter may be sub-optimal

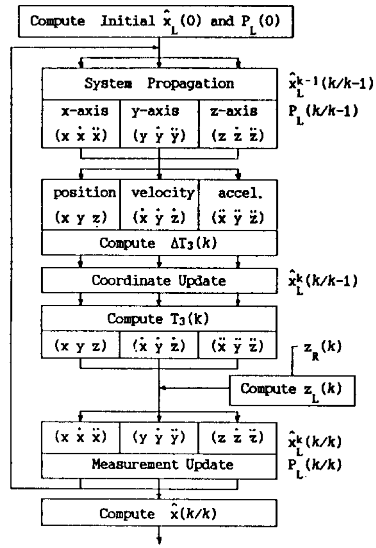


Figure 2: The structure of DMCTF

while other properties such as unbiasedness and stability are maintained.

It has been shown in the previous chapter that the MCTF in the ELCCS is equivalent to a Kalman filter in the RCCS and it is clear that the system model of Eq.(1) is stochastically controllable and observable. Therefore, the MCTF should be a unbiased, stable estimator. The DMCTF is an approximation of the MCTF as shown in Eqs.(28) and (33). Because of the approximations, there may be an destabilizing region in the DMCTF. Hence, stochastic properties of the filter, *i.e.*, unbiasedness and stability should be checked.

For stability check of the stochastic nonlinear system, a Lyapunov-like condition is required. Motivated by the criterion in the probabilistic Hilbert space L_2 that is frequently used in the estimation theory, *i.e.*, minimum mean square, the following definition is introduced.[5]

Definition **1** : A discrete stochastic process $\underline{\mathbf{x}}(k)$ is said to be ***exponentially bounded*** in mean square sense with exponent δ if there exist constants $0 < \delta < 1$, $K_1 \geq 0$ and $K_2 > 0$ such that

$$
\|\underline{\mathbf{x}}(k)\|^2 \leq K_1 + K_2(1-\delta)^k \qquad \forall k \subset Z_+ \tag{34}
$$

Then, the origin of $\underline{\mathbf{x}}(k)$ is said to be ***asymptotically stable*** in mean square sense.

In the above definition, the norm of $\underline{\mathbf{x}}(k)$ in L_2 is defined by

$$
\|\underline{\mathbf{x}}(k)\|^2 = \int_{-\infty}^{\infty} \underline{\mathbf{x}}^T(k)\underline{\mathbf{x}}(k) f_{\underline{\mathbf{x}}}(x)\,dx \tag{35}
$$

where $f_{\underline{\mathbf{x}}}(x)$ is the probability density function of $\underline{\mathbf{x}}(k)$. From Eqs.(10), (12), (15) and (18)-(19), the estimation errors of the DMCTF in the kth ELCCS can be written by

$$
\begin{aligned}
\tilde{\underline{\mathbf{x}}}^k_L(k) &\equiv \underline{\mathbf{x}}^k_L(k) - \hat{\underline{\mathbf{x}}}^k_L(k/k) \\
&= L(k)[\underline{\mathbf{x}}^k_L(k) - \hat{\underline{\mathbf{x}}}^k_L(k/k-1)] - K_L(k)H\underline{v}^k_L(k) \\
&= L(k)\Delta T_9(k/k-1)[\underline{\mathbf{x}}^{k-1}_L(k) - \hat{\underline{\mathbf{x}}}^{k-1}_L(k/k-1)] \\
&\quad -K_L(k)H\Delta T_9(k/k-1)\underline{v}^{k-1}_L(k) \\
&= L(k)\Delta T_9(k/k-1)F\tilde{\underline{\mathbf{x}}}^{k-1}_L(k-1) + L(k)\Delta T_9(k/k-1) \\
&\quad \underline{w}^{k-1}_L(k-1) - K_L(k)H\Delta T_9(k/k-1)\underline{v}^{k-1}_L(k) \\
&= L(k)\Delta T_9(k/k-1)F\tilde{\underline{\mathbf{x}}}^{k-1}_L(k-1) + L(k)T_9(k)\underline{w}(k-1) \\
&\quad -K_L(k)HT_9(k)\underline{v}(k)
\end{aligned} \tag{36}
$$

where $L(k) = I - K_L(k)H$.

Remark 1 : $\Delta T_9(k/k-1)$ and $T_9(k)$ are functions of $\tilde{\mathbf{x}}_L^{k-1}(k/k-1) = T_9(k-1)\tilde{\mathbf{x}}(k/k-1)$, *i.e.*, $\mathbf{z}_L^{k-1}(k-1) = T_9(k-1)\mathbf{z}_R(k-1)$. Therefore, it is clear that they are uncorrelated with $\mathbf{v}_L^{k-1}(k)$ and $\mathbf{w}_L^{k-1}(k-1)$.

In DMCTF, Eq.(16), the coordinate update for an error covariance matrix is omitted by approximation. Hence, the superscript representing the ELCCS can be dropped for the error covariance matrix. Now, define a scalar Lyapunov function $V(\tilde{\mathbf{x}}_L^k(k))$ as

$$V(\tilde{\mathbf{x}}_L^k(k)) = \{\tilde{\mathbf{x}}_L^k(k)\}^T P_L^{-1}(k/k)\tilde{\mathbf{x}}_L^k(k) \tag{37}$$

Under the following assumptions, a sufficiency theorem for the stability of the DMCTF can be established.

Assumptions :

1. The system gain matrix F is invertible and $\|F\| < \infty$.
2. $0 < Q(k) < \infty$ and $0 < R(k) < \infty$.
3. $P_L(k/k) > 0$, *i.e.*, positive definite.
4. $P_L^{-1}(k/k)$ is bounded from below such that

$$\|V(\tilde{\mathbf{x}}_L^k(k))\| = \|(\tilde{\mathbf{x}}_L^k(k))^T P_L^{-1}(k/k)\tilde{\mathbf{x}}_L^k(k)\| \geq c\|\tilde{\mathbf{x}}_L^k(k)\|^2 \qquad c > 0 \tag{38}$$

Theorem 1 : Under the *Assumptions* 1–4, the estimation errors of the DMCTF is exponentially bounded in mean square sense with exponent δ if the following condition is satisfied.

$$\begin{aligned} &F^T \Delta T_9^T(k+1/k) L^T(k+1) P_L^{-1}(k+1/k+1) L(k+1) \\ &\Delta T_9(k+1/k) F - P_L^{-1}(k/k) < 0 \end{aligned} \tag{39}$$

Proof : Take the conditional expectation over $V(\tilde{\mathbf{x}}_L^{k+1}(k+1)) - V(\tilde{\mathbf{x}}_L^k(k))$ given a set of estimation errors, $\tilde{X}_k \equiv \{\tilde{\mathbf{x}}_L^0(0), \ldots, \tilde{\mathbf{x}}_L^k(k)\}$ and using the *Remark* 1, rearrange it with Eq.(36).

$$\begin{aligned} &E_{\tilde{X}_k}\{V(\tilde{\mathbf{x}}_L^{k+1}(k+1)) - V(\tilde{\mathbf{x}}_L^k(k))\} \\ &= (\tilde{\mathbf{x}}_L^k(k))^T E_{\tilde{X}_k}\{F^T \Delta T_9^T(k+1/k) L^T(k+1) P_L^{-1}(k+1/k+1) \\ &\quad L(k+1)\Delta T_9(k+1/k)F - P_L^{-1}(k/k)\}\tilde{\mathbf{x}}_L^k(k) + E_{\tilde{X}_k}\{tr[L^T(k+1) \\ &\quad P_L^{-1}(k+1/k+1)L(k+1)Q(k) + H^T K_L^T(k+1) \\ &\quad P_L^{-1}(k+1/k+1)K_L(k+1)HR_L(k)]\} \end{aligned} \tag{40}$$

where $E_{\tilde{X}_k}\{\cdot\}$ is a conditional expectation operator given $\tilde{X}_k$. Note that the terms in the trace operator $tr(\cdot)$ should be strictly positive definite from the *Assumptions* 2 and 4. From Eq.(39), the term inside the first $E_{\tilde{X}_k}\{\cdot\}$ operator of Eq.(40) is negative. Therefore, there exist $0 < \rho_k < 1$ such that

$$\begin{aligned} &(\tilde{\mathbf{x}}_L^k(k))^T F^T \Delta T_9^T(k+1/k) L^T(k+1) P_L^{-1}(k+1/k+1) L(k+1) \\ &\Delta T_9(k+1/k) F \tilde{\mathbf{x}}_L^k(k) = \rho_k (\tilde{\mathbf{x}}_L^k(k))^T P_L^{-1}(k+1/k+1)\tilde{\mathbf{x}}_L^k(k) \end{aligned} \tag{41}$$

Define the terms inside the $tr(\cdot)$ operator of Eq.(40) as

$$\begin{aligned} M_{1_k} &\equiv E_{\tilde{X}_k}\{tr[L^T(k+1)P_L^{-1}(k+1/k+1)L(k+1)Q(k)] \\ &+ tr[H^T K_L^T(k+1) P_L^{-1}(k+1/k+1) K_L(k+1) H R_L(k)]\} \end{aligned} \tag{42}$$

From Eqs.(41)-(42), Eq.(40) can be written by

$$\begin{aligned} &E_{\tilde{X}_k}\{V(\tilde{\mathbf{x}}_L^{k+1}(k+1)) - V(\tilde{\mathbf{x}}_L^k(k))\} \\ &\leq M_1 - (1-\rho_k) E_{\tilde{X}_k}\{V(\tilde{\mathbf{x}}_L^k(k))\} \end{aligned} \tag{43}$$

where $M_1 \equiv sup_{k \subset Z_+}\{M_{1_k}\} > 0$ and is finite from the *Assumptions* 2 and 4. By the nesting property of conditional expectation, Eq.(43) becomes

$$\begin{aligned} E_{\tilde{X}_0}\{V(\tilde{\mathbf{x}}_L^{k+1}(k+1))\} &= E_{\tilde{X}_0}\{E_{\tilde{X}_k}\{V(\tilde{\mathbf{x}}_L^{k+1}(k+1))\}\} \\ &\leq M_1 + \rho_k E_{\tilde{X}_0}\{E_{\tilde{X}_k}\{V(\tilde{\mathbf{x}}_L^k(k))\}\} \\ &= M_1 + \rho_k E_{\tilde{X}_0}\{V(\tilde{\mathbf{x}}_L^k(k))\} \end{aligned} \tag{44}$$

By applying Eq.(44) recursively for time stages from 0 to k, we have

$$E_{\tilde{X}_0}\{V(\tilde{\mathbf{x}}_L^{k+1}(k+1))\} \leq M_1 \sum_{i=0}^{k} \rho^i + \rho^{k+1} E_{\tilde{X}_0}\{V(\tilde{\mathbf{x}}_L^0(0))\} \tag{45}$$

where $0 < \rho \equiv sup_{k \subset Z_+}\{\rho_k\} < 1$. Take an unconditional expectation over $E_{\tilde{X}_0}\{\cdot\}$ in Eq.(45). From Eq.(38),

$$\|V(\tilde{\mathbf{x}}_L^{k+1}(k+1))\|^2 \leq M_2 + M_3 \rho^{k+1} \tag{46}$$

where $M_2 = M_1 \sum_{i=0}^{k} \rho^i / c = M_1/c(1-\rho)$, $M_3 = E\{V(\tilde{\mathbf{x}}_L^0(0))\}/c$ and c is a constant in Eq.(38). Hence, that the estimation errors of the DMCTF are exponentially bounded, *i.e.*, the DMCTF is asymptotically stable in a stochastic environment if Eq.(39), the condition for a set of nondestabilizing deviation holds.

Q.E.D.

Remark 2 : Eq.(39), the sufficient condition for the stability of the DMCTF can be written by

$$\begin{aligned} &F^T \Delta T_9^T(k+1/k) L^T(k+1) P_L^{-1}(k+1/k+1) L(k+1) \Delta T_9(k+1/k) F - P_L^{-1}(k/k) \\ &= F^T \Delta T_9^T(k+1/k)[L^T(k+1) P_L^{-1}(k+1/k+1) L(k+1) - \Delta T_9(k+1/k) \\ &\quad (F P_L(k/k) F^T)^{-1} \Delta T_9^T(k+1/k)] \Delta T_9(k+1/k) F \end{aligned} \tag{47}$$

where $L(k) = I - K_L(k)H$. Using the matrix inversion lemma and Eqs.(11), (20)-(21), and (33), the terms inside the bracket of Eq.(47) becomes

$$\begin{aligned} &L^T(k+1) P_L^{-1}(k+1/k+1) L(k+1) - \Delta T_9(k+1/k)(F P_L(k/k) F^T)^{-1} \Delta T_9^T(k+1/k) \\ &= P_L^{-1}(k+1/k) P_L(k+1/k+1) P_L^{-1}(k+1/k) - \Delta T_9(k+1/k) \\ &\quad [P_L(k+1/k) - Q(k)]^{-1} \Delta T_9^T(k+1/k) \\ &= P_L^{-1}(k+1/k)[P_L(k+1/k+1) - P_L(k+1/k)] P_L^{-1}(k+1/k) \\ &\quad + P_L^{-1}(k+1/k) - \Delta T_9(k+1/k)[P_L^{-1}(k+1/k) - P_L^{-1}(k+1/k) \\ &\quad \{P_L^{-1}(k+1/k) - Q^{-1}(k)\}^{-1} P_L^{-1}(k+1/k)] \Delta T_9^T(k+1/k) \\ &= -\Delta T_9(k+1/k) P_L^{-1}(k+1/k)\{Q^{-1}(k) - P_L^{-1}(k+1/k)\}^{-1} \\ &\quad P_L^{-1}(k+1/k) \Delta T_9^T(k+1/k) - H^T\{H P_L(k+1/k) H^T + R_L(k+1)\}^{-1} H \\ &\quad + [P_L^{-1}(k+1/k) - \Delta T_9(k+1/k) P_L^{-1}(k+1/k) \Delta T_9^T(k+1/k)] \end{aligned} \tag{48}$$

The last term of Eq.(48) represents the difference between the error covariance matrices that one excludes the coordinate update of Eq.(16) while the other includes it. If $\Delta\hat{\psi}(k)$ and $\Delta\hat{\theta}(k)$ in Eqs.(13)-(14) and the sampling time interval are sufficiently small, this term may have small value. Since the sum of first two terms should be strictly negative definite, it is expected that Eq.(39) may be satisfied.

4. SIMULATION RESULTS

Monte Carlo simulation results of the conventional linear Kalman filter(CKF), the MCTF, and the DMCTF are presented to compare their performance. In the simulation, target scenario and covariance of measurement noise used in [4] are chosen. The target trajectory is a typical maneuver of warcraft and shown in Figure 3. The initial estimate and error covariance matrix are obtained from several initial measurement sequences using the least square estimation method under the assumption that the target moves with constant velocity. Other input conditions are given as follows, *i.e.*

measurement sampling rate $\Delta t = 20Hz$
maneuver constant $\tau = 10$ *sec*
standard deviation of maneuver $\sigma_m = 10m/sec^2$

Under these conditions, the simulation program is run and the the average tracking errors of 500 runs of Monte Carlo simulation are shown in Figure 4. The average tracking error is computed from $\sqrt{E\{\tilde{x}^2(k)\} + E\{\tilde{y}^2(k)\} + E\{\tilde{z}^2(k)\}}$ for the position, velocity, and acceleration respectively. Since the MCTF is equivalent to conventional linear Kalman filter as described in Chapter 2, performances of two filters are exactly same. And, in the figure, the MCTF and DMCTF show similar features except in some short ranges. It is because the assumptions for the approximations used in Eqs.(2.27)-(2.28) and Eq.(2.32) are no longer valid. For other scenario such as pop-up and dive flight trajectory, a typical highly maneuvering scenario, the DMCTF shows a similar performance to the MCTF.

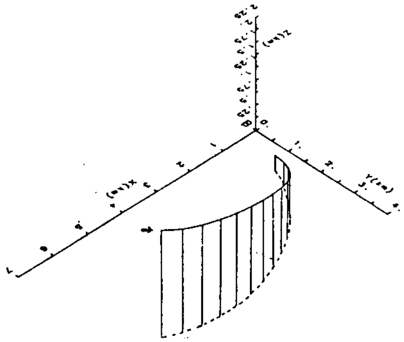
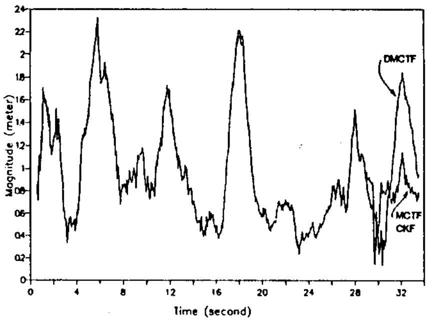
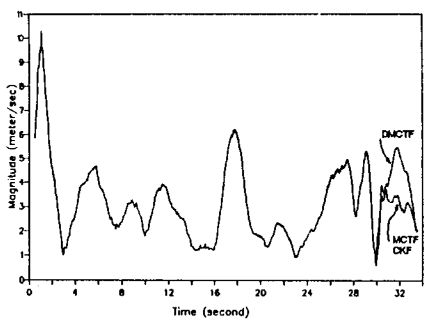


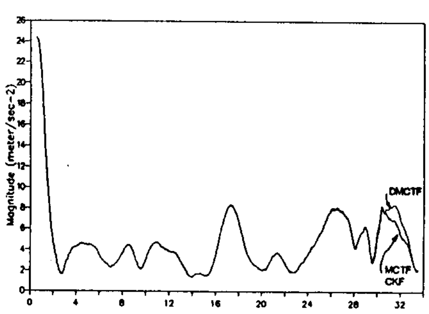
Figure 3: Target Scenario



(a) Average Position Tracking Error .



(b) Average Velocity Tracking Error



(c) Average Acceleration Tracking Error

Figure 4: Average Tracking Error

But the computation time of the DMCTF is much less than that of conventional linear Kalman filter due to decoupling and parallelism. Thus, the filtering can be accomplished more frequently and the performance may be improved consequently.

5. CONCLUSION

This paper shows the exact derivation of a tracking filter in the line-of-sight coordinate and its equivalence to a conventional Kalman filter in the inertial coordinate system. For the reduction of computation time, some approximations are made under several assumptions. It is shown by simulations and stability anaysis that if the sampling time interval is sufficiently small and the range is sufficiently large, the approximated filter may operate within a nondestabilizing region. Since the approximated filter has a decoupled structure for each axis, a parallel architecture can easily be implemented, and consequently the computation time is drastically reduced compared to the conventional Kalman filter. By the parallel architecture of the proposed filter, it is expected that a real-time tracking filter that is robust to high maneuvering target can be implemented if the smoothing technique is combined with the proposed filter.

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